Conceptual Evaluation of Measurement of |B(R)| for determination of q(R) on ITER: Part II

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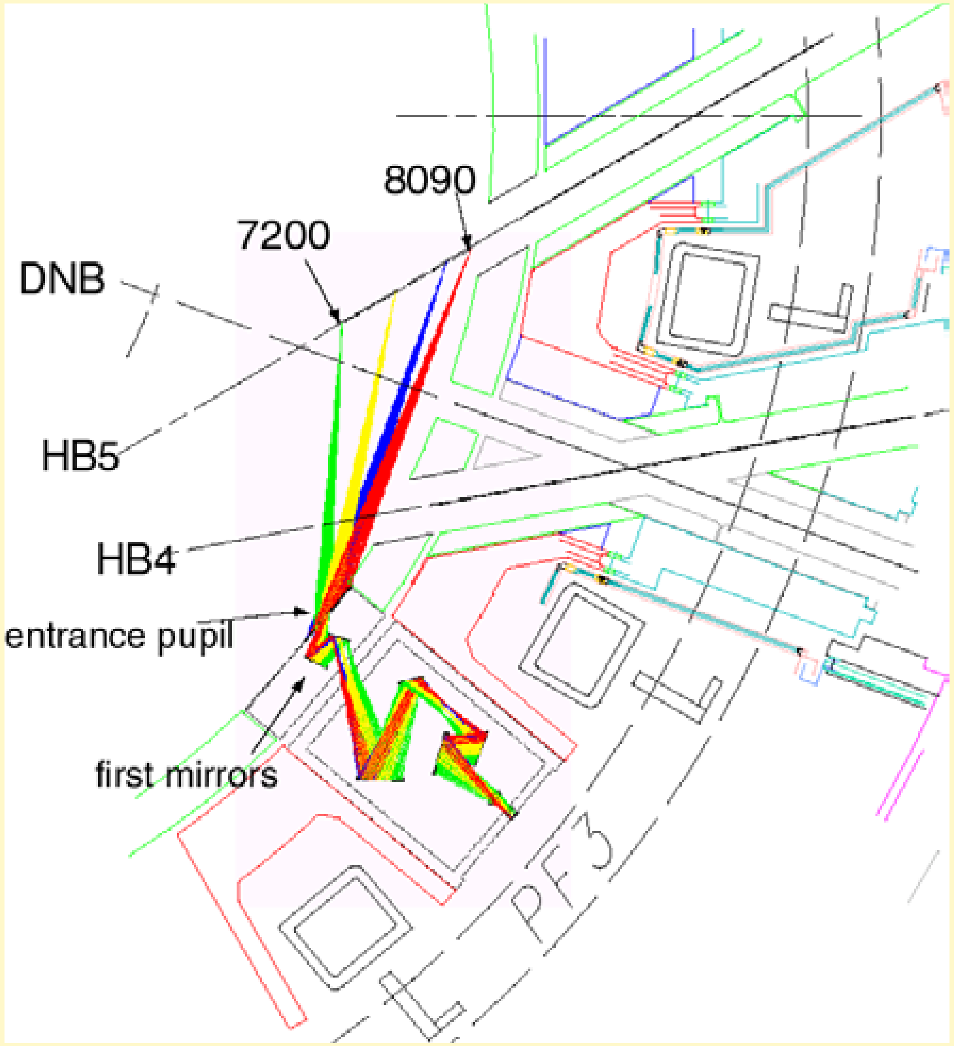
Enter the Labyrinth...



- To what precision can we determine the Stark electric field, for a given photon count?
- What photon count can we expect on ITER?
- How well can we determine the q-profile from the MSE-LS (Line Shift) measurement constraint?
- Is this approach advantageous compared to traditional MSE Line Polarization (MSE-LP) on ITER?

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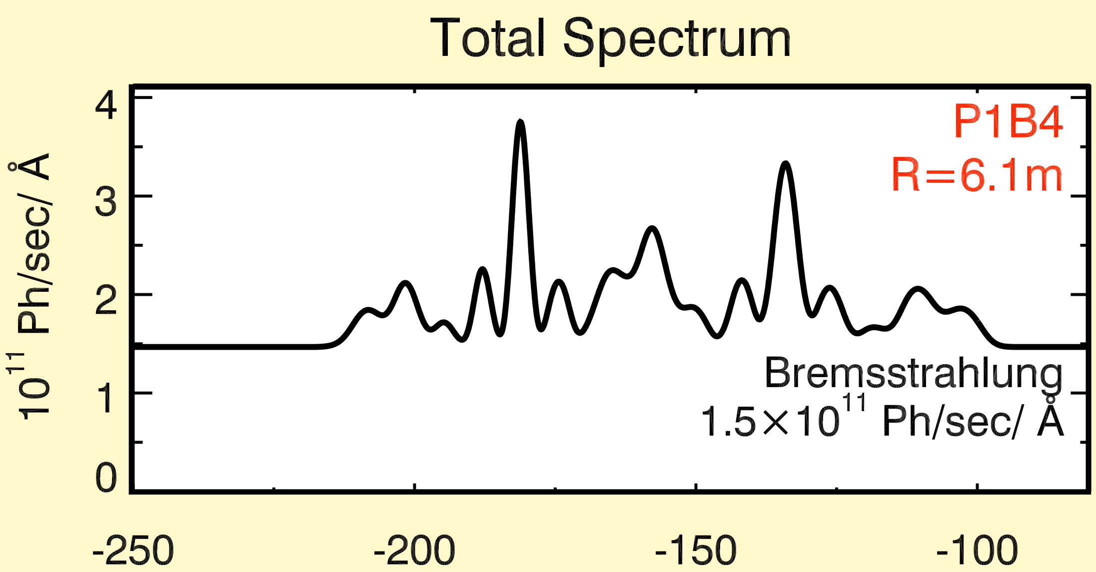
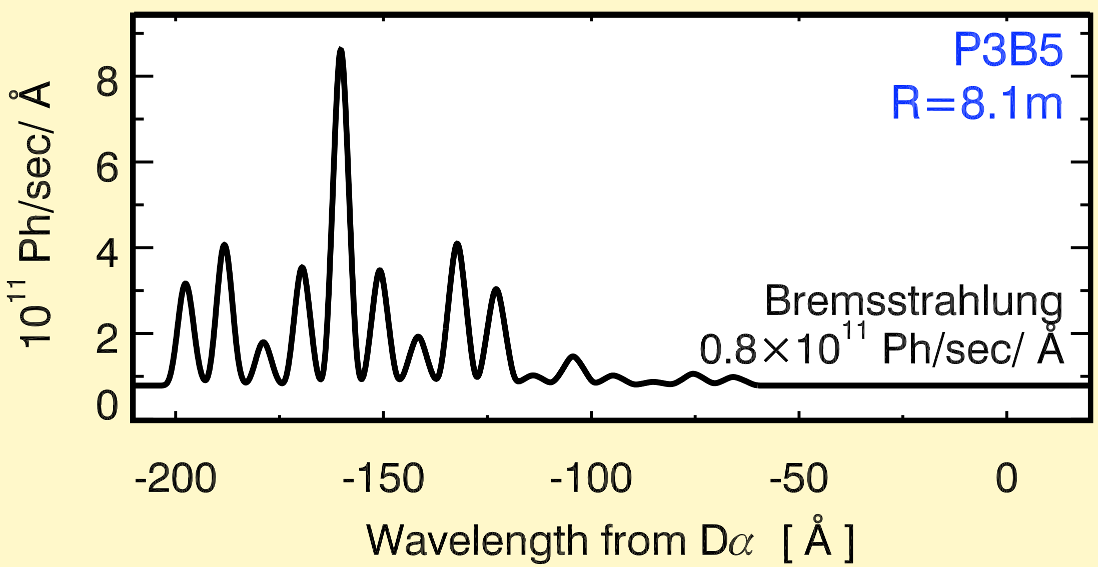
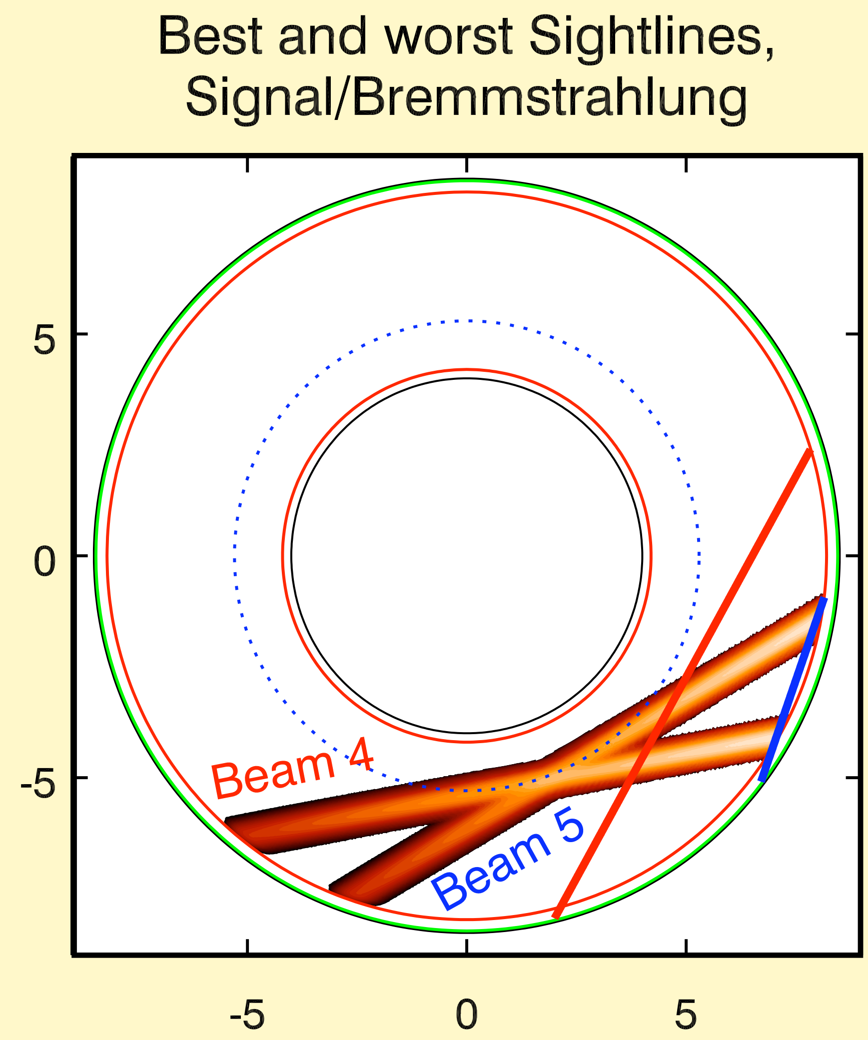
Bracket Measurement Uncertainty by Analyzing Best and Worst Case







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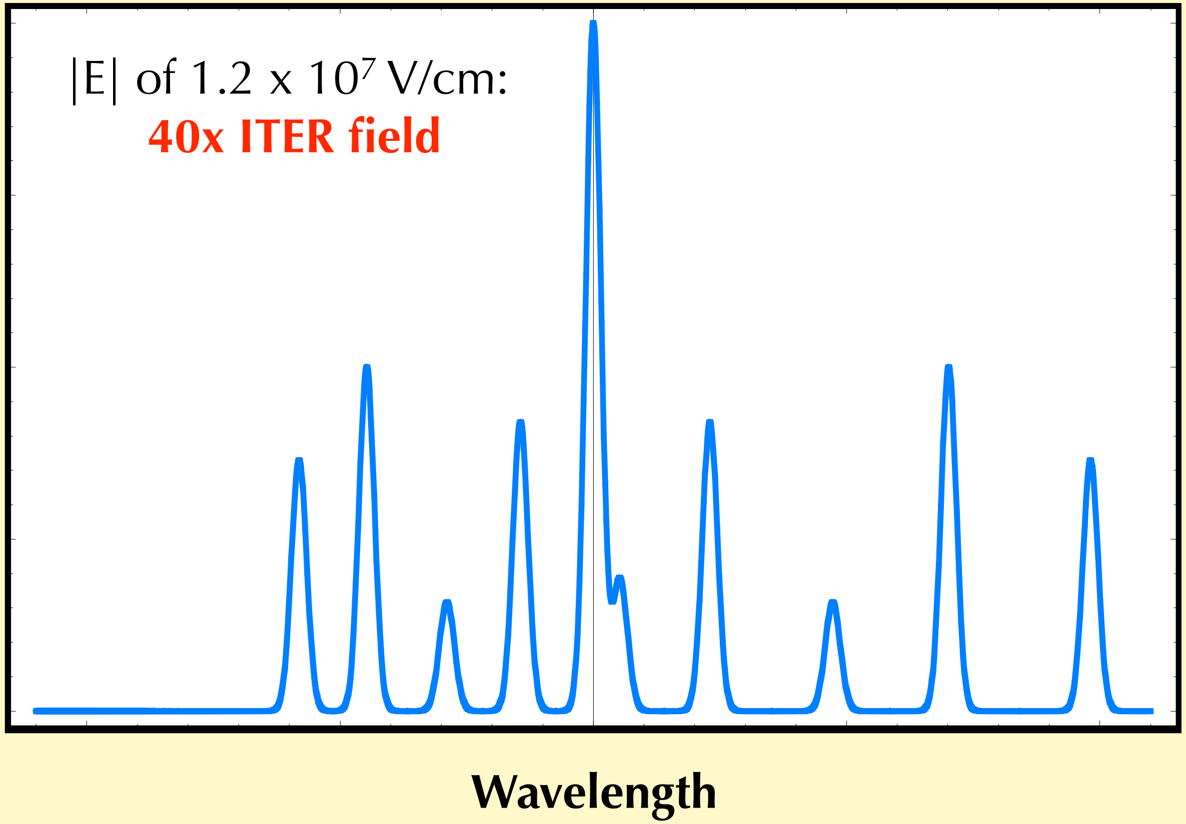
|E| Constrains Line Spacing

Energy Levels in Second Order Perturbation:

$$E_2 = \frac{Z^2}{2n^2} + \frac{3}{2}F\frac{n}{Z}(n_1 - n_2) - \frac{1}{16}F^2(\frac{n}{Z})^4[17n^2 - 3(n_1 - n_2)^2 - 9m^2 + 19]$$

(E is energy, Z is atomic number, n is principal quantum number, F is electric field, n_1, n_2, m are quantum numbers for hydrogen wave function in parabolic coordinates)

10 allowed transitions (σ_0 splits relative to linear Stark)



Spectrum center, line widths and amplitudes determined by Doppler shift and geometry for each channel: Allowed to vary in least-squares fit

|E| constrains spacing

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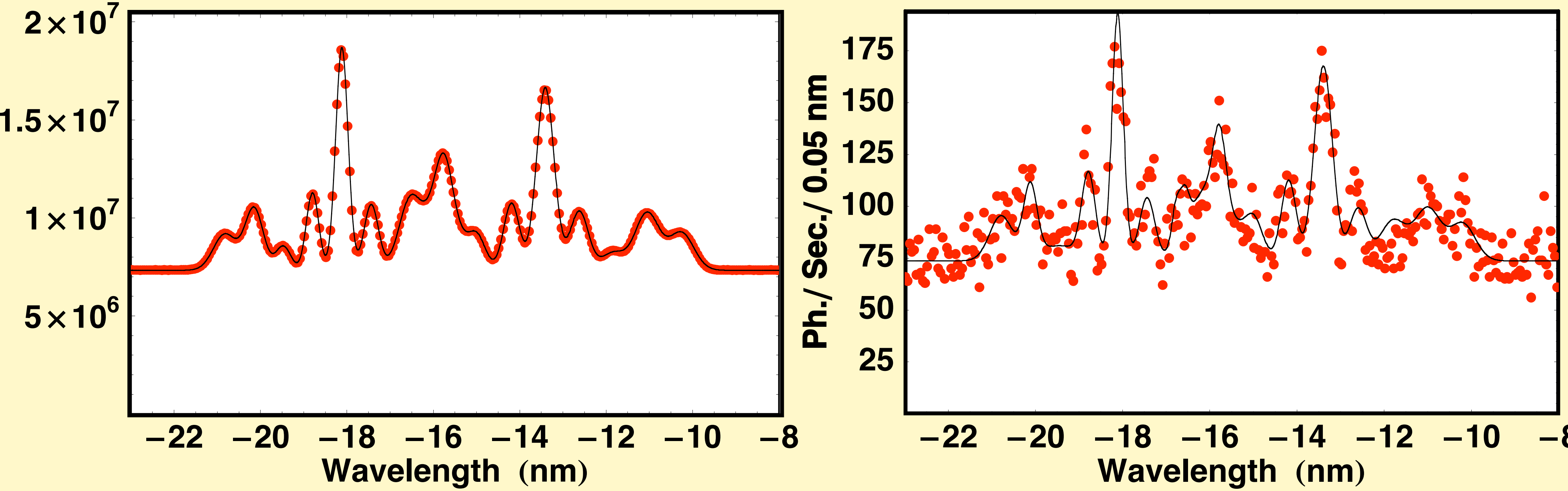
Constrained Fit Applied to Simulated Data



- Shown above are two examples with same sightline (extreme high and low photon count levels considered)
- Add random, Poisson distributed noise to photon count
- Use 0.05 nm resolution
- Apply least-squares fit to both beam spectra independently

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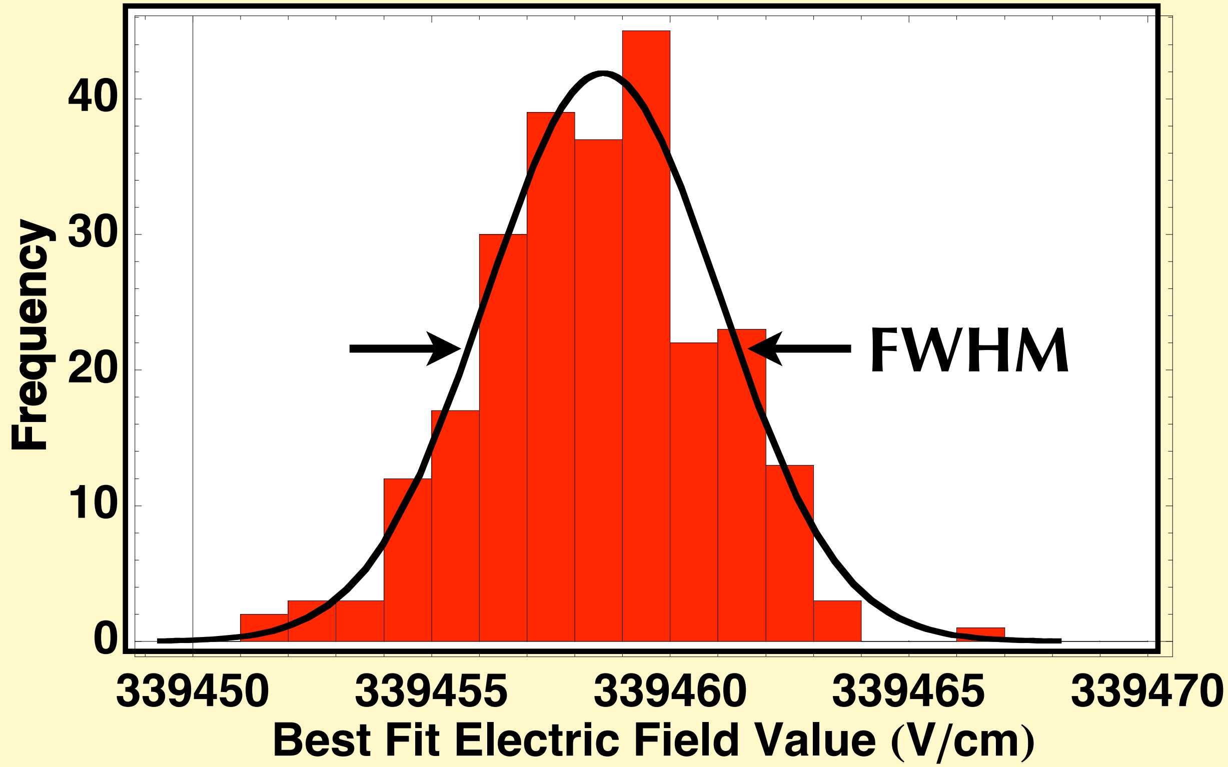
Histogram from Multiple Fits to Derive Uncertainty



- Generate random noise on spectrum of each count rate 250 times
- Fit each case and record electric field
- Bin, plot and fit electric field values for estimate of measurement error

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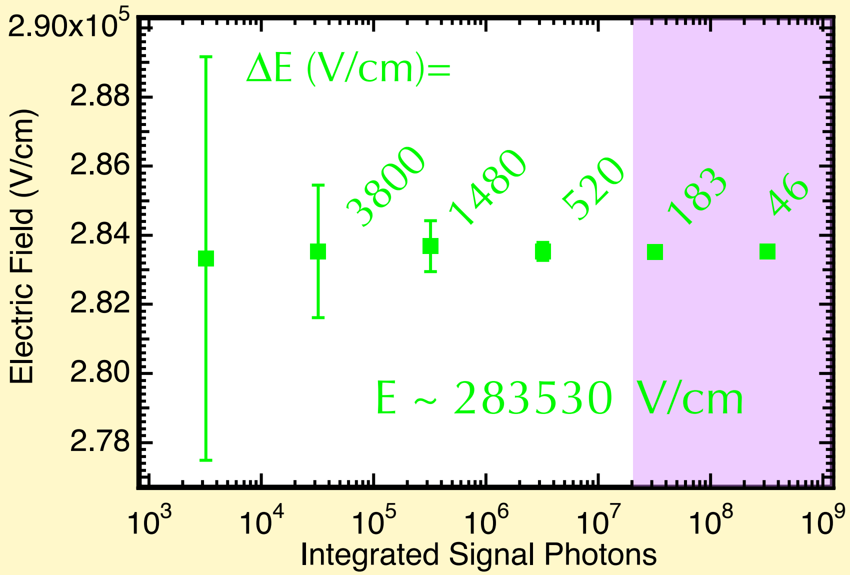
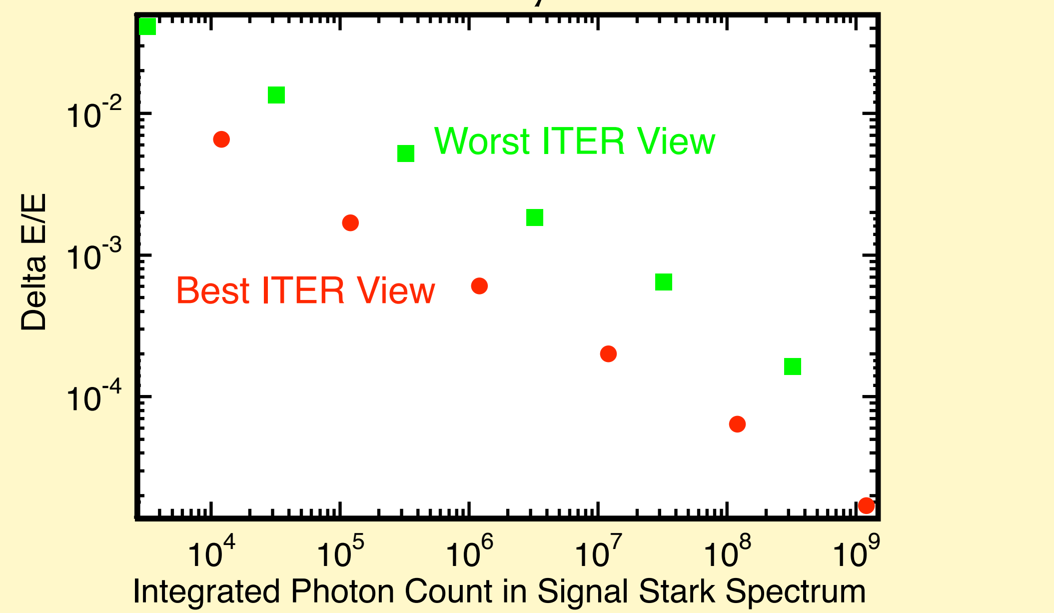
Excellent Precision at High Photon Count

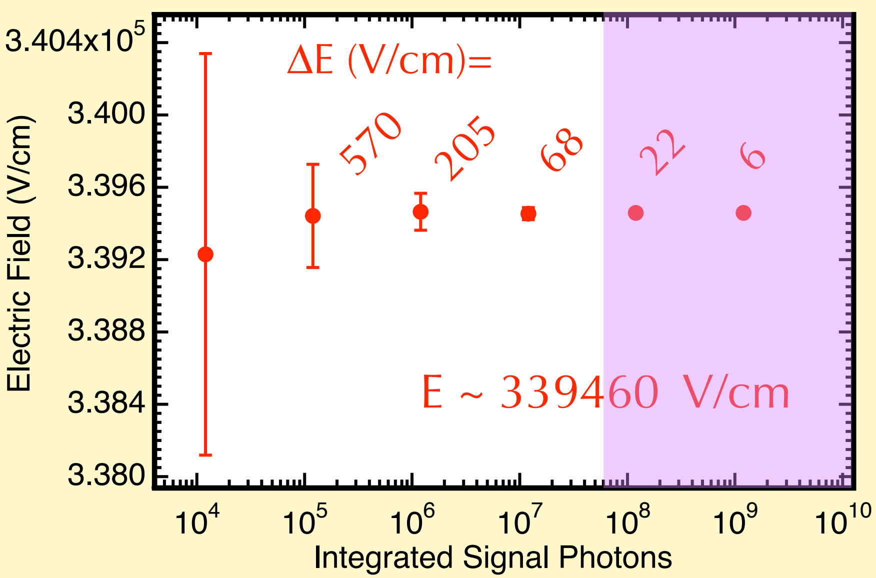
P1B4 'Worst Case' ITER View



Statistical Uncertainty vs. Photon Count



P3B5 'Best Case' ITER View



Calibration is key. Interface with ITER design team to:

- Install two views of beam in duct, to monitor velocity precisely with spectrometer.
- Perform beam-into-gas calibration for baseline vacuum magnetic field.

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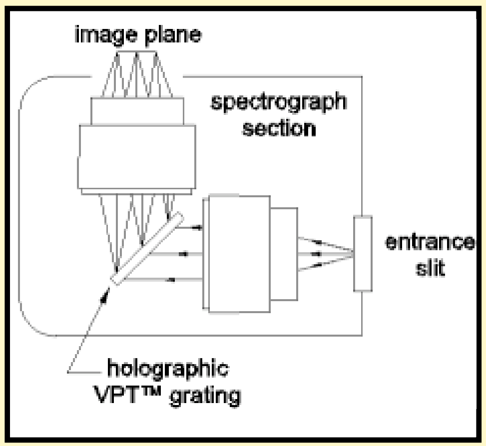
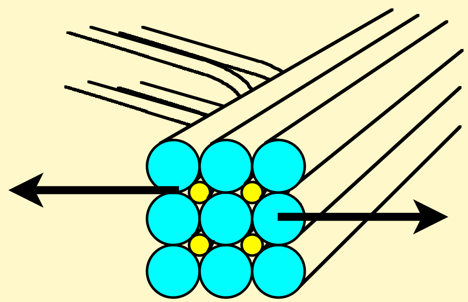
Consider Complimentary Systems for ITER

MSE-LS

Transmission grating spectrometer (Kaiser)



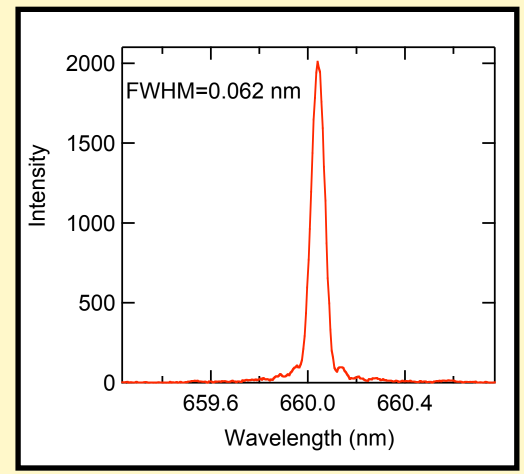


Pack fibers for simultaneous measurement

- Best transmission for full spectrum
- Single-grating presently available
- Multiple-grating could be developed for higher throughput
- Well-matched to labyrinth etendue

MSE-LP

Tunable birefringent filter (Nova Photonics)



- Best transmission for narrow portion of spectrum
- Easily tuned to match variations in beam energy

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Throughput Factor Calculations

Parameter	Reason	Grating Spec. (IBI)	Tunable Filter (γ)
Input Object Size	Max. photon count	4.6 x 4.6 cm^2	10 x 40 cm^2
Input NA	Malaquias '03	0.015	0.015
Output NA	Limited by collection	0.278 (spectrometer)	0.416 (fibers)
Mirror Transmission	50% 1st, following better	0.34	0.34
Fiber Dia. (core/total)	Spect. / Max photons	0.2/0.25 mm	1.0/1.2 mm
Number of Fibers	Spect. input / Image size	100	36
Slitwidth	Chosen resolution	0.025 mm	n/a
Fiber Area (mm^2)	Best match	0.5	28.3
Fiber Packing Fraction	Spect. NA / Image Size	0.081	0.55
Fiber Transmission	typical value	0.9	0.9
Spect. / Filter Trans.	typical value	0.6	0.3
Quantum Efficiency/F	CCD / APD	0.8/1	0.8/2
Bandpass	Appropriate for system	0.05 nm	0.5 nm
Rel. Throughput Factor		0.00063	0.02

Consider also: Mosaic grating of abv x 4

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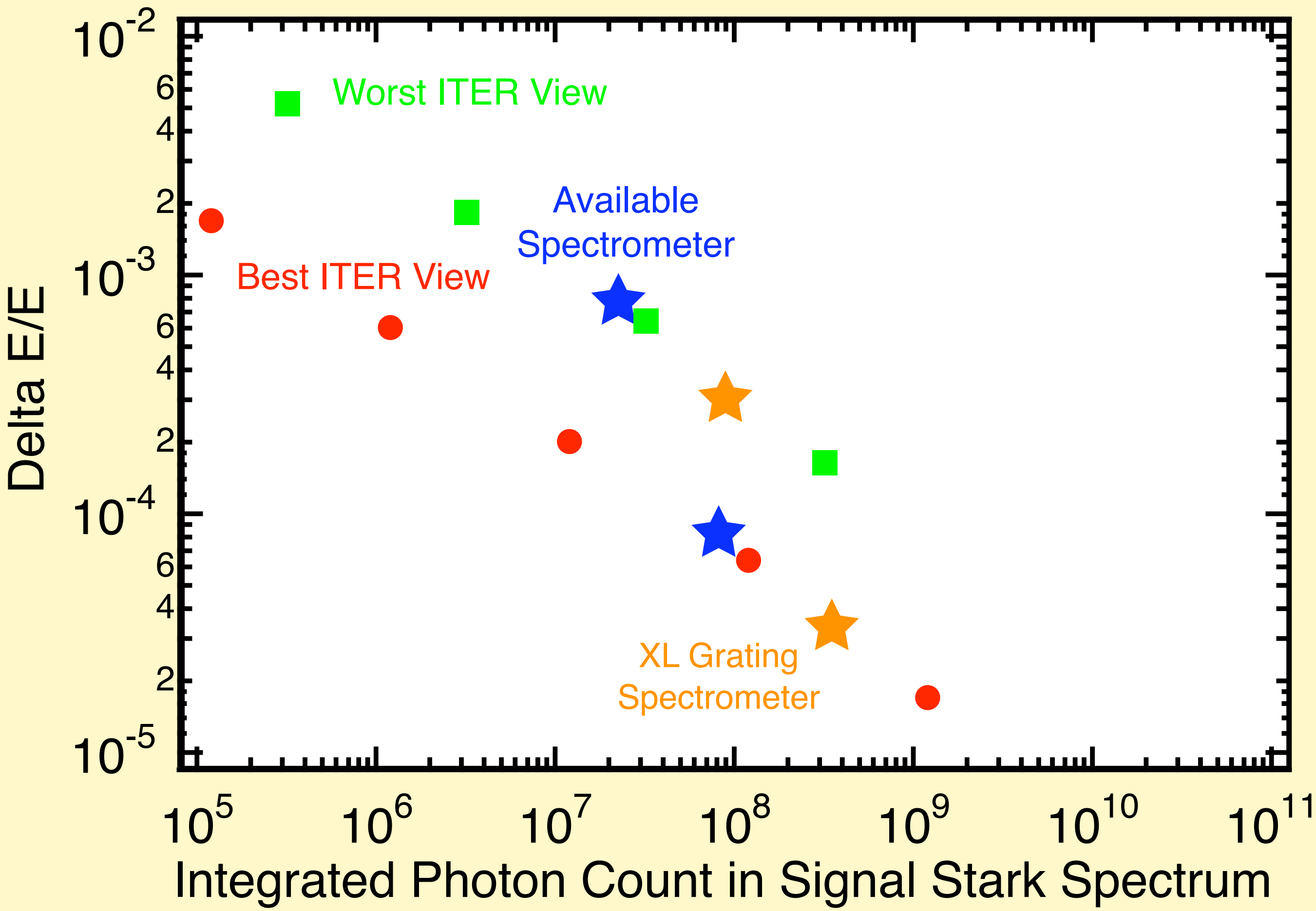
MSE-LS Measurement Ability on ITER

Best Case: (1.2 x 10^{13} ph/sec) x (0.01 sec) x (T. F.)

Worst Case: (3.2 x 10^{12} ph/sec) x (0.01 sec) x (T. F.)



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Analysis of Utility of MSE-LS vs MSE-LP

Theory and code developed by L. Zakharov (PPPL). Further details in breakout session of this meeting. Tues. 3:20 pm B318

Concept: Determine possible variances in reconstructed equilibria given a set of magnetic diagnostics and perturbation of background profiles.

- ESC equilibrium code solves linearized Grad-Shafranov equation to calculate the response of diagnostics to perturbations in plasma position and current density.
- Weight appropriately to account for accuracy of measurements.
- Use SVD analysis to determine possible error in reconstruction of q, p profiles in presence of prescribed perturbations.
- Consider relative merit of MSE-LS and MSE-LP systems for realistic measurement error in ITER

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Details of Powerful Technique

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Grad-Shafranov (GSh) Equation:

$$\Delta^* \bar{\Psi} = -T(\bar{\Psi}) - P(\bar{\Psi}) r^2$$

requires boundary conditions and two 1-D functions.

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Use ESC to solve linearized GSh for N perturbations: $$\bar{\Psi} = \bar{\Psi}_0 + \psi$$ $$\Delta^* \psi + T'_{\bar{\Psi}} \psi + P'_{\bar{\Psi}} \psi = -\delta T(a) - \delta P(a) r^2$$ $$\xi = \sum_{n=0}^{n<N_\xi} A_n \xi^n \quad \delta T = \sum_{n=0}^{n<N_T} T_n f^n \quad \delta P = \sum_{n=0}^{n<N_P} P_n f^n$$ $$N = N_\xi + N_T + N_P$$	

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Result can be cast in matrix form, convenient for SVD analysis:

$$\vec{X} \equiv \{\underbrace{A_0, A_1, ..., A_{N_\xi - 1}}_{N_\xi \ of \ \xi}, \underbrace{T_0, T_1, ..., T_{N_T - 1}}_{N_T \ of \ \delta T}, \underbrace{P_0, P_1, ..., P_{N_P - 1}}_{N_P \ of \ \delta P}\}$$

$$\delta\vec{S} \equiv \{\underbrace{\delta\Psi_0, \delta\Psi_1, ..., \delta\Psi_{M_\Psi - 1}}_{M_\Psi \ of \ \delta\Psi}, \underbrace{\delta B_0, \delta B_1, ..., \delta B_{M_B - 1}}_{M_B \ of \ \delta B_{pol}}, \underbrace{\delta S_0, \delta S_1, ..., \delta S_{M_S - 1}}_{M_S \ of \ \delta \ others}\}$$

$$\delta\vec{S} = \mathbb{A}\vec{X}$$

$$\mathbb{A} = \mathbb{A}_{M \times N}$$

Response matrix $\mathbb{A}$ relates magnetic signals to perturbations

Use ESC to solve linearized GSh for N perturbations:

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$$\xi = \sum_{n=0}^{n<N_\xi} A_n \xi^n \quad \delta T = \sum_{n=0}^{n<N_T} T_n f^n \quad \delta P = \sum_{n=0}^{n<N_P} P_n f^n$$

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$$N = N_\xi + N_T + N_P$$

Working matrix $\bar{\mathbb{A}}$ includes weights of signal errors

SVD technique used to determine variance in reconstructed quantities (eg p, q) that results from perturbations

Result is sorted list of perturbations from least to most 'visible' and RMS error for each desired quantity

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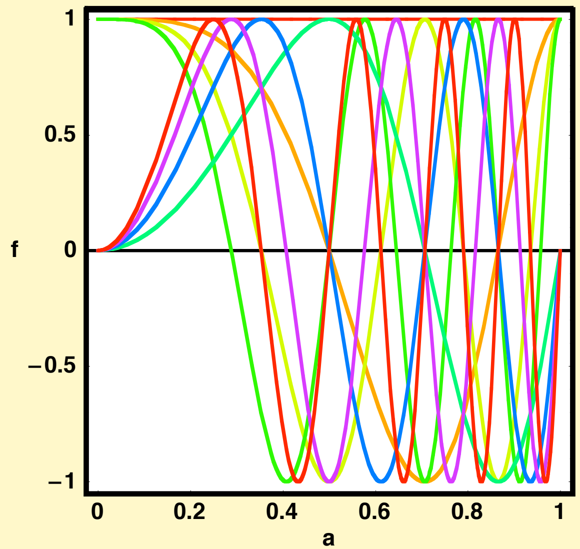
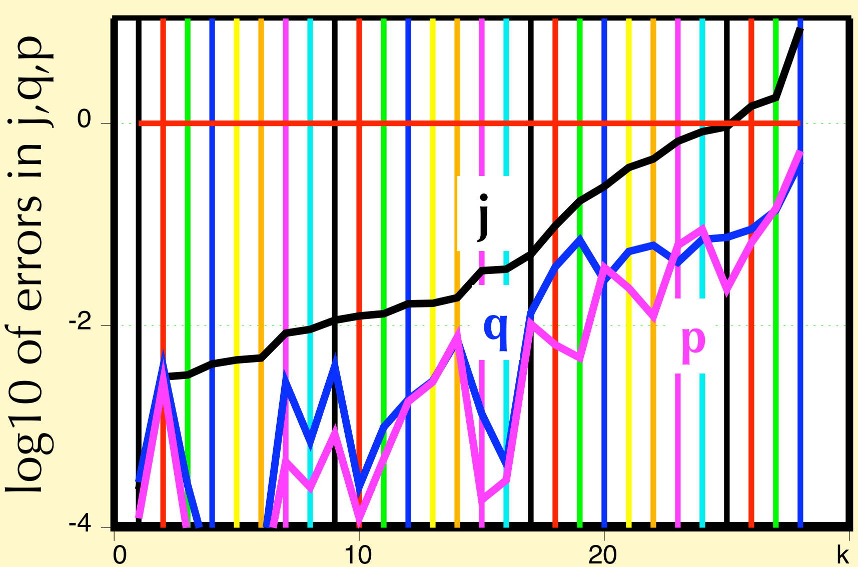
Sample of Results: External B, ψ, and MSE-LP at 0.3° Error

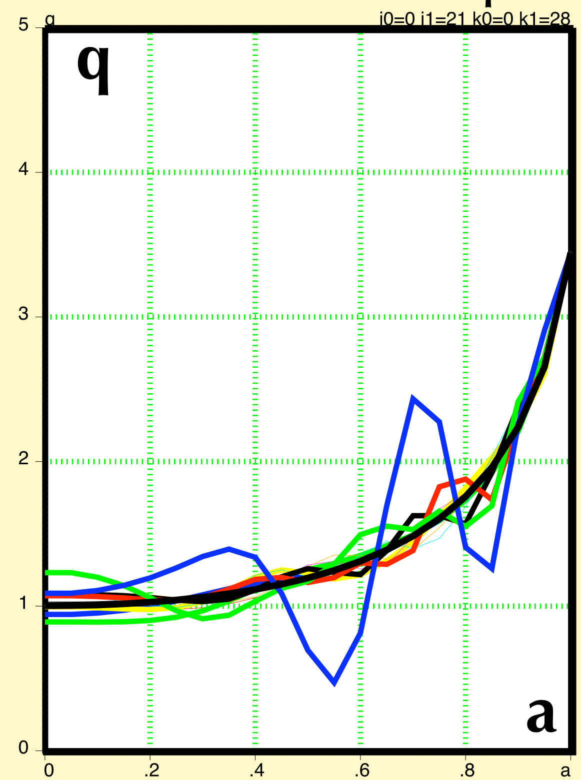
Basis Functions (8 shown):

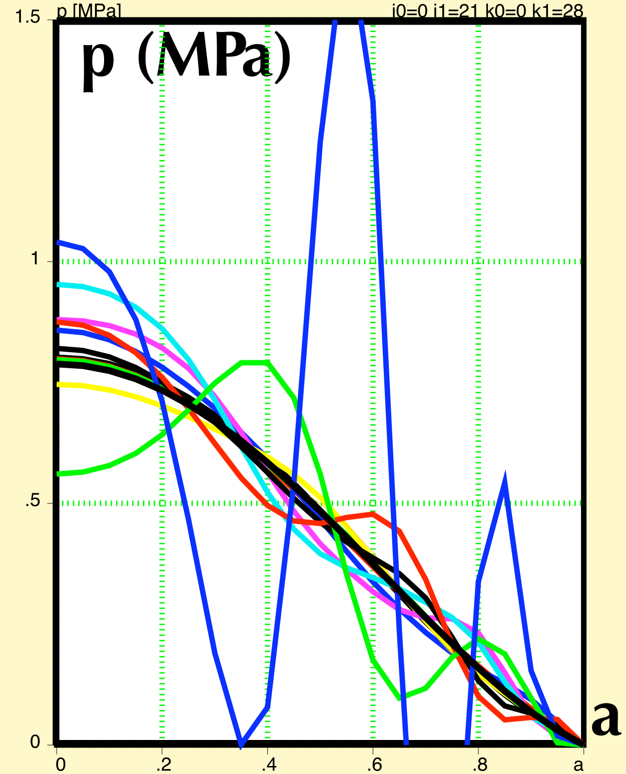


$\sigma_{\rm rms}$ Results



Reconstructed quantities with variances:





Max. Variance:

$\mathrm{Log}(\sigma_{\rm rms}) = -0.38$

$\sigma_{\rm rms} = 0.42$

$\mathrm{Log}(\sigma_{\rm rms}) = -0.27$

$\dfrac{\sigma_{\rm rms}}{p_o} = 0.67$

$$N = N_\xi + N_T + N_P$$
$$= 12 + 10 + 6$$

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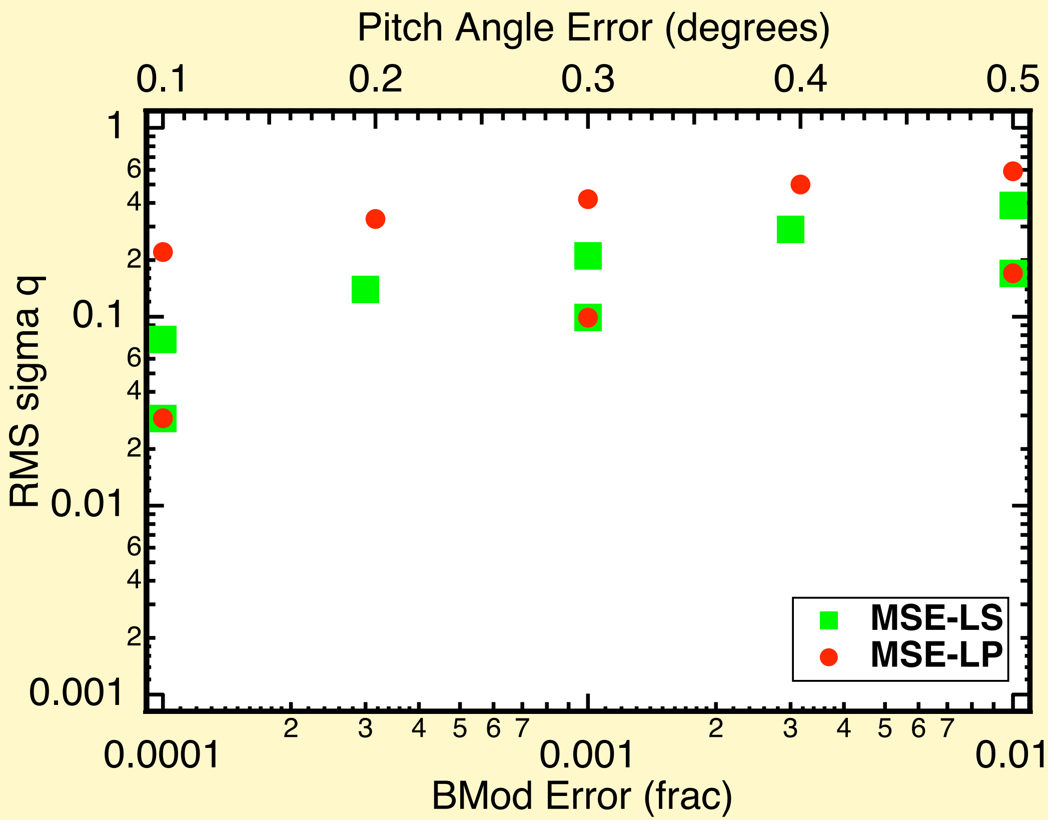
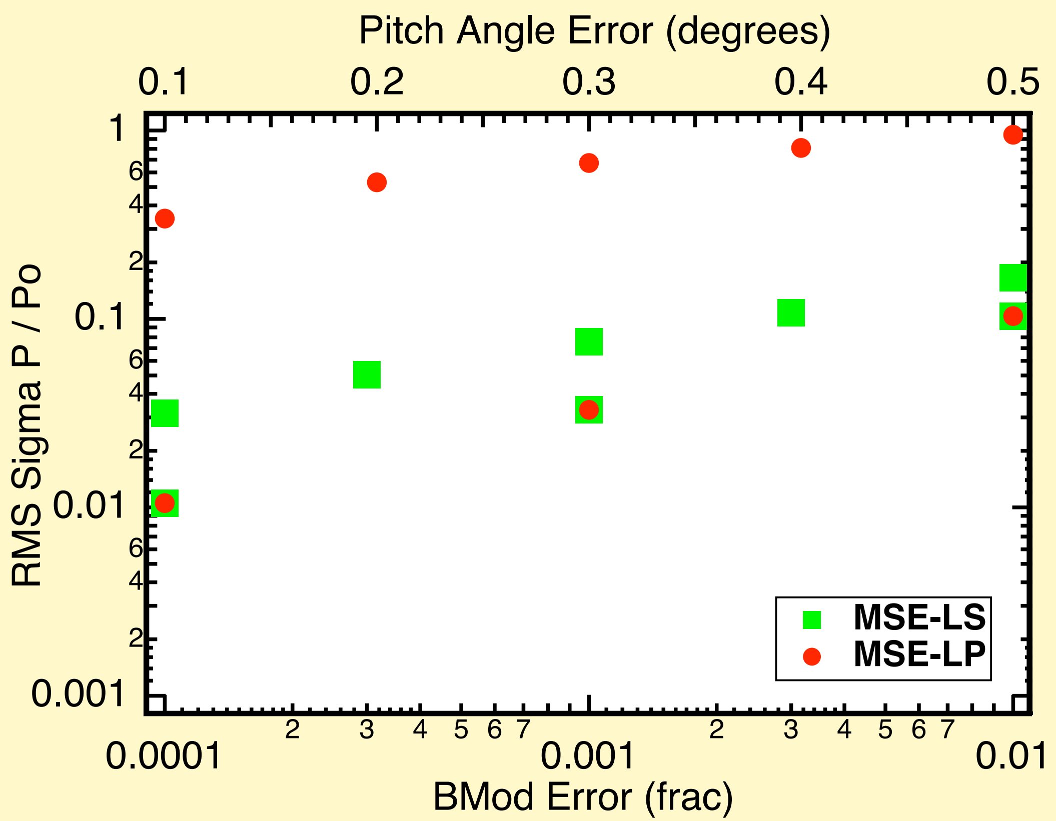
Preliminary Results for MSE-LS and -LP on ITER





MSE-LS appears superior to MSE-LP for pressure profile reconstruction, and comparable to MSE-LP for q-profile reconstruction. Combined system advantageous for both.

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Conclusion of Extended Assessment Study

- Detailed 3D SimMSE model developed
- Fitting of simulated data performed
- Hardware considerations included
- With available technology, could measure |E| in ITER to 0.1%. (statistical uncertainty)
- Preliminary work with L. Zakharov's theory of equilibrium variances suggests MSE-LS may be superior to MSE-LP in ITER
- Possibility of pressure profile reconstruction with MSE-LS alone, or hybrid system

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Recommendations

- Further exploration of parameter variation in SimMSE code: Beam energy, beam steering, aperture location.
- Study of systematic uncertainty for MSE-LS measurement.
- SimMSE model extension to include MSE-LP.
- Study of statistical and systematic uncertainty for MSE-LP measurement
- Further work with L. Zakharov: Consider other profiles, perturbation spectra, effects to ensure MSE-LS vs MSE-LP result is robust, incorporate more realistic error assessments.
- Generate recommendation regarding precision spectroscopy vs. polarimetry for optimal path forward to meet ITER design goal with minimum cost and risk.

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