Computation of MHD Flows using the Lattice Boltzmann Method

Kannan N. Premnath & Martin J. Pattison

MetaHeuristics LLC

Santa Barbara, CA 93105

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Main Topics

- Lattice Boltzmann Models for MHD
- New Lattice Boltzmann Model for high Hartmann number MHD
- Simulation results in 2D and 3D
- Summary and Conclusions

Magnetohydrodynamic (MHD) Equations

Fluid dynamical equations

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \boldsymbol{u}) = 0$$

$$\frac{\partial \boldsymbol{u}}{\partial t} + \nabla \cdot (\boldsymbol{u}\boldsymbol{u}) = -\nabla p + N\boldsymbol{F}_{Lorentz} + \frac{1}{\mathrm{Re}} \nabla \cdot \Pi_{visc}$$

Lorentz force $\boldsymbol{F}_{Lorentz} = \boldsymbol{J} \times \boldsymbol{B}$

Magnetic induction equation

$$\frac{\partial \boldsymbol{B}}{\partial t} + \nabla \cdot (\boldsymbol{u}\boldsymbol{B} - \boldsymbol{B}\boldsymbol{u}) = \frac{1}{\mathrm{Re}_m} \nabla^2 \boldsymbol{B} \qquad \nabla \cdot \boldsymbol{B} = 0$$

Dimensionless numbers

Reynolds number $\mathrm{Re} = \dfrac{u_0 L}{\nu}$

Interaction parameter or Stuart number $N = \dfrac{\sigma B_0^2 L}{\rho u_0}$

Hartmann number $Ha = \sqrt{N\,\mathrm{Re}}$

Magnetic Reynolds number $\mathrm{Re}_m = \dfrac{u_0 L}{\eta}, \quad \eta = \dfrac{1}{\sigma \mu_{mag}}$ Magnetic diffusivity

Lattice Boltzmann Model for MHD

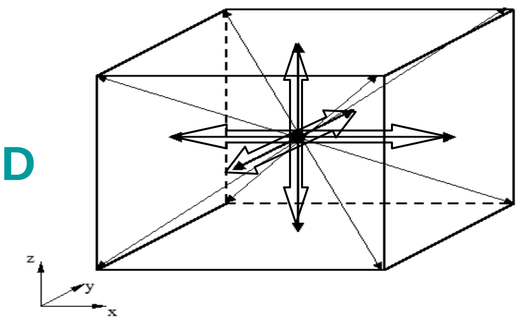
I. Hydrodynamics

Scalar distribution function for hydrodynamics

$$f_\alpha(\boldsymbol{x}+\boldsymbol{e}_\alpha\delta_t, t+\delta_t) - f_\alpha(\boldsymbol{x},t) = -\frac{1}{\tau_f}(f_\alpha - f_\alpha^{eq})$$

streaming collision

3D



Coincident lattices

$\boldsymbol{e}_\alpha,\ \alpha = 0,1,\ldots,14$

$\Xi_\beta,\ \beta = 0,1,\ldots,6$

Equilibrium distribution functions

$$f_\alpha^{eq} = f_\alpha^{eq}\left(\rho, \boldsymbol{u}, \boldsymbol{B}, \mu_{mag}\right)$$

functions of macro fields

Macroscopic fields

$$\rho = \sum_{\alpha=0}^{b} f_\alpha \qquad \rho\boldsymbol{u} = \sum_{\alpha=0}^{b} f_\alpha \boldsymbol{e}_\alpha \qquad p = \frac{1}{3}c^2\rho$$

moments of distribution function

Lattice Boltzmann Model for MHD (cont…)

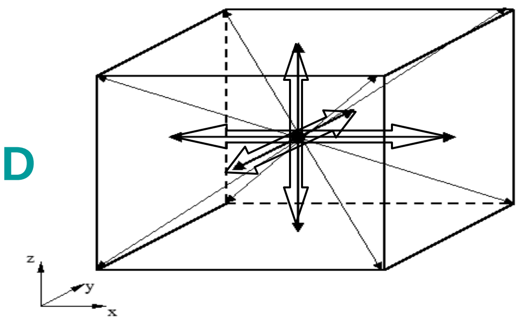
II. Magnetic induction

Vector distribution function for magnetic induction

$$\boldsymbol{g}_\beta(\boldsymbol{x}+\Xi_\beta\delta_t, t+\delta_t)-\boldsymbol{g}_\beta(\boldsymbol{x},t)=-\frac{1}{\tau_m}(\boldsymbol{g}_\beta-\boldsymbol{g}_\beta^{eq})$$

streaming collision

3D



Coincident lattices

$\boldsymbol{e}_\alpha,\ \alpha=0,1,\ldots,14$

$\Xi_\beta,\ \beta=0,1,\ldots,6$

Equilibrium distribution functions

$$\boldsymbol{g}_\alpha^{eq}=\boldsymbol{g}_\alpha^{eq}(\boldsymbol{u},\boldsymbol{B})$$

functions of macro fields

Macroscopic fields

$$\boldsymbol{B}=\sum_{\beta=0}^{b_m}\boldsymbol{g}_\beta$$

$$(\boldsymbol{J})_k=\frac{1}{\mu_{mag}}(\nabla\times\boldsymbol{B})_k=-\frac{N_m}{c^2\tau_m\mu_{mag}}\varepsilon_{ijk}\left[\sum_{\beta=0}^{b_m}\boldsymbol{e}_{\alpha i}\boldsymbol{g}_{\beta j}-\left(\boldsymbol{u}_i\boldsymbol{B}_j-\boldsymbol{B}_i\boldsymbol{u}_j\right)\right]$$

$$N_m=\begin{cases}3, 2D\\4, 3D\end{cases}$$

moments of distribution function

Lattice Boltzmann Model for MHD (cont…)

Transport coefficients (Diffusivities)

The emergent behavior of LBM is macrodynamical MHD equations with transport coefficients dependent on relaxation time and lattice parameters

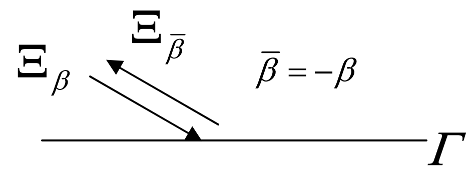
$$\nu = \frac{c^2}{3}\left(\tau_f - \frac{1}{2}\right)\delta_t \qquad \eta = \frac{c^2}{N_m}\left(\tau_m - \frac{1}{2}\right)\delta_t$$

Boundary conditions

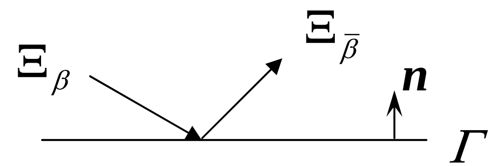
$$\boldsymbol{B} = \boldsymbol{B}^a + \boldsymbol{B}^i$$

Special cases

Bounce-back $\quad \boldsymbol{B}^i(\Gamma) = \boldsymbol{0} \quad$ Insulating



Specular reflection $\quad \dfrac{\partial \boldsymbol{B}^i}{\partial \boldsymbol{n}}(\Gamma) = \boldsymbol{0} \quad$ Conducting



General Case

Extrapolation method (proposed)

Electromagnetic domain extending outside fluid flow domain

Fluid boundary

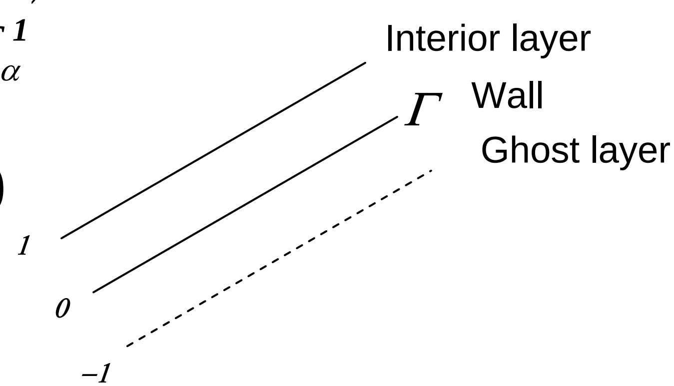
$$f_\alpha^{eq}\Big|_{\Gamma_f} = f_\alpha^{eq}\left(\boldsymbol{u}_{\Gamma_f}\right)$$

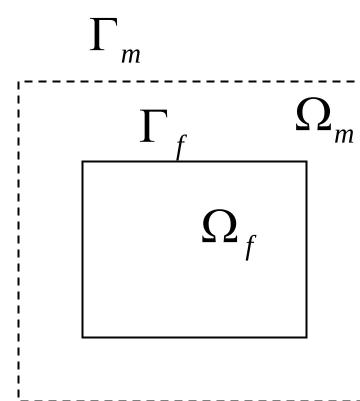
Post-collision $\quad f_\alpha^{-1} = 2 f_\alpha^{0} - f_\alpha^{1}$

Electromagnetic boundary

$$\boldsymbol{g}_\alpha^{eq}\Big|_{\Gamma_m} = \boldsymbol{g}_\alpha^{eq}(\boldsymbol{B}_{\Gamma_m})$$

Post-collision $\quad \boldsymbol{g}_\beta^{-1} = 2\boldsymbol{g}_\beta^{0} - \boldsymbol{g}_\beta^{1}$





$$\boldsymbol{B} = \boldsymbol{B}^a + \boldsymbol{B}^i (= \boldsymbol{0})$$

Advantages of LBM for MHD flows

- Avoids *time-consuming* solution of Poisson-type pressure equation

  All information obtained ***locally***

- Naturally amenable for implementation on parallel computers

  Efficient calculation procedure for **handling large problems**

- Well suited to MHD flows in complex geometries

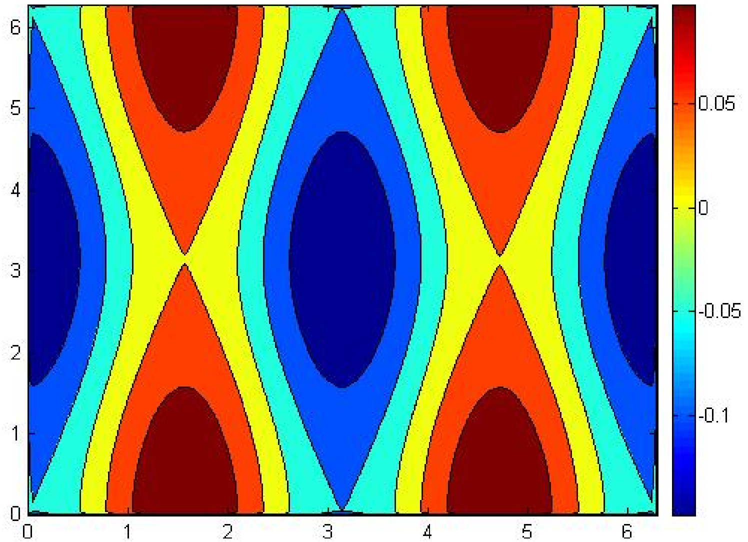
- Other advantages

  - Complete field formulation
  - Calculated fields *solenoidal* to machine round-off error
  - Current density as higher moment of the distribution function (no finite differencing)

MHD Results in 2D

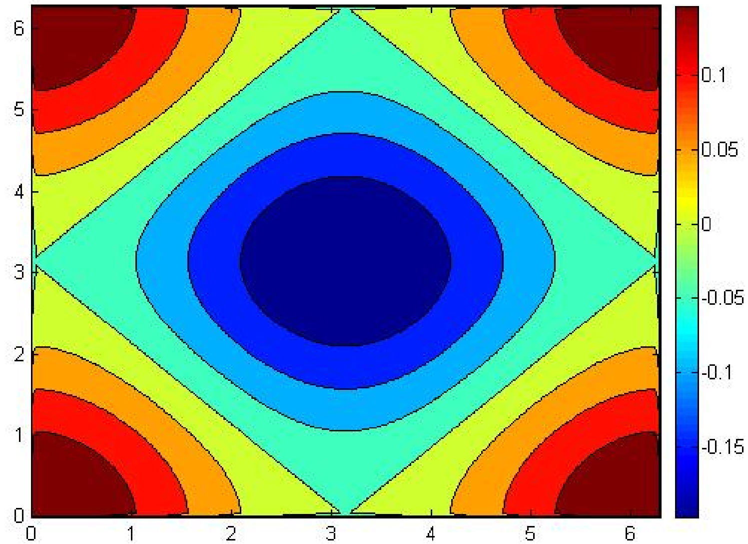
Orszag -Tang vortex

t=0



$$\boldsymbol{J} = j\hat{\boldsymbol{k}} = \frac{1}{\mu_{mag}} \nabla \times \boldsymbol{B}$$

Current density



$$\boldsymbol{\omega} = \omega \boldsymbol{k} = \nabla \times \boldsymbol{u}$$

Vorticity

Initial conditions

$$u_x(x, y, 0) = u_0 \sin y \qquad B_x(x, y, 0) = -b_0 \sin y$$

$$u_y(x, y, 0) = u_0 \sin x \qquad B_y(x, y, 0) = -b_0 \sin(2x)$$

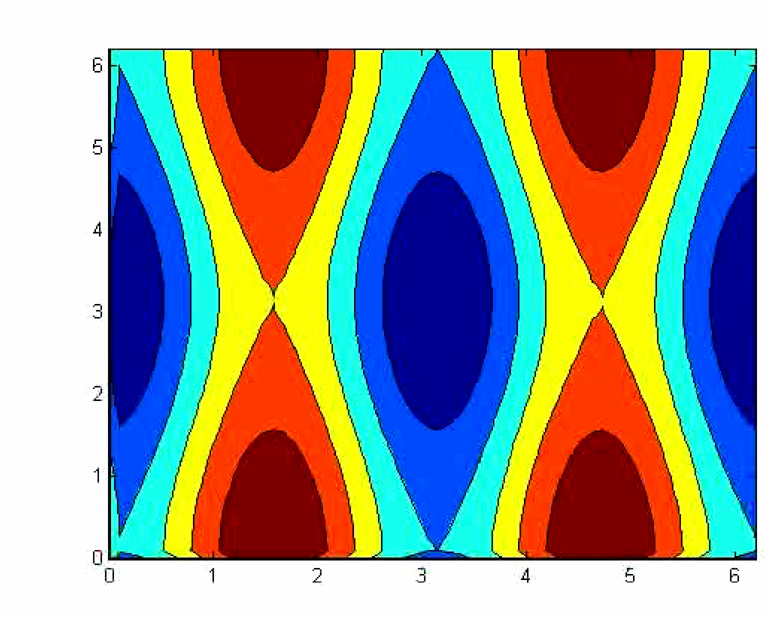
(Orszag and Tang, J. Fluid Mech., 90: 129 (1979))

$$\mathrm{Re} = \mathrm{Re}_m = 765$$

MHD Results in 2D

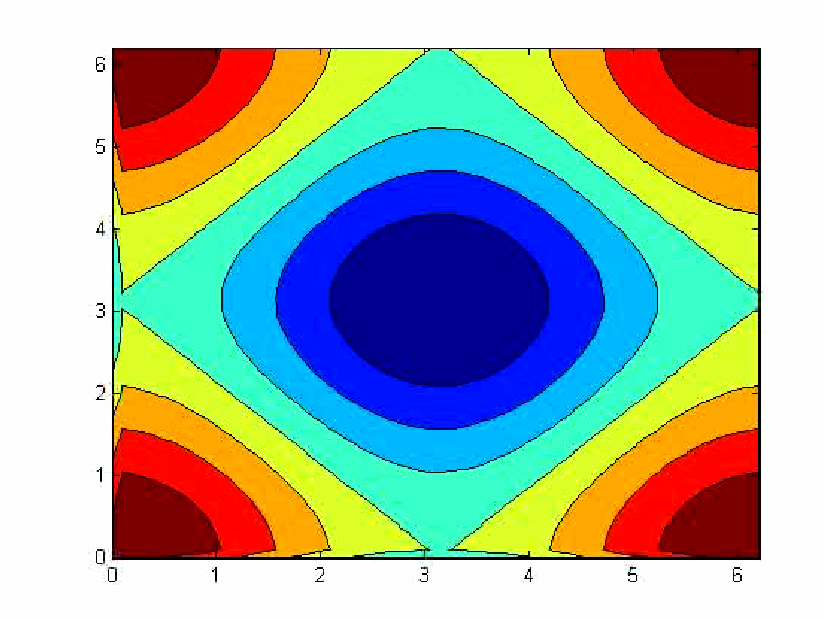
Orszag -Tang vortex

Time evolution



$$\boldsymbol{J} = j\hat{\boldsymbol{k}} = \frac{1}{\mu_{mag}} \nabla \times \boldsymbol{B}$$

Current density

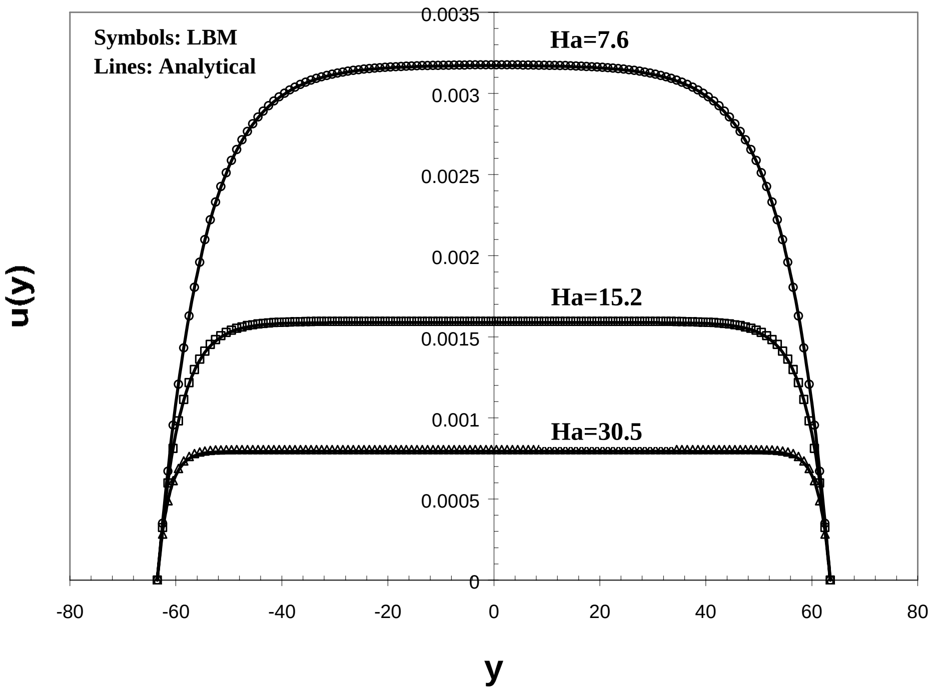


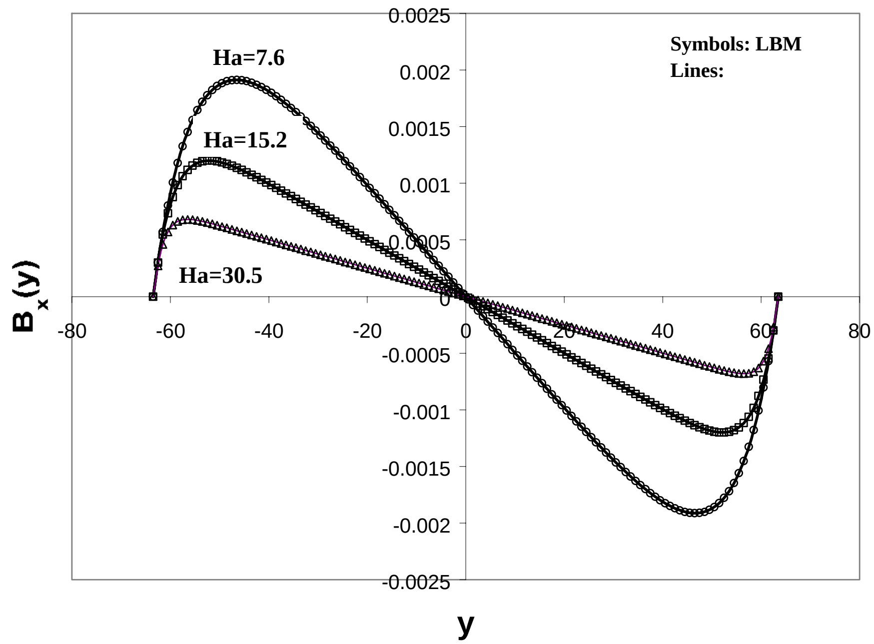
$$\boldsymbol{\omega} = \omega\hat{\boldsymbol{k}} = \nabla \times \boldsymbol{u}$$

Vorticity

MHD Results in 2D - II

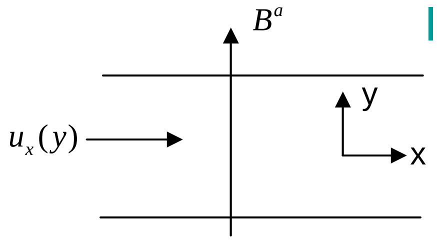
Hartmann Flow





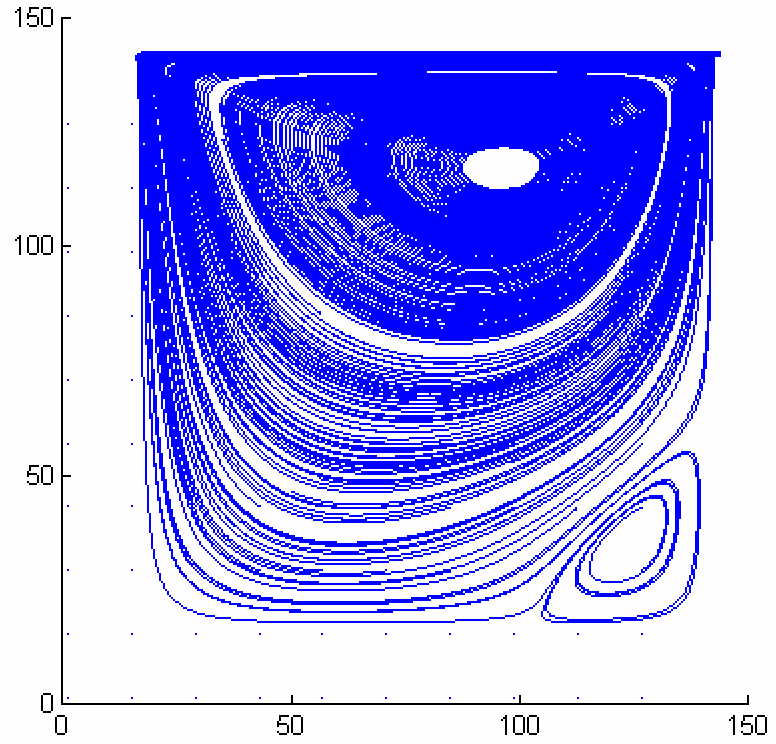
Velocity profiles

Induced magnetic field profiles



MHD Results in 2D -III

MHD Lid-driven Cavity



Fluid flow Domain: 128×128

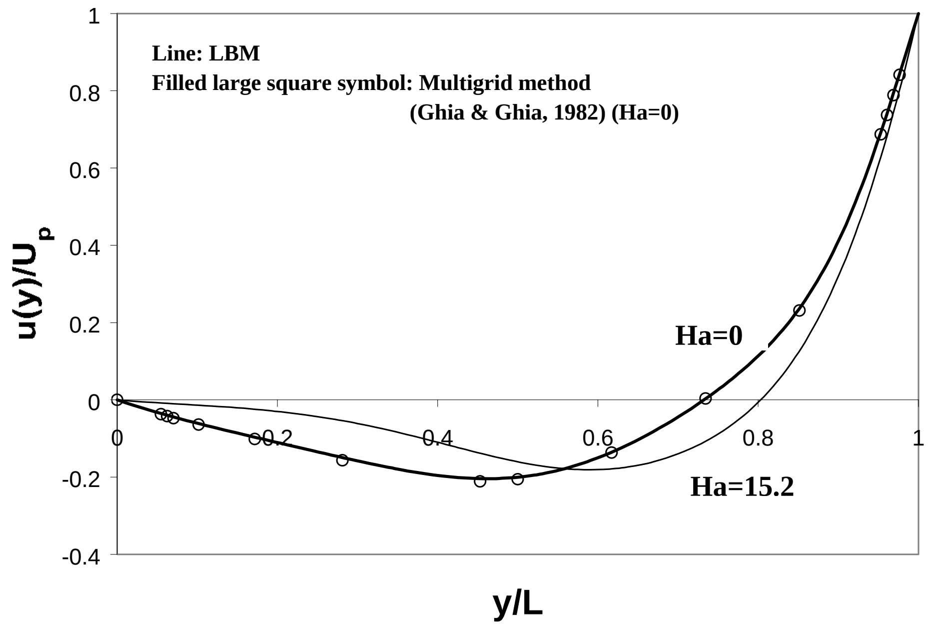
Electromagnetic domain: 162×162

Induced magnetic fields set to zero on the electromagnetic boundary

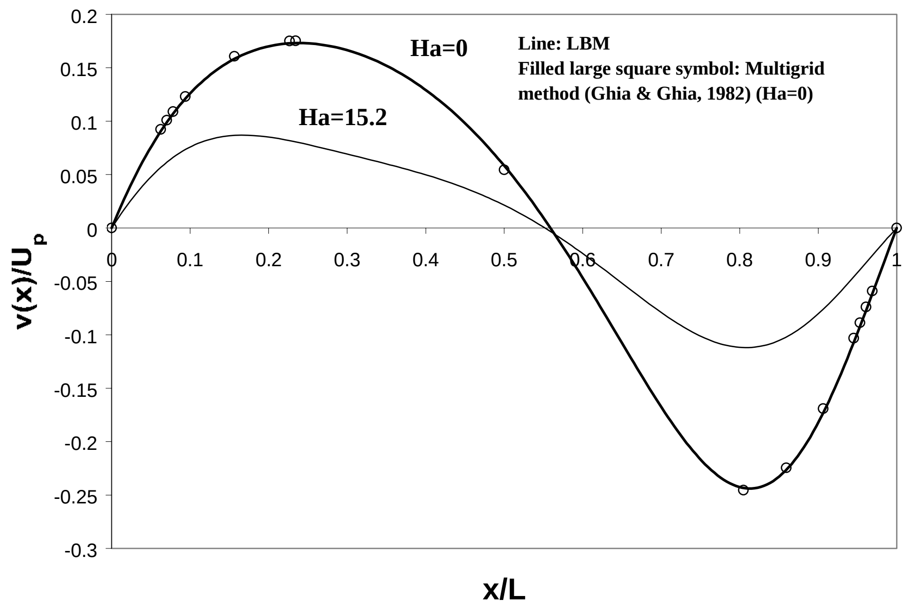
Streamlines: Re = 100, Ha = 15.2, Re_m = 100

MHD Results in 2D -III

MHD Lid-driven Cavity



u-velocity profiles



v-velocity profiles

A new LB model for high *Ha* MHD flows - I

- High Hartmann number (*Ha*) flows require the resolution of various thin viscous boundary or shear layers
  - Hartmann layers $\delta_{m_1} \sim \dfrac{1}{Ha}$
  - Side layers $\delta_{m_2} \sim \dfrac{1}{\sqrt{Ha}}$
  - Ludford free shear layers from sharp bends
- Standard LB MHD model restricted to uniform lattice grids
- Standard LB MHD model uses a single relaxation time (SRT), which restricts stability for a given resolution
- A new Multiple Relaxation Time (MRT) Interpolation Supplemented Lattice Boltzmann Model (ISLBM) developed for *non-uniform* or *stretched* grids with *improved stability*

A new LB model for high *Ha* MHD flows - II

Scalar distribution function for hydrodynamics

$$f_\alpha(\boldsymbol{x}+\boldsymbol{e}_\alpha\delta_t, t+\delta_t) - f_\alpha(\boldsymbol{x},t) = -\Lambda_{\alpha\beta}(f_\beta - f_\beta^{eq}) + \left(I_{\alpha\beta} - \frac{1}{2}\Lambda_{\alpha\beta}\right)S_\beta$$

streaming — collision — Forcing term representing Lorentz force

$\Lambda_{\alpha\beta}$ Components of the MRT matrix

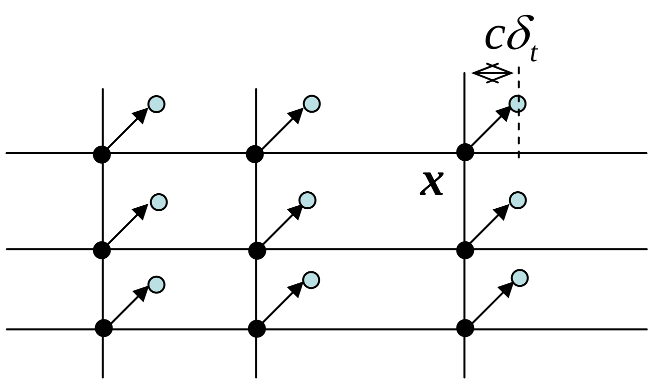
Vector distribution function for magnetic induction

$$\boldsymbol{g}_\beta(\boldsymbol{x}+\Xi_\beta\delta_t, t+\delta_t) - \boldsymbol{g}_\beta(\boldsymbol{x},t) = -\frac{1}{\tau_m}(\boldsymbol{g}_\beta - \boldsymbol{g}_\beta^{eq})$$

Interpolation step

$$f_\alpha(\boldsymbol{x}, t+\delta_t) = F\left(f_\alpha(\{\boldsymbol{x}_{neighbors}\} + \boldsymbol{e}_\alpha\delta_t, t+\delta_t)\right)$$

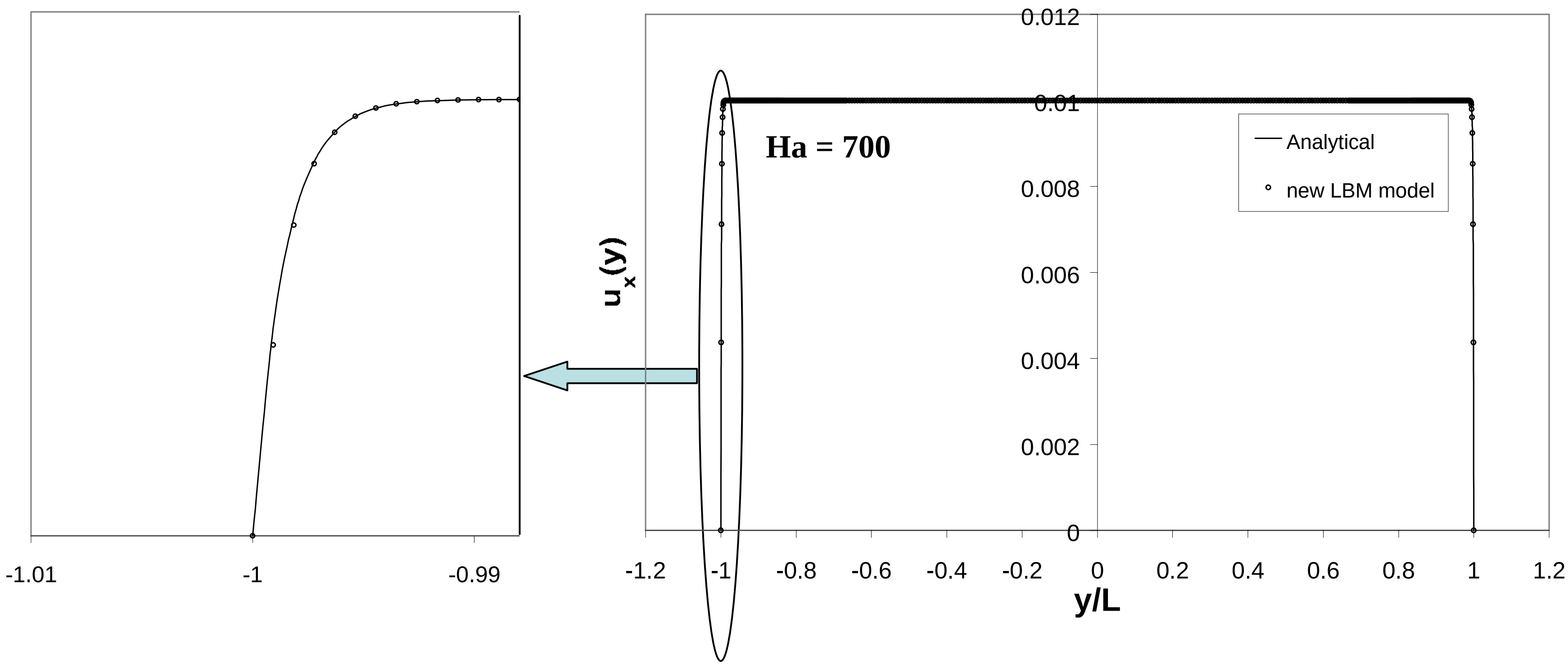
$$\boldsymbol{g}_\beta(\boldsymbol{x}, t+\delta_t) = G\left(\boldsymbol{g}_\beta(\{\boldsymbol{x}_{neighbors}\} + \boldsymbol{e}_\alpha\delta_t, t+\delta_t)\right)$$



Second order Interpolation of distribution functions

A new LB model for high *Ha* MHD flows - III

Hartmann Flow

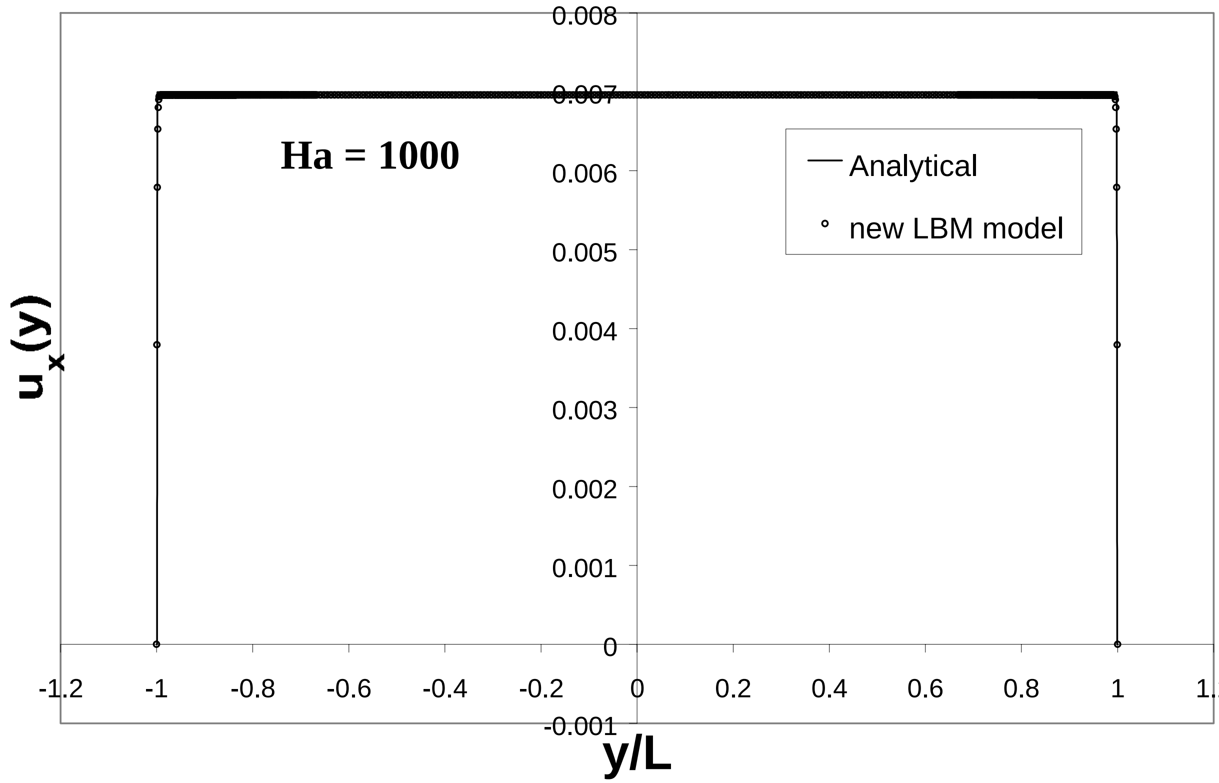


Velocity profile (*Ha* = 700)

Non-uniform grid with simple step-changes in grid resolutions

A new LB model for high *Ha* MHD flows - IV

Hartmann Flow

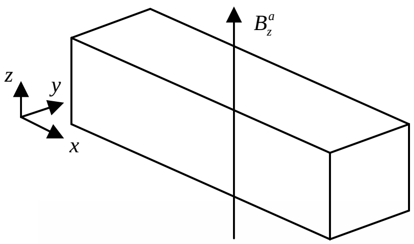


Non-uniform grid with simple step-changes in grid resolutions

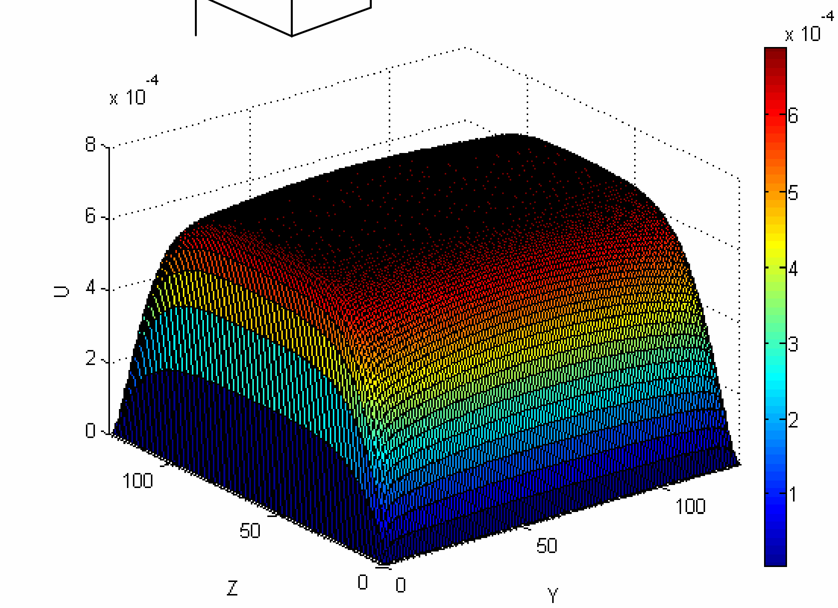
Velocity profile (*Ha* = 1000)

Boundary layer stretching transformations (e.g. Roberts transformation) can be used to further increase *Ha*

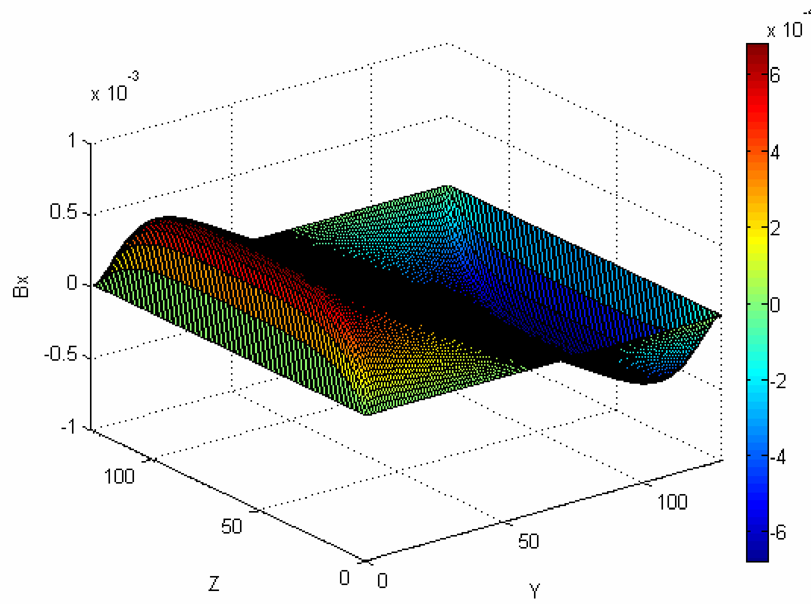
3D MHD Flows - I



Developed MHD duct flow



Velocity profile



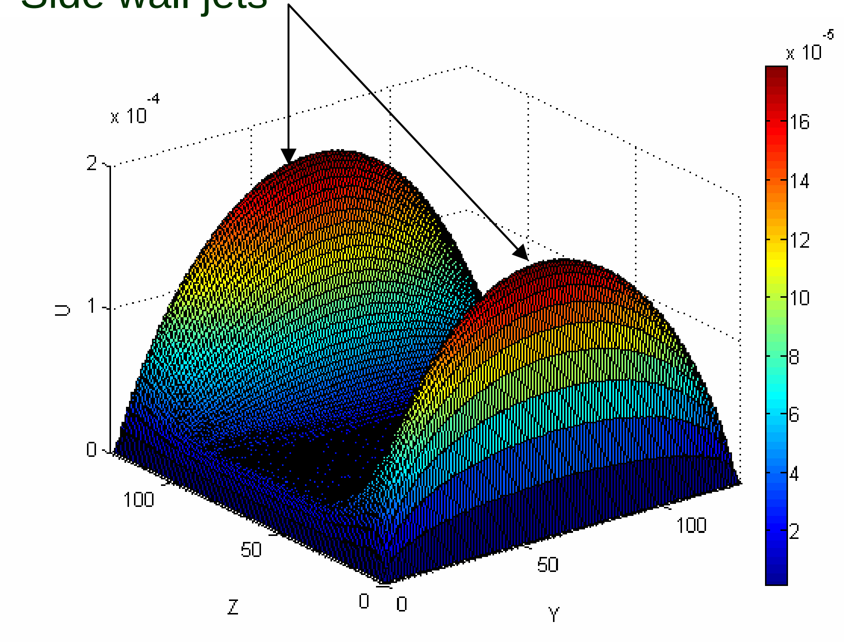
Induced magnetic field profile

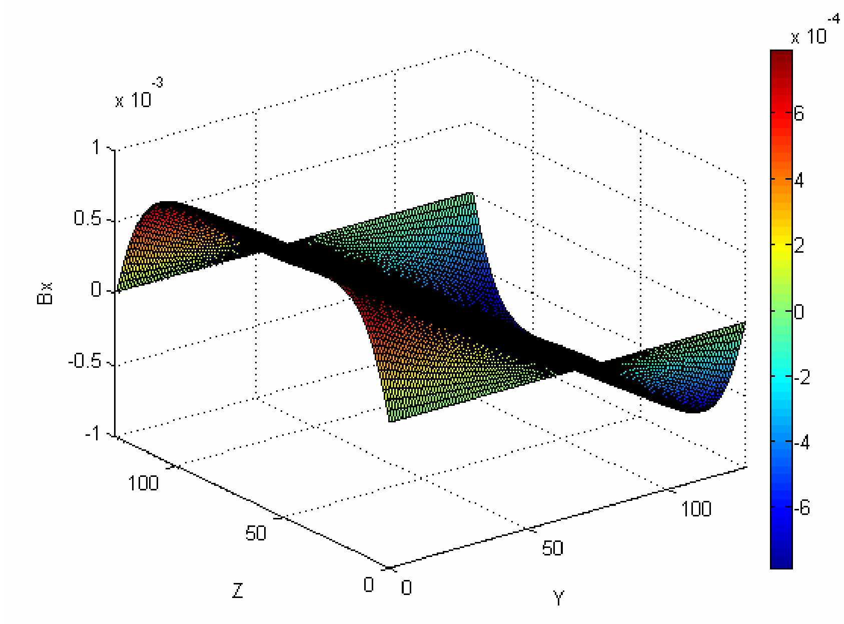
Hartmann walls – perfectly insulating,
Side walls - perfectly insulating (*Ha* = 30)

3D MHD Flows - I

Developed MHD duct flow

Side wall jets





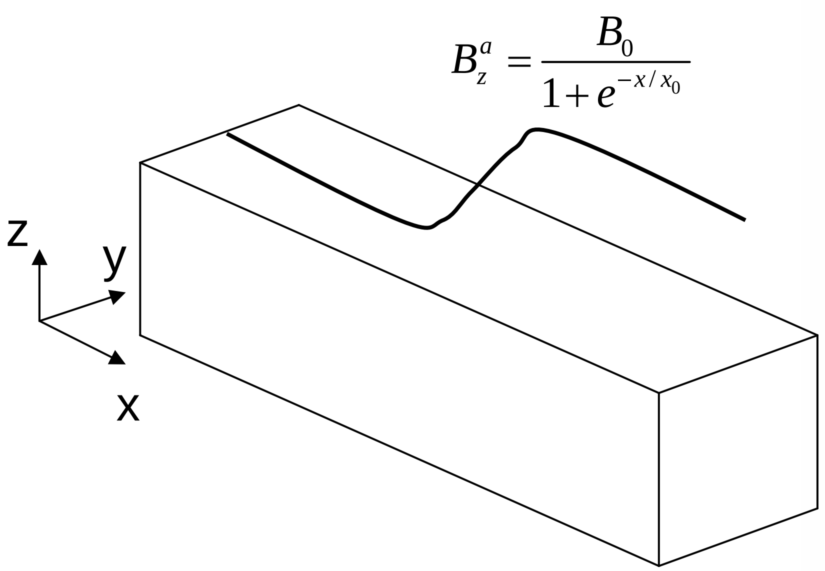
Velocity profile

Induced magnetic field profile

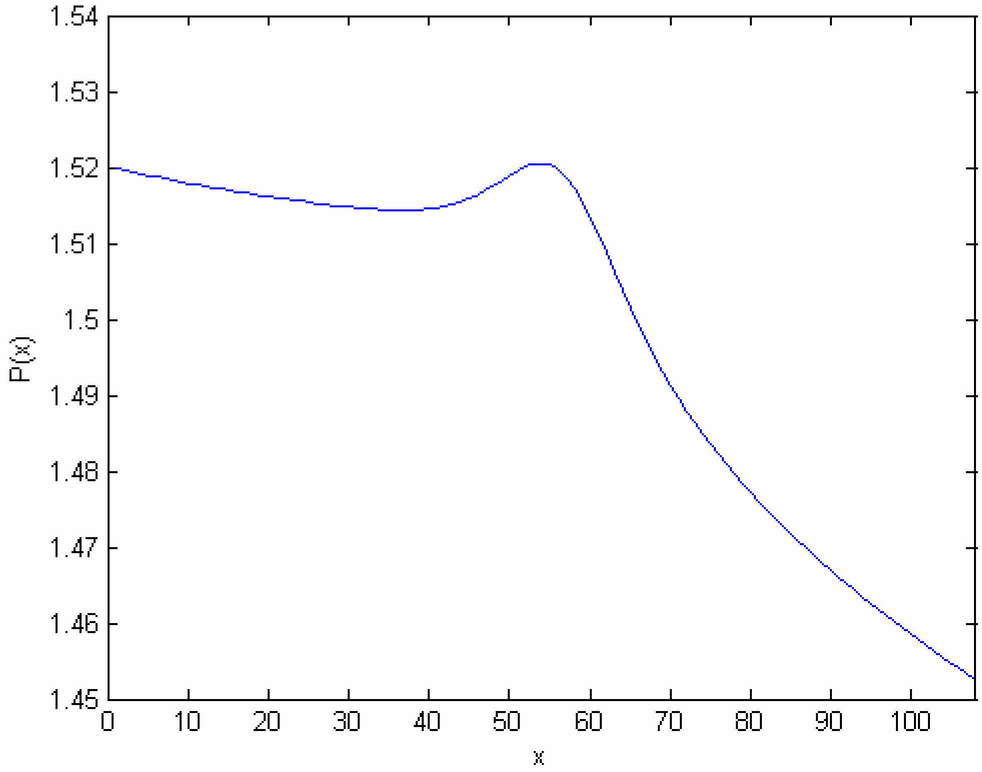
Hartmann walls – conducting,
Side walls - perfectly insulating (*Ha* = 30)

3D MHD Flows - II

3D Developing MHD Duct Flow – Sterl problem



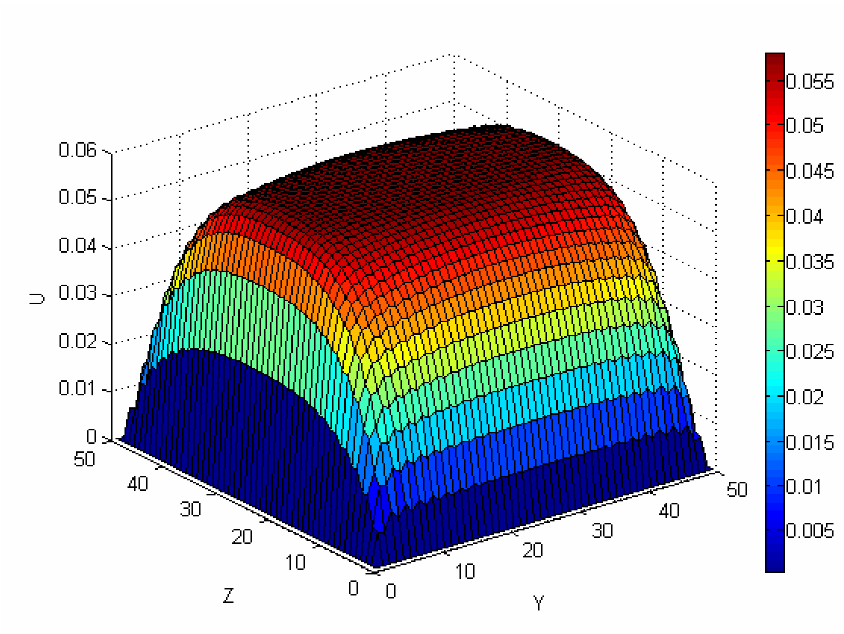
Streamwise sharp gradient in the applied magnetic field



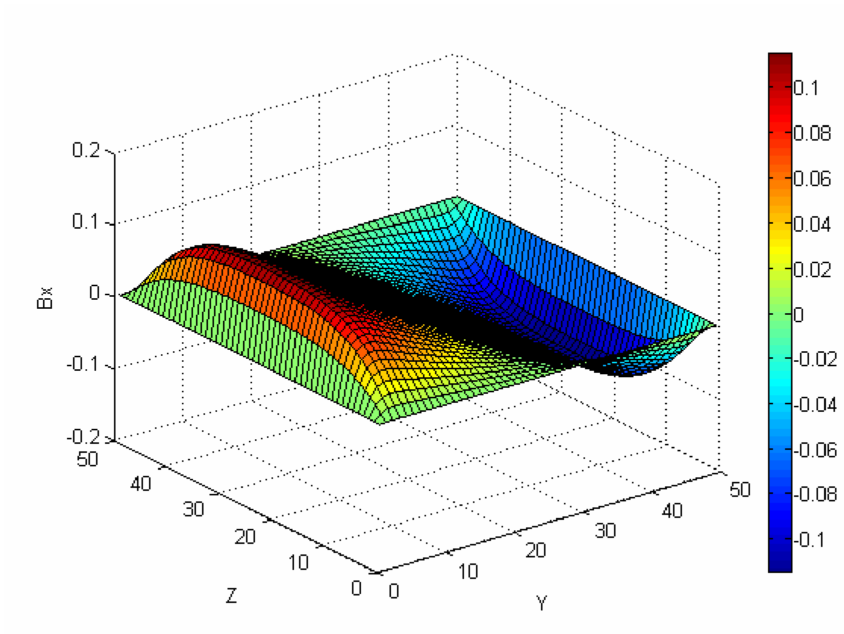
Pressure Variation along streamwise direction (*Ha* = 44)

3D MHD Flows - II

3D Developing MHD Duct Flow – Sterl problem



Velocity profile at the exit plane



Induced magnetic field at the exit plane

Summary and Conclusions

- Lattice Boltzmann simulations for for 2D and 3D MHD performed
- Simulations of MHD test problems in 2D and 3D show qualitative and quantitative agreement
- A new multiple relaxation time (MRT) interpolation supplemented lattice Boltzmann model (ISLBM) for simulating high *Ha* MHD flows

Ongoing and Future Work

Code Version 1 Capabilities

- MHD flows at intermediate Hartmann numbers
- Multiphase flows
- Heat transfer with non-uniform thermal conductivities
- Complex geometries
- Parallel processing using MPI
- Pre-processor: Cart3D from NASA
- Post-processor: FieldView

Code Release by end of June, 2005

Code will be implemented on a smaller cluster at MetaHeuristics and a larger cluster at National Center for Supercomputing Applications (NCSA)

On-going and Future Work

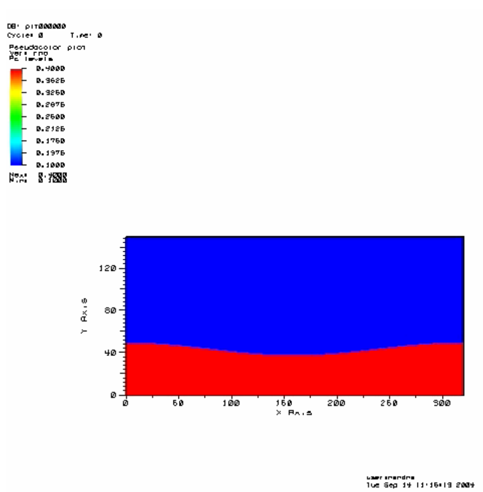
Code Version 2 Additional Capabilities

- MHD flows at high Hartmann numbers
- Non-uniform grids
- Turbulence modeling capability using Smagorinsky type large eddy simulation (LES) model

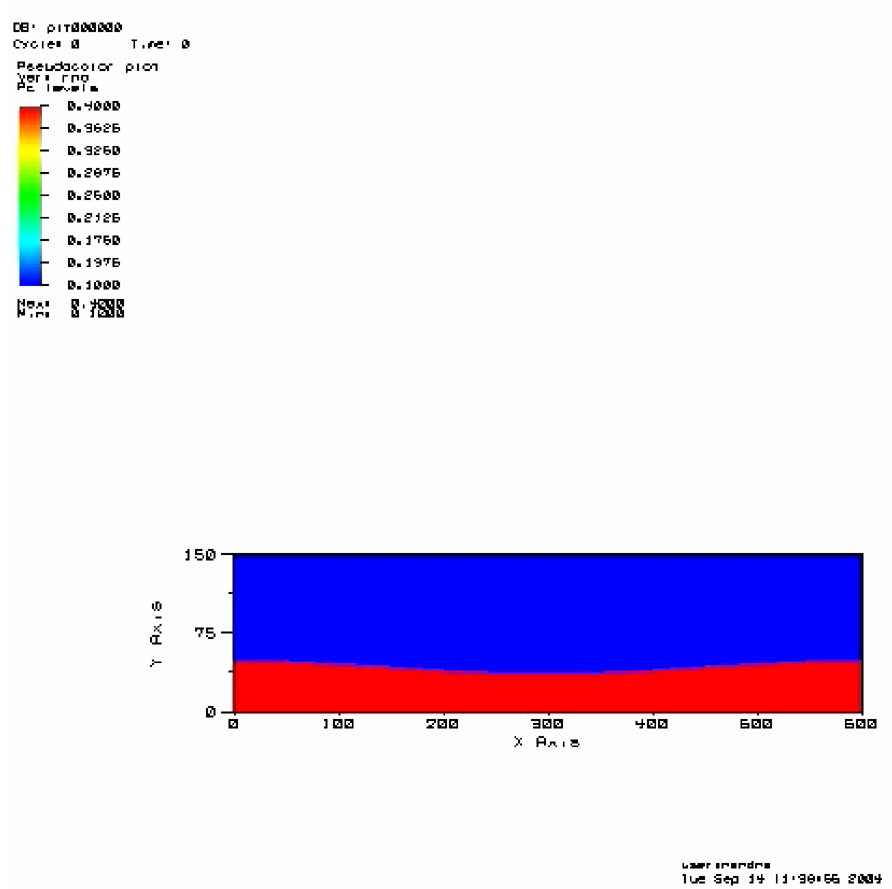
Code Release by end of October, 2005

Multiphase Flow Capabilities

Example Problem - I: Rayleigh Instability and Satellite Droplet Formation

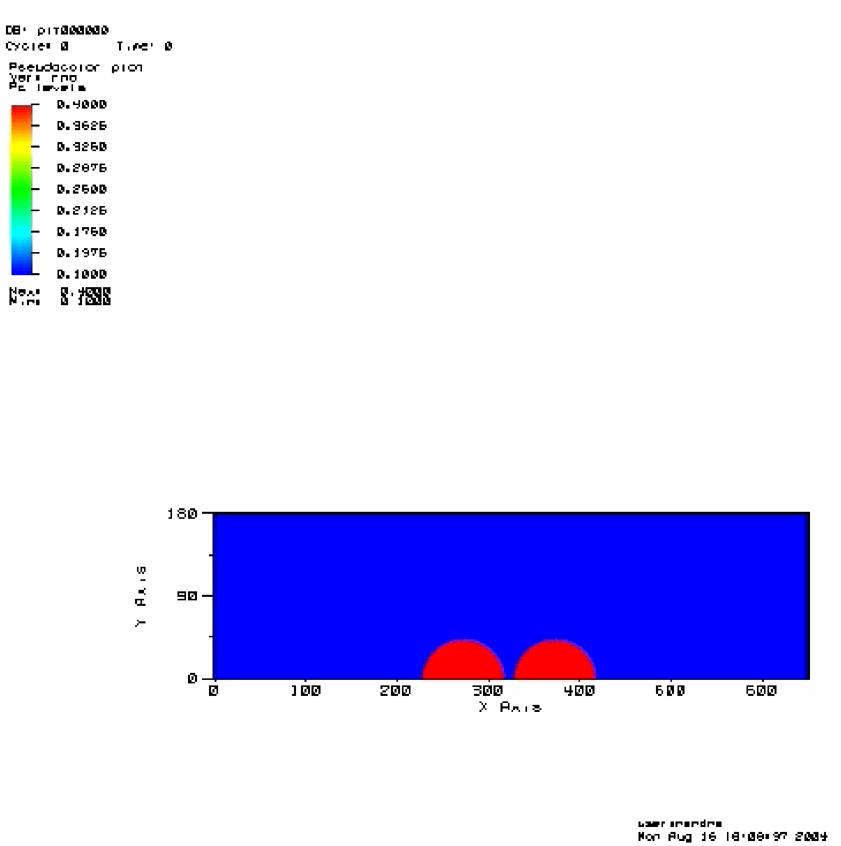


Liquid Cylindrical Column perturbed by *shorter* wavelength surface disturbance

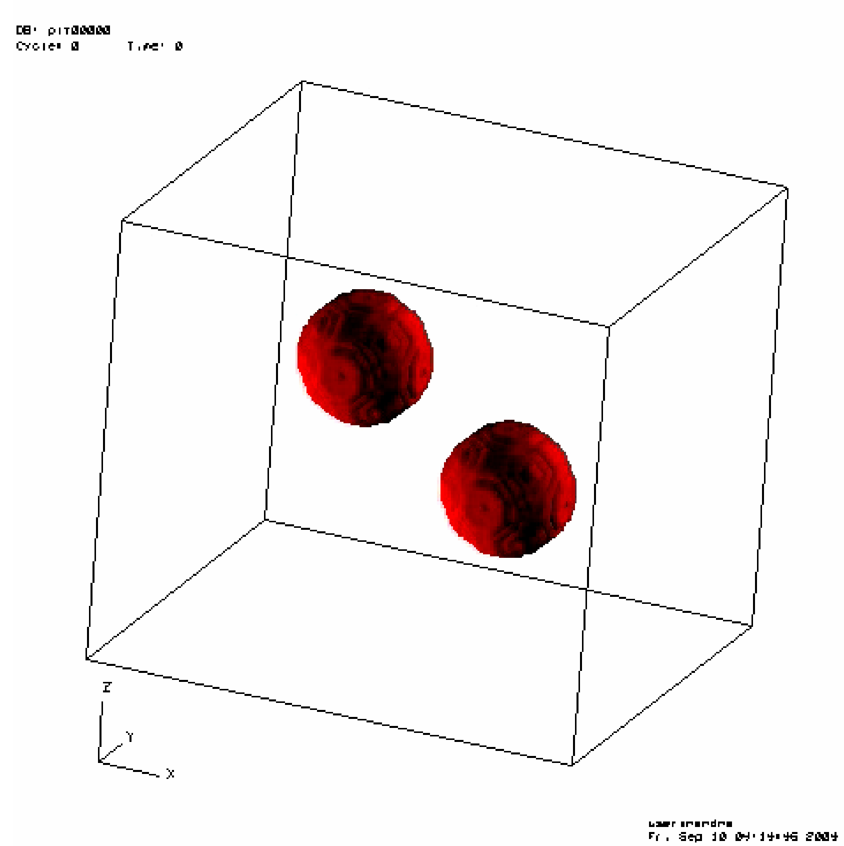


Liquid Cylindrical Column perturbed by *longer* wavelength surface disturbance

Example Problem - II: Drop Collisions



Head-on collision resulting in reflexive separation



Off-center collision resulting in stretching separation

Supplementary Slides

Pre-conditioning LBM for Accelerating Convergence to Steady State

New Pre-conditioned LB MHD Model for Acceleration to Steady-State

Evolution equations

$$\bar{f}_\alpha(\boldsymbol{x}+\boldsymbol{e}_\alpha\delta_t, t+\delta_t) - \bar{f}_\alpha(\boldsymbol{x},t) = -\frac{1}{\tau}(\bar{f}_\alpha - f_\alpha^{eq}) + \left(1-\frac{1}{2\tau}\right)S_\alpha\delta_t \qquad S_\alpha = f_\alpha^{eq}\frac{(\boldsymbol{e}_\alpha - \boldsymbol{u})\cdot\boldsymbol{F}}{\rho c_s^2}\frac{1}{\gamma_f}$$

$$\boldsymbol{g}_\alpha(\boldsymbol{x}+\Xi_\alpha\delta_t, t+\delta_t) - \boldsymbol{g}_\alpha(\boldsymbol{x},t) = -\frac{1}{\tau_m}(\boldsymbol{g}_\alpha - \boldsymbol{g}_\alpha^{eq})$$

Equilibrium distribution functions

$$f_\alpha^{eq} = w_\alpha\rho\left[1+\frac{\boldsymbol{e}_\alpha\cdot\boldsymbol{u}}{c_s^2} + \frac{1}{\gamma_f}\left\{\frac{(\boldsymbol{e}_\alpha\cdot\boldsymbol{u})^2}{2c_s^4} - \frac{\boldsymbol{u}\cdot\boldsymbol{u}}{2c_s^2}\right\}\right]$$

$$\boldsymbol{g}_\alpha^{eq} = W_\alpha\left[\boldsymbol{B} + \frac{1}{\gamma_m}\frac{\Xi_\alpha\cdot\underline{\underline{\Lambda}}^{(0)}}{\theta}\right] \qquad \underline{\underline{\Lambda}}^{(0)} = \boldsymbol{uB} - \boldsymbol{Bu}$$

Pre-conditioning parameters:

$$0 < \gamma_f \le 1$$

$$0 < \gamma_m \le 1$$

Macroscopic Fields

$$\rho = \sum_{\alpha=0}^{b} f_\alpha \qquad \rho\boldsymbol{u} = \sum_{\alpha=0}^{b} f_\alpha\boldsymbol{e}_\alpha + \frac{1}{2}\frac{1}{\gamma_f}\boldsymbol{F}\delta_t \qquad p = \gamma_f\frac{c^2}{3}\rho \qquad \boldsymbol{B} = \sum_{\beta=0}^{b_m}\boldsymbol{g}_\beta$$

Transport coefficients

$$\nu = \frac{c^2}{3}\gamma_f\left(\tau_f - \frac{1}{2}\right)\delta_t \qquad \eta = \frac{c^2}{N_m}\gamma_m\left(\tau_m - \frac{1}{2}\right)\delta_t$$

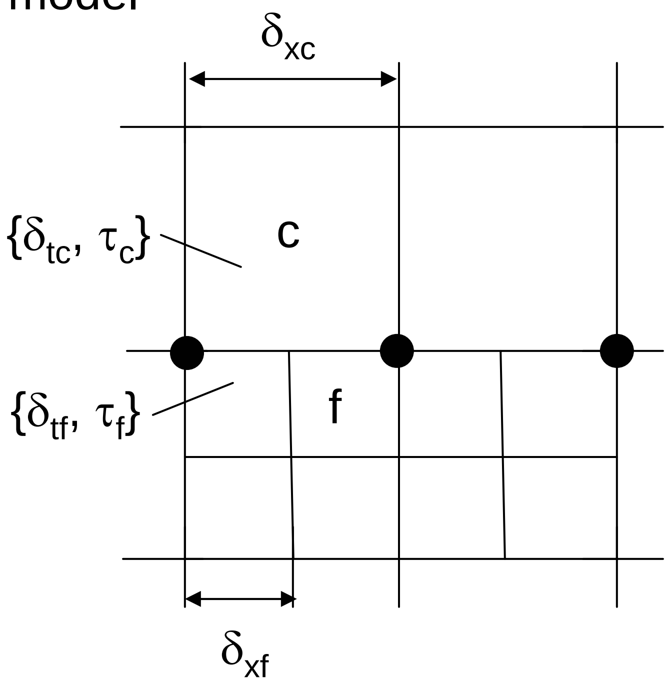
Local Grid Refinement Technique for LB MHD model

New Local Grid Refinement Schemes for LBM with Forcing Terms and SRT/MRT Models - I

LBE with forcing term with single relaxation time (SRT) model

$$\bar{f}_\alpha(\boldsymbol{x}+\boldsymbol{e}_\alpha\delta_t, t+\delta_t) - \bar{f}_\alpha(\boldsymbol{x},t) = -\frac{1}{\tau}(\bar{f}_\alpha - f_\alpha^{eq}) + \left(1-\frac{1}{2\tau}\right)S_\alpha\delta_t$$

where forcing term is given by $S_\alpha = f_\alpha^{eq}\dfrac{(\boldsymbol{e}_\alpha - \boldsymbol{u})\cdot\boldsymbol{F}}{\rho c_s^2}$



Grid refinement factors

$$m = \frac{\delta_{xc}}{\delta_{xf}} = \frac{\delta_{tc}}{\delta_{tf}} \qquad \tau_f = \frac{1}{2} + m\left(\tau_c - \frac{1}{2}\right) \qquad p = \frac{\tau_c - 1}{\tau_f - 1} \qquad q = \frac{\tau_c}{\tau_f}$$

Transformation Relations

$$\tilde{\bar{f}}_\alpha^{(c)} = f_\alpha^{(eq,f)} + mp\left(\tilde{\bar{f}}_\alpha^{(f)} - f_\alpha^{(eq,f)}\right) + \frac{1}{2}(1-p)S_\alpha\delta_{tc}$$

$$\tilde{\bar{f}}_\alpha^{(f)} = f_\alpha^{(eq,c)} + \frac{1}{m}p^{-1}\left(\tilde{\bar{f}}_\alpha^{(c)} - f_\alpha^{(eq,c)}\right) + \frac{1}{2}(1-p^{-1})S_\alpha\delta_{tf}$$

Here, tilde refers to post-collision value

$$\bar{f}_\alpha^{(c)} = f_\alpha^{(eq)} + mq\left(\bar{f}_\alpha^{(f)} - f_\alpha^{(eq)}\right) + \frac{1}{2}(q-1)S_\alpha\delta_{tc}$$

$$\bar{f}_\alpha^{(f)} = f_\alpha^{(eq)} + \frac{1}{m}q^{-1}\left(\bar{f}_\alpha^{(c)} - f_\alpha^{(eq)}\right) + \frac{1}{2}(q^{-1}-1)S_\alpha\delta_{tf}$$

Similar transformation relations can be developed for the vector distribution function representing magnetic induction

New Local Grid Refinement Schemes for LBM with Forcing Terms and SRT/MRT Models - II

LBE with forcing term with multi relaxation time (MRT) model

$$\bar{f}_\alpha(\boldsymbol{x}+\boldsymbol{e}_\alpha\delta_t, t+\delta_t) - \bar{f}_\alpha(\boldsymbol{x}, t) = -\Lambda_{\alpha\beta}(\bar{f}_\beta - f_\beta^{eq}) + \left(I_{\alpha\beta} - \frac{1}{2}\Lambda_{\alpha\beta}\right) S_\beta \delta_t$$

where forcing term is given by $S_\alpha = f_\alpha^{eq} \dfrac{(\boldsymbol{e}_\alpha - \boldsymbol{u})\cdot \boldsymbol{F}}{\rho c_s^2}$

$$\hat{\Lambda} = \boldsymbol{T}^{-1}\Lambda\boldsymbol{T} = diag(s_0, s_1, s_2, \ldots, s_8)$$

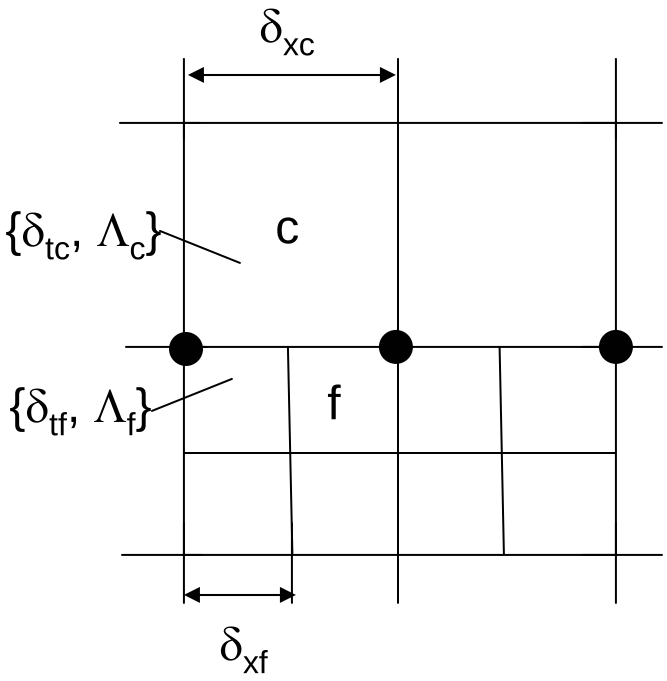
Grid refinement factors

$$m = \frac{\delta_{xc}}{\delta_{xf}} = \frac{\delta_{tc}}{\delta_{tf}}$$

$$\frac{1}{s_{i\,f}} = \frac{1}{2} + m\left(\frac{1}{s_{ic}} - \frac{1}{2}\right), i = 0, 1, 2, \ldots, b$$

$$P = \left(\Lambda_c^{-1} - I\right)\left(\Lambda_f^{-1} - I\right)^{-1}$$

$$Q = \Lambda_c^{-1}\Lambda_f$$



New Local Grid Refinement Schemes for LBM with Forcing Terms and SRT/MRT Models - III

LBE with forcing term with multi relaxation time (MRT) model (cont…)

Transformation Relations

$$\tilde{\bar{\boldsymbol{f}}}^{(c)} = \boldsymbol{f}^{(eq,f)} + mP\left(\tilde{\bar{\boldsymbol{f}}}^{(f)} - \boldsymbol{f}^{(eq,f)}\right) + \frac{1}{2}\left(I - P\right)\boldsymbol{S}\delta_{tc}$$

$$\tilde{\bar{\boldsymbol{f}}}^{(f)} = \boldsymbol{f}^{(eq,c)} + \frac{1}{m}P^{-1}\left(\tilde{\bar{\boldsymbol{f}}}^{(c)} - \boldsymbol{f}^{(eq,c)}\right) + \frac{1}{2}\left(I - P^{-1}\right)\boldsymbol{S}\delta_{tf}$$

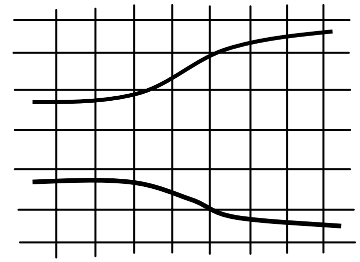
$$\bar{\boldsymbol{f}}^{(c)} = \boldsymbol{f}^{(eq)} + mQ\left(\bar{\boldsymbol{f}}^{(f)} - \boldsymbol{f}^{(eq)}\right) + \frac{1}{2}\left(Q - 1\right)\boldsymbol{S}\delta_{tc}$$

$$\bar{\boldsymbol{f}}^{(f)} = \boldsymbol{f}^{(eq)} + \frac{1}{m}Q^{-1}\left(\bar{\boldsymbol{f}}^{(c)} - \boldsymbol{f}^{(eq)}\right) + \frac{1}{2}\left(Q^{-1} - 1\right)\boldsymbol{S}\delta_{tf}$$

Here, tilde refers to post-collision value

Curved Boundary Treatment for MHD Flows using LBM

New Curved Boundary Treatment - I

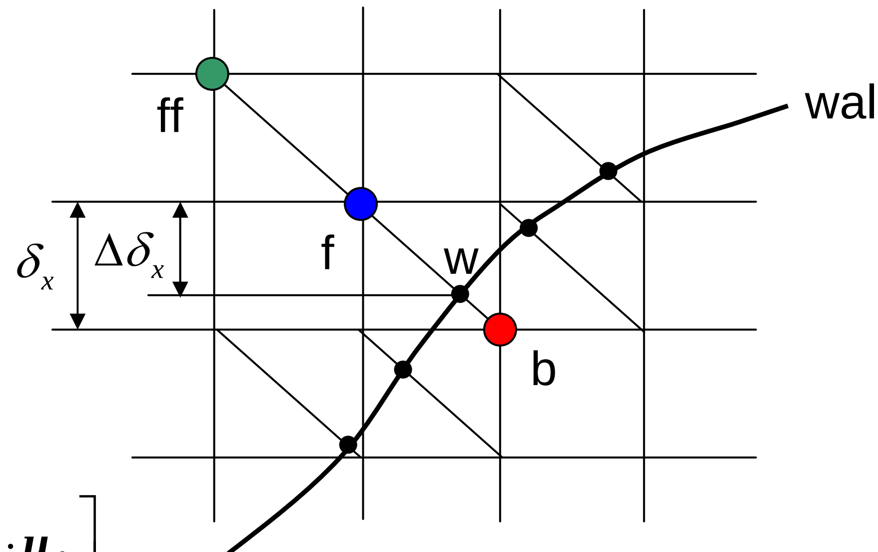


Scalar LBE with forcing term

$$\bar{f}_\alpha(\boldsymbol{x}+\boldsymbol{e}_\alpha\delta_t, t+\delta_t) = \tilde{\bar{f}}_\alpha(\boldsymbol{x},t) \equiv \bar{f}_\alpha(\boldsymbol{x},t) - \frac{1}{\tau}(\bar{f}_\alpha - f_\alpha^{eq}) + \left(1-\frac{1}{2\tau}\right)S_\alpha\delta_t$$

Here, tilde refers to post-collision value

Reconstructed distribution function from the solid side



$$\tilde{\bar{f}}_{\bar{\alpha}}(\boldsymbol{x}_b,t) \equiv (1-\chi)\tilde{\bar{f}}_\alpha(\boldsymbol{x}_f,t) + \chi \bar{f}_\alpha^{(*)}(\boldsymbol{x}_b,t) + \frac{2\rho w_\alpha}{c_s^2}\boldsymbol{e}_{\bar{\alpha}}\cdot\boldsymbol{u}_w + \left(\tau-\frac{1}{2}\right)\left[S_{\bar{\alpha}} - (1-\chi)S_\alpha\right]\Big|_{(\boldsymbol{x}_f,t)}\delta_t$$

where

$$\bar{f}_\alpha^{(*)}(\boldsymbol{x}_b,t) = w_\alpha\rho(\boldsymbol{x}_f,t)\left[1 + \frac{\boldsymbol{e}_\alpha\cdot\boldsymbol{u}_{bf}}{c_s^2} + \frac{(\boldsymbol{e}_\alpha\cdot\boldsymbol{u}_f)^2}{2c_s^4} - \frac{\boldsymbol{u}_f\cdot\boldsymbol{u}_f}{2c_s^2}\right]$$

$$\Delta = \frac{|\boldsymbol{x}_f - \boldsymbol{x}_w|}{|\boldsymbol{x}_f - \boldsymbol{x}_b|} \qquad \boldsymbol{e}_{\bar{\alpha}} = -\boldsymbol{e}_\alpha$$

$$\left.\begin{aligned} \boldsymbol{u}_{bf} &= \left(1-\frac{1}{\Delta}\right)\boldsymbol{u}_f + \frac{1}{\Delta}\boldsymbol{u}_w \\ \chi &= \frac{2\Delta-1}{\tau} \end{aligned}\right\}\Delta \geq \frac{1}{2} \qquad \left.\begin{aligned} \boldsymbol{u}_{bf} &= \boldsymbol{u}_{ff} \\ \chi &= \frac{2\Delta-1}{\tau-2} \end{aligned}\right\}\Delta < \frac{1}{2}$$

$$c_s = \frac{1}{\sqrt{3}}c$$

New Curved Boundary Treatment - II

Vector LBE

$$\boldsymbol{g}_\alpha(\boldsymbol{x}+\Xi_\alpha\delta_t, t+\delta_t) = \tilde{\boldsymbol{g}}_\alpha(\boldsymbol{x},t) \equiv \boldsymbol{g}_\alpha(\boldsymbol{x},t) - \frac{1}{\tau_m}(\boldsymbol{g}_\alpha - \boldsymbol{g}_\alpha^{eq})$$

Here, tilde refers to post-collision value

Reconstructed distribution function from the solid side

$$\tilde{\boldsymbol{g}}_{\bar{\alpha}}(\boldsymbol{x}_b,t) \equiv (1-\chi_m)\tilde{\boldsymbol{g}}_\alpha(\boldsymbol{x}_f,t) + \chi_m \boldsymbol{g}_\alpha^{(*)}(\boldsymbol{x}_b,t) + \frac{2W_\alpha}{\theta}\Xi_{\bar{\alpha}} \cdot \underline{\underline{\Lambda}}_w^{(0)} + W_\alpha \zeta_m \left[(\boldsymbol{B}_w - \boldsymbol{B}_f) + s(\boldsymbol{B}_{bf} - \boldsymbol{B}_f)\right]$$

where

$$\boldsymbol{g}_\alpha^{(*)}(\boldsymbol{x}_b,t) = W_\alpha \left[\boldsymbol{B}_{bf} + \frac{\Xi_\alpha \cdot \underline{\underline{\Lambda}}_{bf}^{(0)}}{\theta}\right] \qquad \underline{\underline{\Lambda}}^{(0)} = \boldsymbol{uB} - \boldsymbol{Bu}$$

$$\left.\begin{aligned}
\boldsymbol{B}_{bf} &= \left(1-\frac{1}{\Delta}\right)\boldsymbol{B}_f + \frac{1}{\Delta}\boldsymbol{B}_w \\
\underline{\underline{\Lambda}}_{bf}^{(0)} &= \left(1-\frac{1}{\Delta}\right)\underline{\underline{\Lambda}}_f^{(0)} + \frac{1}{\Delta}\underline{\underline{\Lambda}}_w^{(0)} \\
\chi_m &= \frac{2\Delta-1}{\tau_m} \\
\zeta_m &= \frac{\tau_m + (1-\chi_m)(\tau_m-1) - \chi_m}{\Delta+s}
\end{aligned}\right\} \Delta \geq \frac{1}{2}$$

$$\left.\begin{aligned}
\boldsymbol{B}_{bf} &= \boldsymbol{B}_{ff} \\
\underline{\underline{\Lambda}}_{bf}^{(0)} &= \underline{\underline{\Lambda}}_{ff}^{(0)} \\
\chi_m &= \frac{2\Delta-1}{\tau_m-2} \\
\zeta_m &= \frac{\tau_m + (1-\chi_m)(\tau_m-1) - \chi_m}{\Delta-s}
\end{aligned}\right\} \Delta < \frac{1}{2}$$

$s \neq 0$ is a free parameter

Sub Grid Scale (SGS) Turbulence Modeling for MHD Flows using LBM

Sub Grid Scale (SGS) Modeling of MHD Turbulent Flows For LES using LBM

Evolution equation of "coarse-grained" LBE

$$f_\alpha(\boldsymbol{x}+\boldsymbol{e}_\alpha\delta_t, t+\delta_t) - f_\alpha(\boldsymbol{x},t) = -\frac{1}{\tau}(f_\alpha - f_\alpha^{eq})$$

Total relaxation time $\tau = \tau_0 + \tau_t$

Laminar kinematic viscosity $$\nu_o = \frac{c^2}{3}\left(\tau_o - \frac{1}{2}\right)\delta_t$$

Smagorinsky SGS eddy viscosity $$\nu_{Smag} = \left(C_s\Delta\right)^2 \bar{S}, \quad \bar{S} = \sqrt{S_{ij}S_{ij}}, C_s \sim 0.09$$

Effective Eddy viscosity due to magnetic field $$\nu_{eddy} = \nu_{Smag} \times \exp\left[-\frac{\left|\boldsymbol{B}^a\right|^2}{\rho\eta\left(C_m\Delta\right)^2 \nu_{Smag}}\right], C_m \sim 0.2$$

Magnetic damping factor

(Shimomura, Phys. Fluids., 3: 3098 (1991))

Total kinematic "viscosity" $$\nu_{total} = \nu_0 + \nu_{eddy} = \frac{c^2}{3}\left(\tau - \frac{1}{2}\right)\delta_t$$