Kinetic Effects on MHD Modes in NSTX

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Outline

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- MHD Model: Advantage and Limitation
- Characteristic Scales in NSTX Plasmas
- Kinetic Stabilization of Ballooning Modes
- Destabilization of TAEs by Energetic Particles
- Kinetic-Fluid Model
- Summary

Motivation

- Disparate scales in plasmas: traditionally global-scale phenomena are studied using MHD or multi-fluid models, while small-scale phenomena are described by kinetic theories.
- Multiscale coupling: small-scale particle kinetic physics couples with large-scale MHD phenomena.
- Energetic particle physics: global instabilities (TAE and fishbone modes) are driven by fast ions via wave-particle interaction and can cause serious fast ion loss in toroidal plasmas.
- Kinetic-MHD model and simulation codes: through joint theory-experimental studies, understanding of energetic particle physics phenomena in major tokamak experiments has improved.
- Thermal particle kinetic physics: thermal particle kinetic effects are as important as fast ions in determining global phenomena in burning plasmas – e.g., kinetic ballooning modes, TAEs,

➔ Need to include kinetic effects of both fast and thermal particles in studying MHD phenomena in NSTX.

Global Phenomena: MHD Model

- Momentum Equation:

$$\rho\,[\partial/\partial t + \mathbf{V}\cdot\nabla]\,\mathbf{V} = -\nabla P + \mathbf{J}\times\mathbf{B}$$

- Continuity Equation:

$$[\partial/\partial t + \mathbf{V}\cdot\nabla]\,\rho + \rho\nabla\cdot\mathbf{V} = 0$$

- Maxwell's Equations:

$$\partial B/\partial t = -\nabla\times\mathbf{E}, \quad \mathbf{J} = \nabla\times\mathbf{B}, \quad \nabla\cdot\mathbf{B} = 0$$

- Ohm's Law: $\mathbf{E} + \mathbf{V}\times\mathbf{B} = \eta\mathbf{J}$
- Adiabatic Pressure Law: $[\partial/\partial t + \mathbf{V}\cdot\nabla]\,(P/\rho^{5/3}) = 0$

➔ important to understand advantages and limitations of MHD model.

Advantages of MHD Model

- Retains properly global geometrical effects such as pressure gradient, magnetic field gradient & curvature.

- Covers most long wavelength, low-frequency EM waves and instabilities:

  -- Fast Magnetosonic branch ($\omega \simeq kV_A$): compressional Alfven waves (CAEs), mirror modes, etc.

  -- Shear Alfven branch ($\omega = k_{||}V_A$): shear Alfven waves (TAEs, GAEs), ballooning modes, kink modes, tearing modes, etc.

  -- slow magnetosonic modes ($\omega = k_{||}C_s$)

- Simpler to perform theoretical analysis and simulations than gyrokinetic/Vlasov models.

Limitations of MHD Model

- Ohm's law: for small resistivity, plasma fluid is almost frozen in **B** and moves perpendicularly with **E**×**B** velocity, and parallel electric field is negligible except in resistive boundary layer.
- Pressure law: pressure changes adiabatically via **E**×**B** convection and plasma compression.
- Gyroviscosity are ignored.

➔ No particle kinetic physics!

Characteristic Scales in NSTX Plasmas

- Characteristic scales of particle dynamics and low-frequency (ω, $\mathbf{k}$) perturbations:

  For $B = 1$ T, $T_{e,i} \sim 1$ keV, $\varepsilon_h \sim 100$ keV, L_B, $L_p \sim 0.5$ m, then we have $\rho_i \simeq 0.3$ cm, $v_i \simeq 10^8$ cm/s, $\omega_{ci} \simeq 10^8$ sec^{-1}, $\omega_{bi} \simeq 10^6$ sec^{-1}, $\omega_{di} \simeq 10^5$ sec^{-1}

  -- temporal scale ordering:

  $$\omega_{ci} \sim \omega_{be} \geq \omega_{bh} > \omega_{bi} \sim \omega_{*i,e} \geq \omega_{di,e}$$

  -- spatial scale ordering:

  $$\Delta_{bh} > \Delta_{bi} > \rho_h > \rho_i \sim c/\omega_{pi} > \rho_e$$

- To describe low-frequency (ω, $\mathbf{k}$) phenomena, MHD model is a good approximation only if (a) $\omega_{ci} >> \omega >> \omega_t, \omega_b, \omega_d$ and (b) $k_\perp \rho_i << 1$ are satisfied for all particle species that have significant contributions in density or momentum or pressure.

Kinetic Effects on Ballooning Modes

- Consider finite $\delta E_{||}$ due to kinetic effects of finite ion gyroradii and trapped electron dynamics.
- A finite $\delta E_{||}$ enhances $\delta J_{||}$ which provides a strong stabilizing field line bending effect.
- Particle kinetic effects increase (decrease) the first (second) critical β_C for ballooning instability over the MHD prediction.

Kinetic Ballooning Mode Equations

Consider finite $dE_{\|}$ due to kinetic effects of finite ion gyroradii and trapped electron dynamics, and the approximate KBM equation is

$$\vec{B}\bullet\nabla\left(\frac{k_\perp^2}{B^2}\vec{B}\bullet\nabla S_c\Phi\right)+\omega(\omega-\omega_{*pi})\frac{k_\perp^2}{V_A^2}\Phi+\beta\left(\frac{\vec{B}\times\vec{\kappa}\bullet\vec{k}}{B}\right)\left(\frac{\vec{B}\times\nabla P\bullet\vec{k}}{BP}\right)\Phi\approx 0$$

$$S_c\equiv 1-\Psi/\Phi\approx 1+\frac{n_e}{n_{eu}}\left[\frac{\beta_e}{2}\left(\frac{\omega_{*pi}-\omega_{*pe}}{\omega}\right)^2-\frac{q_iT_e}{q_eT_i}\left(\frac{\omega-\omega_{*pi}}{\omega-\omega_{*e}}\right)b_i\right.$$
$$\left.-\frac{3}{2}\left(\frac{\omega-\omega_{*pe}}{\omega-\omega_{*e}}\right)\frac{\langle\hat{\omega}_{de}\rangle}{\omega}+\left(\frac{\omega-\omega_{*pi}}{\omega-\omega_{*e}}\right)\frac{\hat{\omega}_{Be}+\hat{\omega}_{\kappa e}}{2\omega}\right]$$

$$\delta E_{\|}=-\nabla_{\|}\Psi,\quad \delta J_{\|}=\frac{i}{\omega}\nabla_\perp^2\nabla_{\|}(\Phi-\Psi),\qquad b_i=\frac{k_\perp^2T_i}{m_i\omega_{ci}^2}$$

Stabilizing Kinetic Effects on Ballooning Mode

$$(V_e > \omega / k_{\|} > V_i)$$

- For a given electric field perturbation electrons move across **B** differently from ions due to finite ion gyroradius effect and charge separation is created.
- Ions move much slower than the wave phase velocity along **B** and is essentially quasi-static.
- Electrons move much faster than the wave phase velocity along **B** and will play the role of keeping charge quasi-neutral.
- Trapped electrons do not contribute much to charge redistribution due to fast bounce motion.
- Untrapped electrons play the dominant role of maintaining charge quasi-neutrality.
- Untrapped electron density is much smaller than trapped electron’s and thus an enhanced parallel electric field is created to move the untrapped electrons to maintain charge quasi-neutrality.
- Enhanced parallel electric field produces enhanced parallel current and thus enhanced field line tension, which stabilizes ballooning modes.

KBM Stability

Local dispersion relation

$$\frac{\omega(\omega-\omega_{*pi})}{V_A^2}+\beta\left(\frac{\vec{B}\times\vec{\kappa}\bullet\vec{k}_\perp}{k_\perp B}\right)\left(\frac{\vec{B}\times\nabla P\bullet\vec{k}_\perp}{k_\perp BP}\right)\approx S_c k_\parallel^2$$

At marginal stability $\omega \simeq \omega_{*pi}$ and critical β is given by

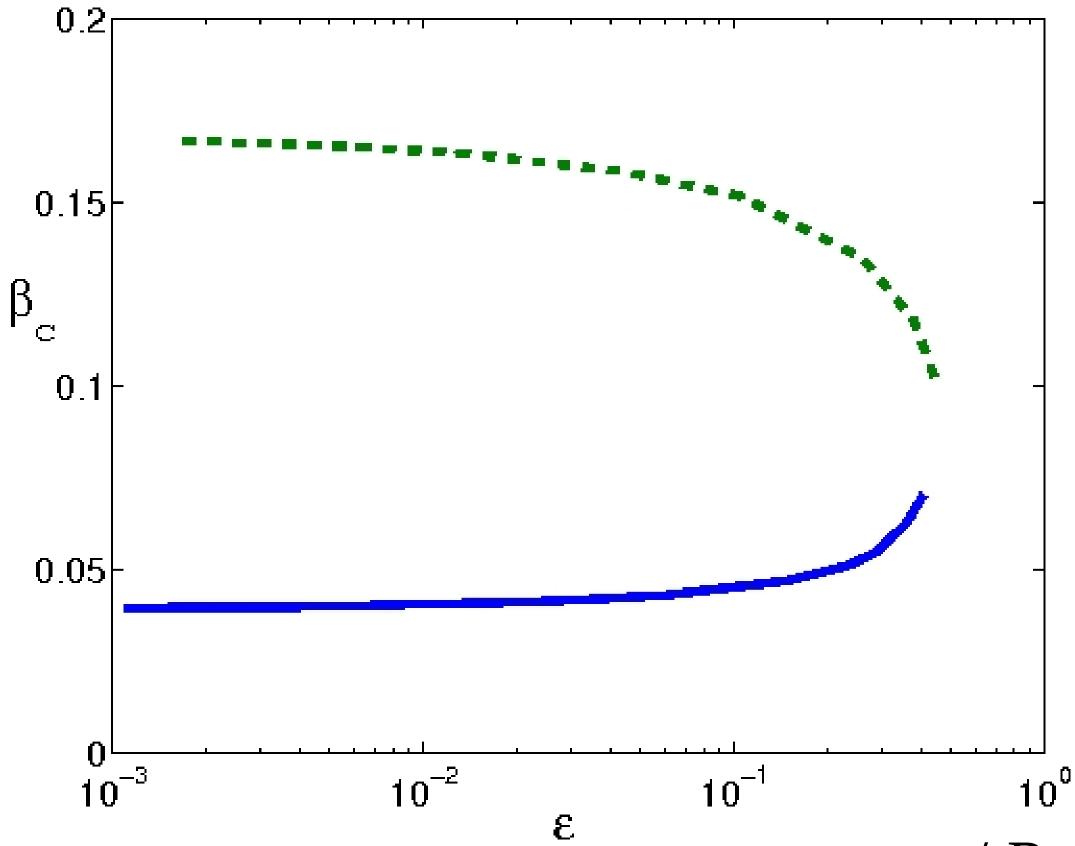
$$\beta_c \approx S_c\beta_c^{MHD}-\frac{\omega(\omega-\omega_{*pi})R_c L_p}{V_A^2}$$

For low aspect ratio toroidal plasmas

$$\frac{n_{eu}}{n_e}\approx\frac{(R-r)}{2(R+r)} << 1$$

$$For \quad R/r=1.5, \qquad \frac{n_{eu}}{n_e}\approx 0.1, \qquad S_c >> 1 \ \text{ and } \quad \beta_c >> \beta_c^{MHD}$$

Trapped Electron Stabilization of KBMs in Large Aspect Ratio Tokamaks

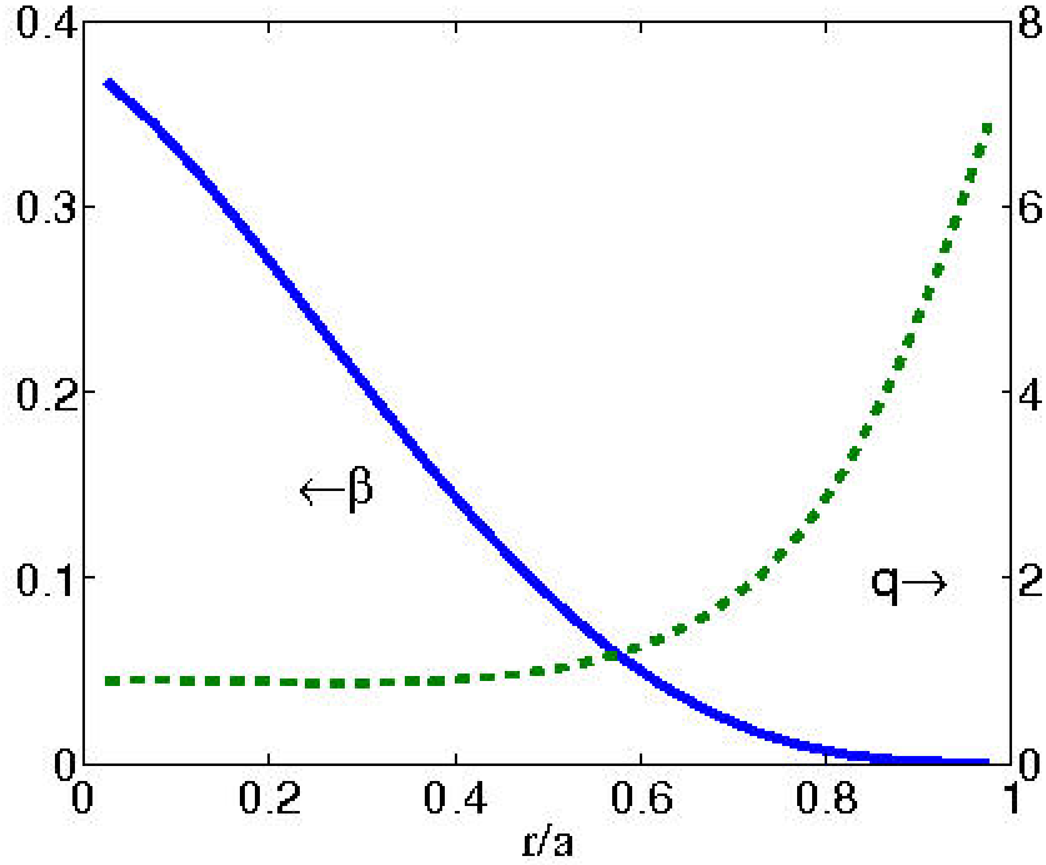


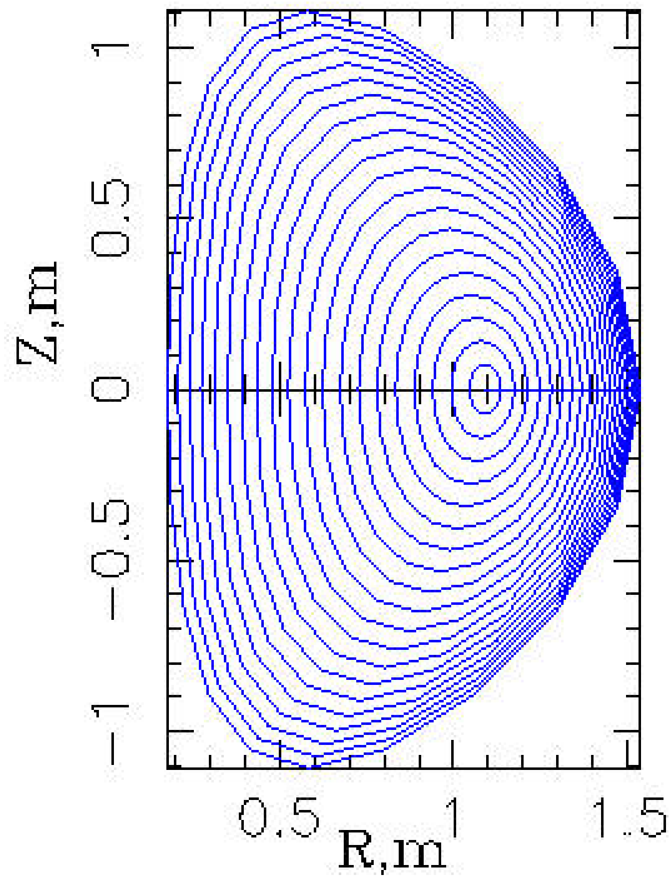
$$\varepsilon = r / R$$

$$b_\theta = k_\theta^2 \rho_i^2 / 2 = 0.1, \quad L_n / R = 0.1,$$

$$\hat{s} = 0.5, \quad q = 1, \quad T_e = T_i, \quad \eta_e = \eta_i = 0$$

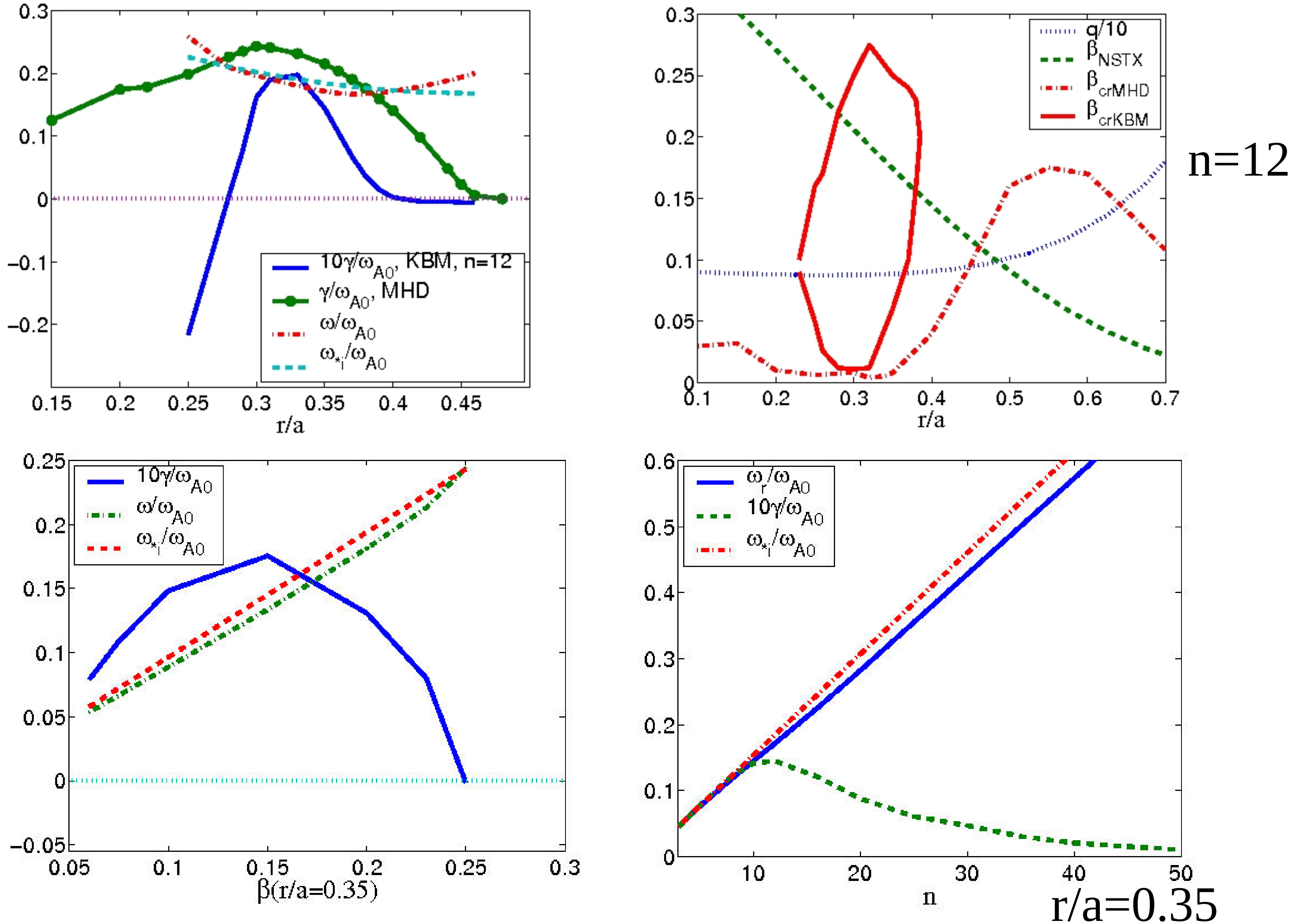
Small Aspect RatioTorus: NSTX





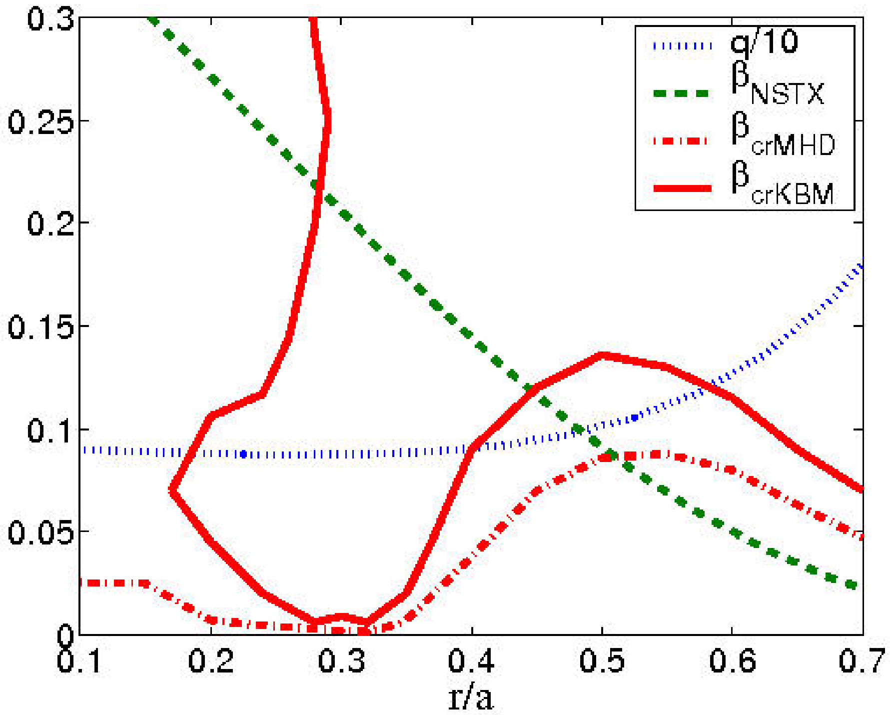
R/a = 1.27, a = 0.68 m, κ = 1.63, δ = 0.417, $T_e = T_i$ = 1 keV, $\langle\beta\rangle$ = 9%, q_{min} = 0.93 at r/a = 0.3

KBM Stability in NSTX (η_i=η_e=0)

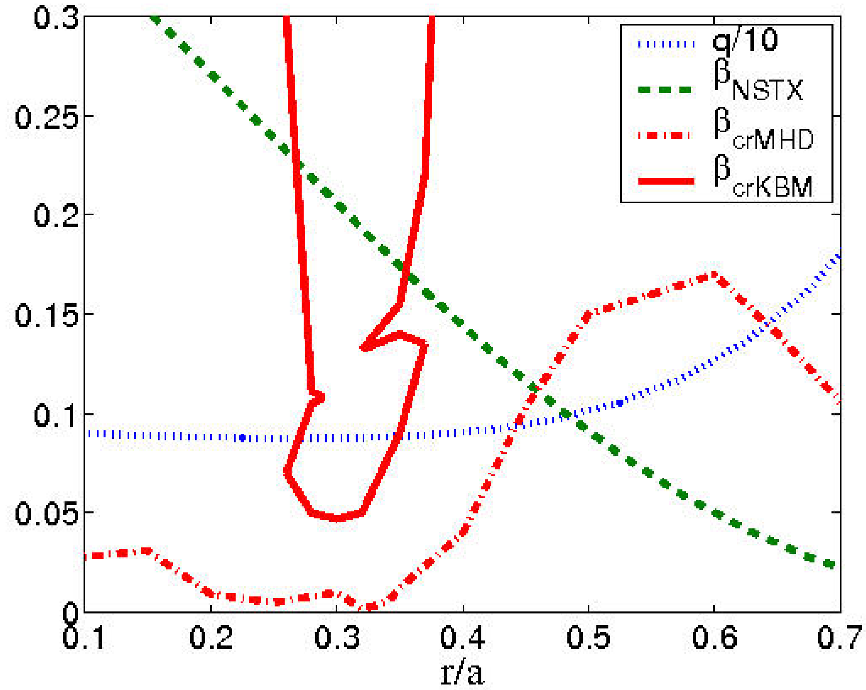


Stability of KBMs in NSTX

Aspect ratio effect:
R/a=1.67, $\eta_i=\eta_e=\mathbf{0}$



Temperature gradient effect:
R/a=1.27, $\eta_i=\eta_e=\mathbf{1}$



Prediction of TAEs Based on MHD Model

- TAEs are discrete toroidal Alfven eigenmodes due to nonuniform q-profile and nonuniform magnetic field intensity along **B**.

High-n TAE equation

$$\left[\frac{d^2}{d\theta^2} + \left(\frac{\omega}{\omega_A}\right)^2 (1 - 2\epsilon\cos\theta) - \frac{s^2}{\left(1 + s^2\theta^2\right)^2}\right]\Phi = 0$$

$\varepsilon = r/R$, $s = rq'/q$, $\omega_A = V_A/qR$

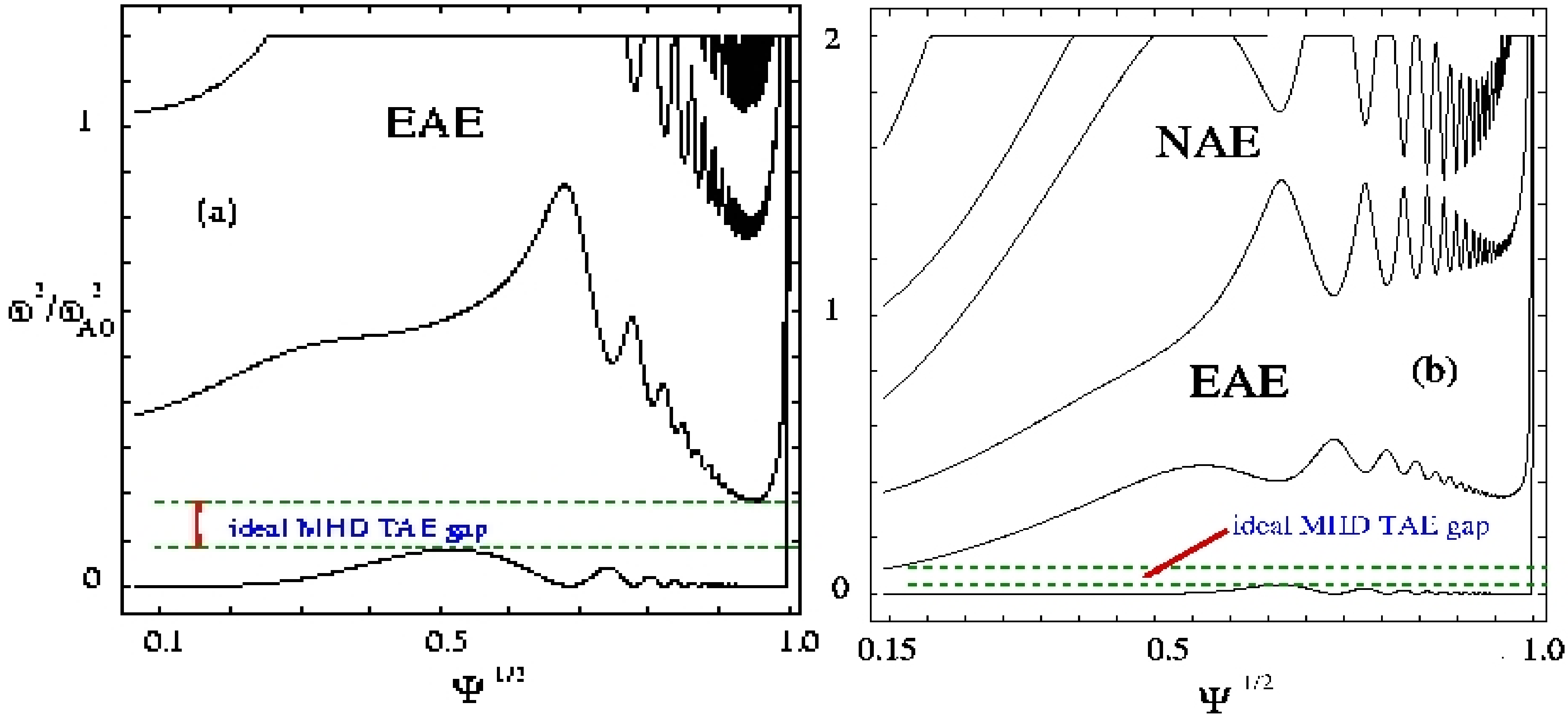
- Coupling between neighboring poloidal harmonics produces Alfven continuous spectrum gap bounded by $\omega_\pm{}^2 \simeq (1 \pm \varepsilon)\, \omega_A{}^2/4$
- Magnetic shear allows discrete TAEs with frequencies in the gap:

$\omega^2 \to \omega_-{}^2$ as $s \to 0$; $\omega^2 \to \omega_+{}^2$ as $s \to \infty$

- TAEs exist because the periodicity in the wave potential is broken by magnetic shear – similar to discrete energy states in a periodic lattice due to periodicity breaking by impurity in solid state physics.

NOVA: Alfvén Continuum (n = 3) and TAEs in NSTX

$\langle\beta\rangle = 10\ \%$ $(q_0 = 0.7,\ q_1 = 16)$ $\langle\beta\rangle = 33\ \%$



- Large continuum gaps due to low aspect ratio
- Many TAEs with different n's
- TAEs can be driven unstable by fast ions if $nq(V_h/V_A) \leq rL_h/R\rho_h$

Kinetic-MHD Model: Energetic Particle Physics

- Two-component plasma: core and hot components with $n_h \ll n_c$, $n \simeq n_c$, $P_c \sim P_h$
- Core plasmas are treated as MHD-fluid
- Hot particles are governed by kinetic models such as gyrokinetic equations or full Vlasov equations
- Coupling between core plasmas and hot particles is via pressure (or current) term in momentum equation
- No parallel electric field

Kinetic-MHD Model

- Momentum Equation:

$$\rho\,[\partial/\partial t + \mathbf{V}\cdot\nabla]\,\mathbf{V} = -\nabla P_c - \nabla\cdot\mathbf{P}_h + J \times B$$

- Continuity Equation:

$$[\partial/\partial t + \mathbf{V}\cdot\nabla]\,\rho + \rho\nabla\cdot\mathbf{V} = 0$$

- Maxwell's Equations:

$$\partial B/\partial t = -\nabla\times\mathbf{E}, \quad J = \nabla\times\mathbf{B}, \quad \nabla\cdot\mathbf{B} = 0$$

- Ohm's Law: $\mathbf{E} + \mathbf{V}\times\mathbf{B} = 0$, $\mathbf{E}\cdot\mathbf{B} = 0$
- Adiabatic Pressure Law: $[\partial/\partial t + \mathbf{V}\cdot\nabla]\,(P_c/\rho^{5/3}) = 0$
- Hot Particle Pressure Tensor:

$$\mathbf{P}_h = \{m_h/2\} \int d^3v\, \mathbf{vv}\, f_h(\mathbf{x},\mathbf{v})$$

where f_h is governed by gyrokinetic or Vlasov equation.

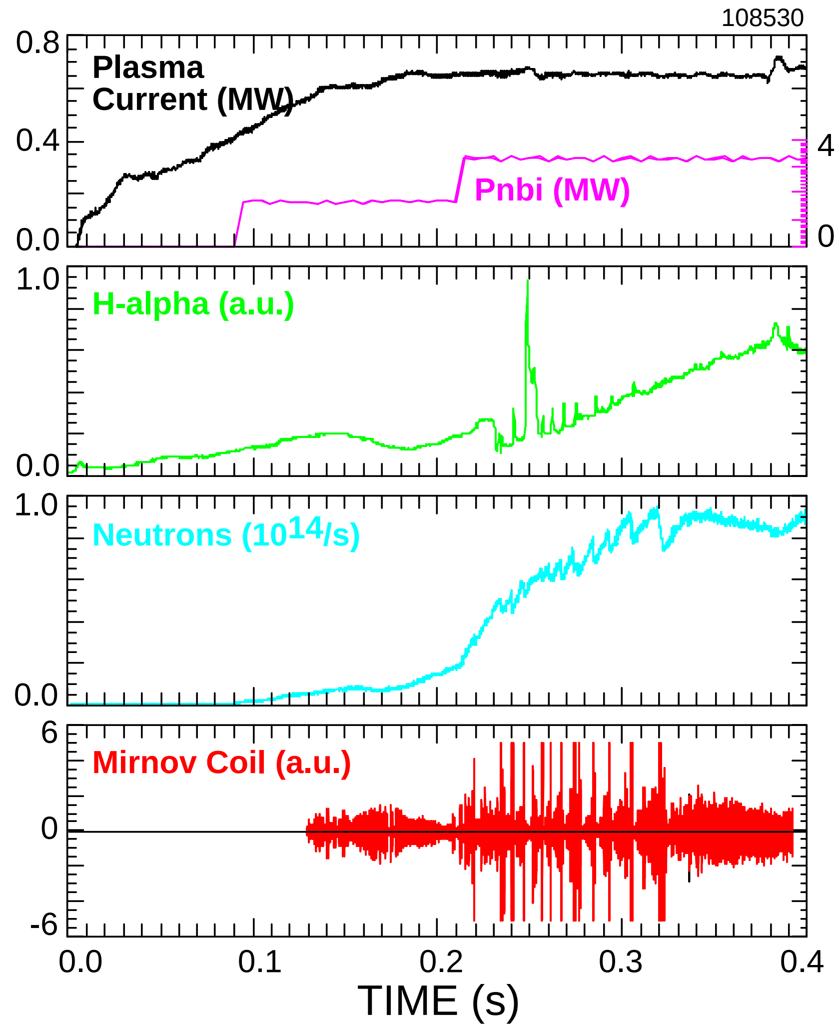
PPPL Kinetic-MHD Codes

- Linear Stability Codes
  - -- NOVA-K: global TAE stability code with perturbative treatment of thermal particle and fast ion kinetic physics
  - -- NOVA-2: global kinetic-MHD code with non-perturbative treatment of fast ion kinetic effects
  - -- HINST: high-n kinetic-MHD code with non-perturbative treatment of fast ion kinetic effects
- Nonlinear Simulation Codes
  - -- M3D-K: global kinetic-MHD code including fast ion kinetic physics which is determined by gyrokinetic equation.
  - -- HYM-1: global kinetic-MHD code with fast ion kinetic physics determined by full Vlasov equation of motion.
  - -- HYM-2: global hybrid code with ions treated by full equation of motion and electrons treated as massless fluid.

➔ Through joint theory-experiment efforts, we have gained understanding of energetic particle physics phenomena in major tokamak experiments.

Fast ions excite large amplitude bursting TAEs, which cause fast ion loss in NSTX

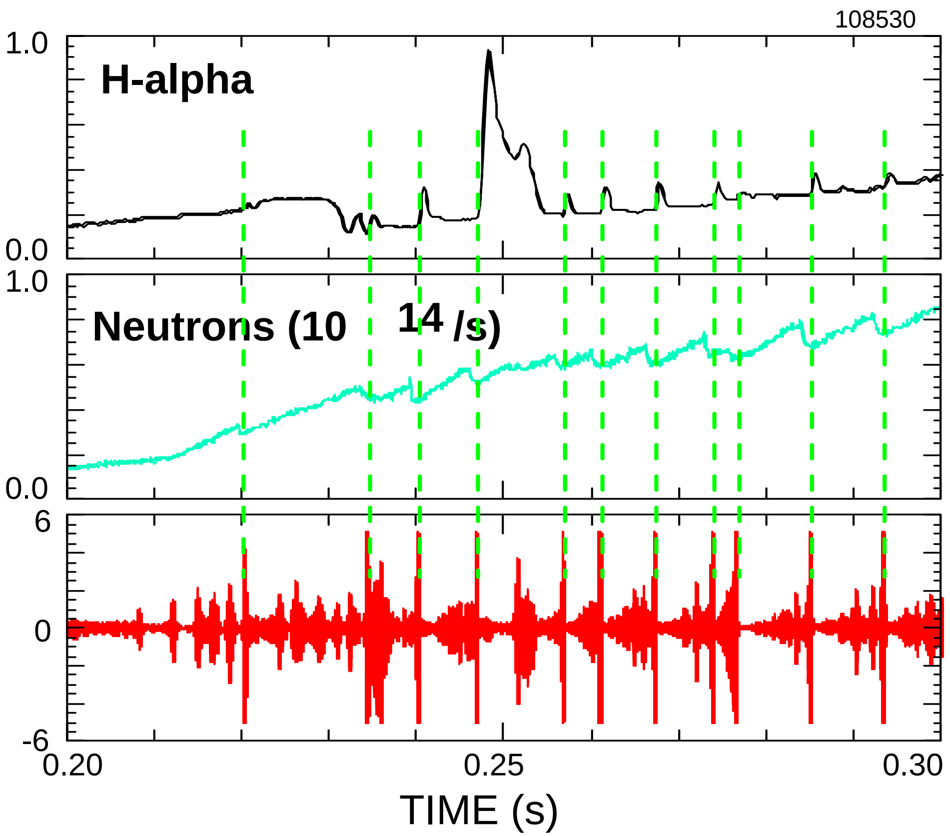
- Fast neutron drops correlated with H-alpha bursts; fast ions hitting wall?
- Small impact on soft x-ray emission.

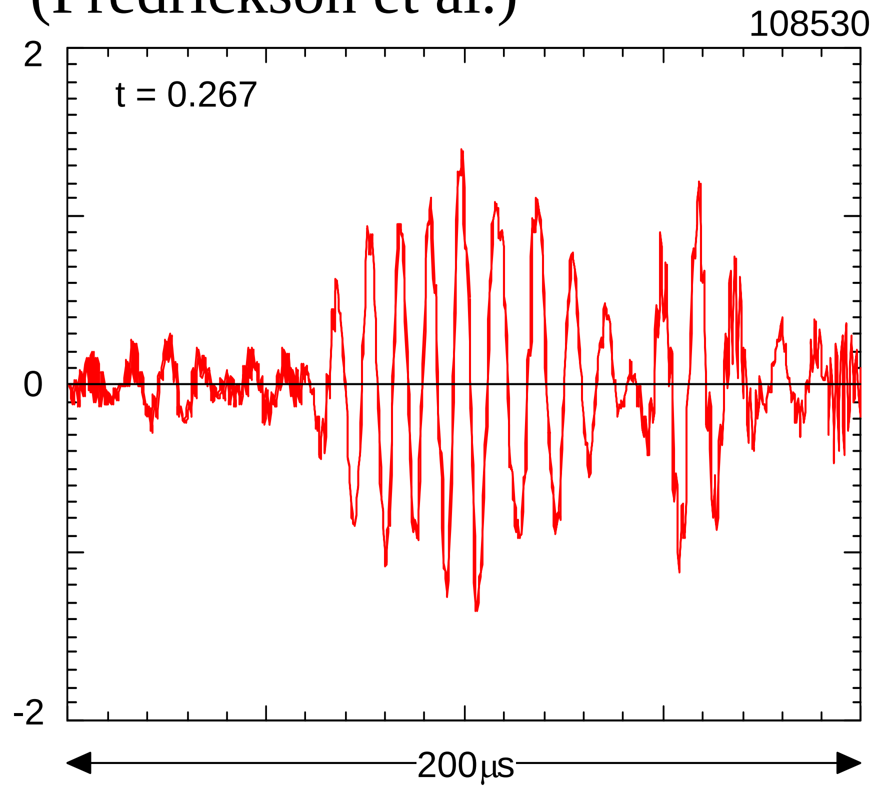


(Fredrickson et al.)

Bursting TAEs in NSTX NBI Experiments

(Fredrickson et al.)

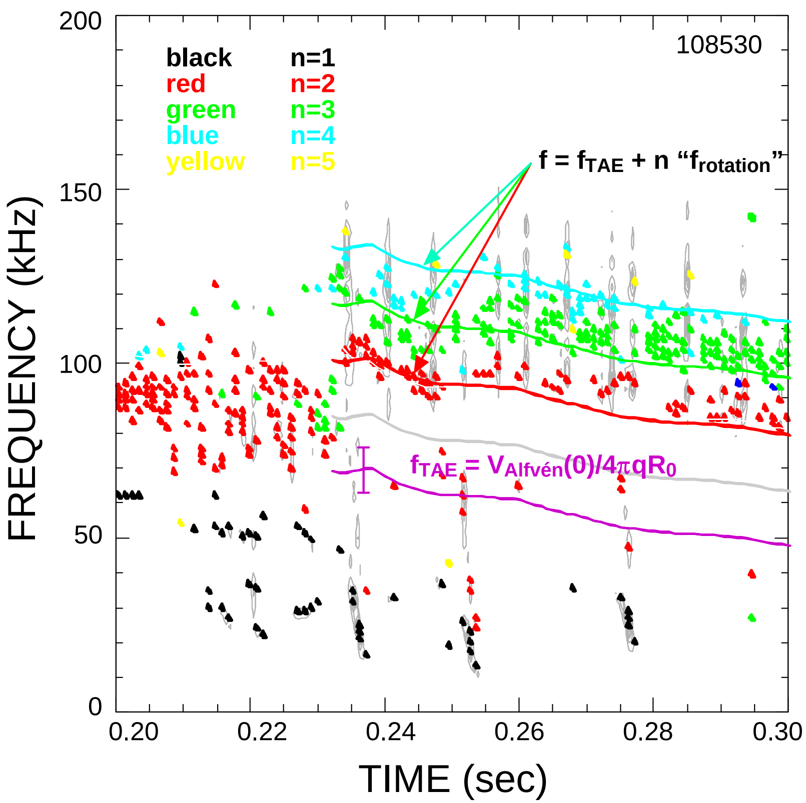




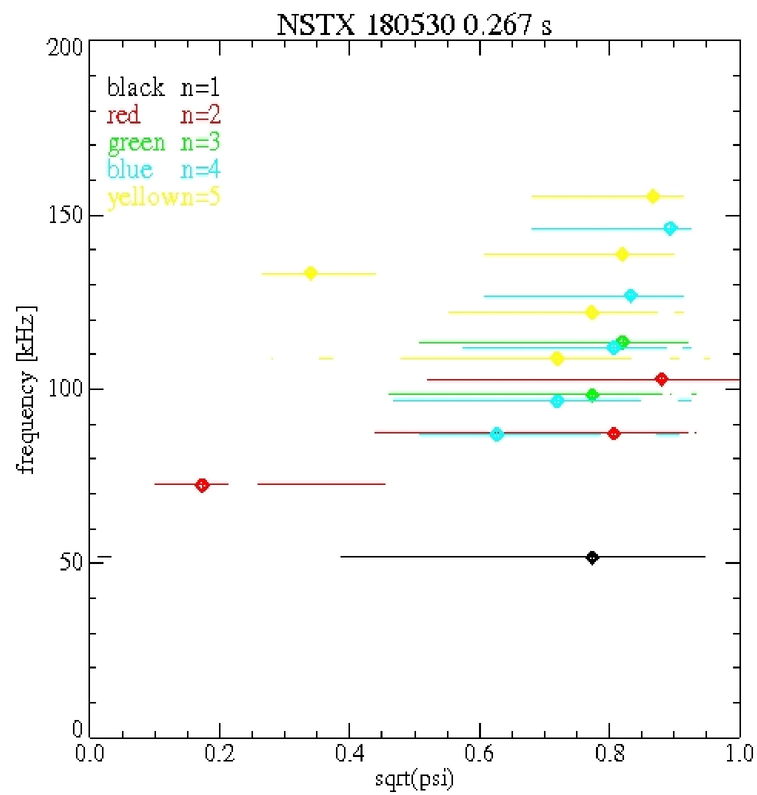
- NSTX shot with B = 0.434T, R = 87 cm, a = 63cm, P_{NB} = 3.2MW.
- Single dominant mode being n=2 or 3, mode amplitude modulation represents "beating" of multiple modes.
- Bursting TAEs lead to neutron drop and cause 5 – 10% fast ion loss.

NOVA-K Study of TAEs in NSTX

NSTX Results

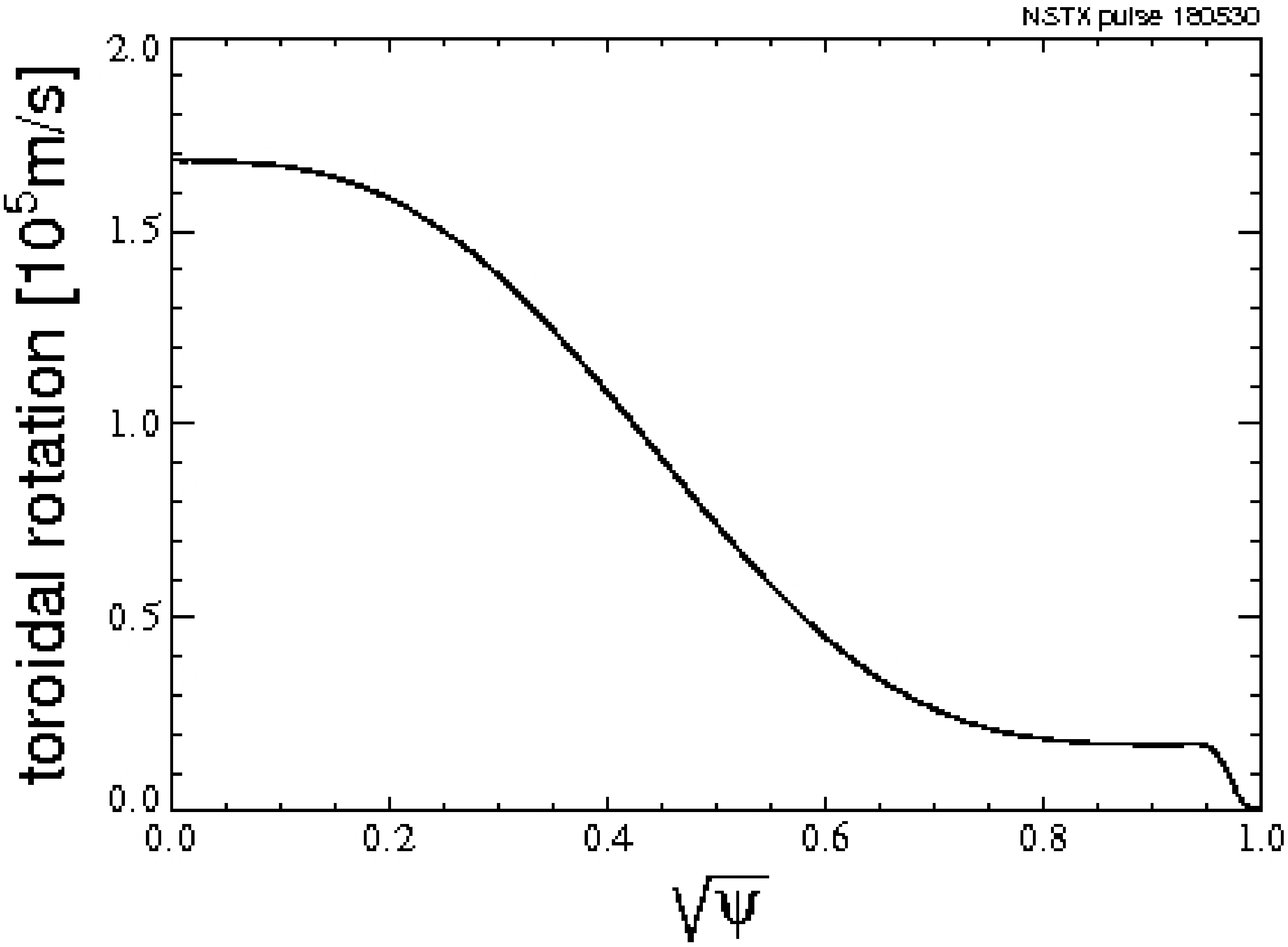


NOVA-K Results



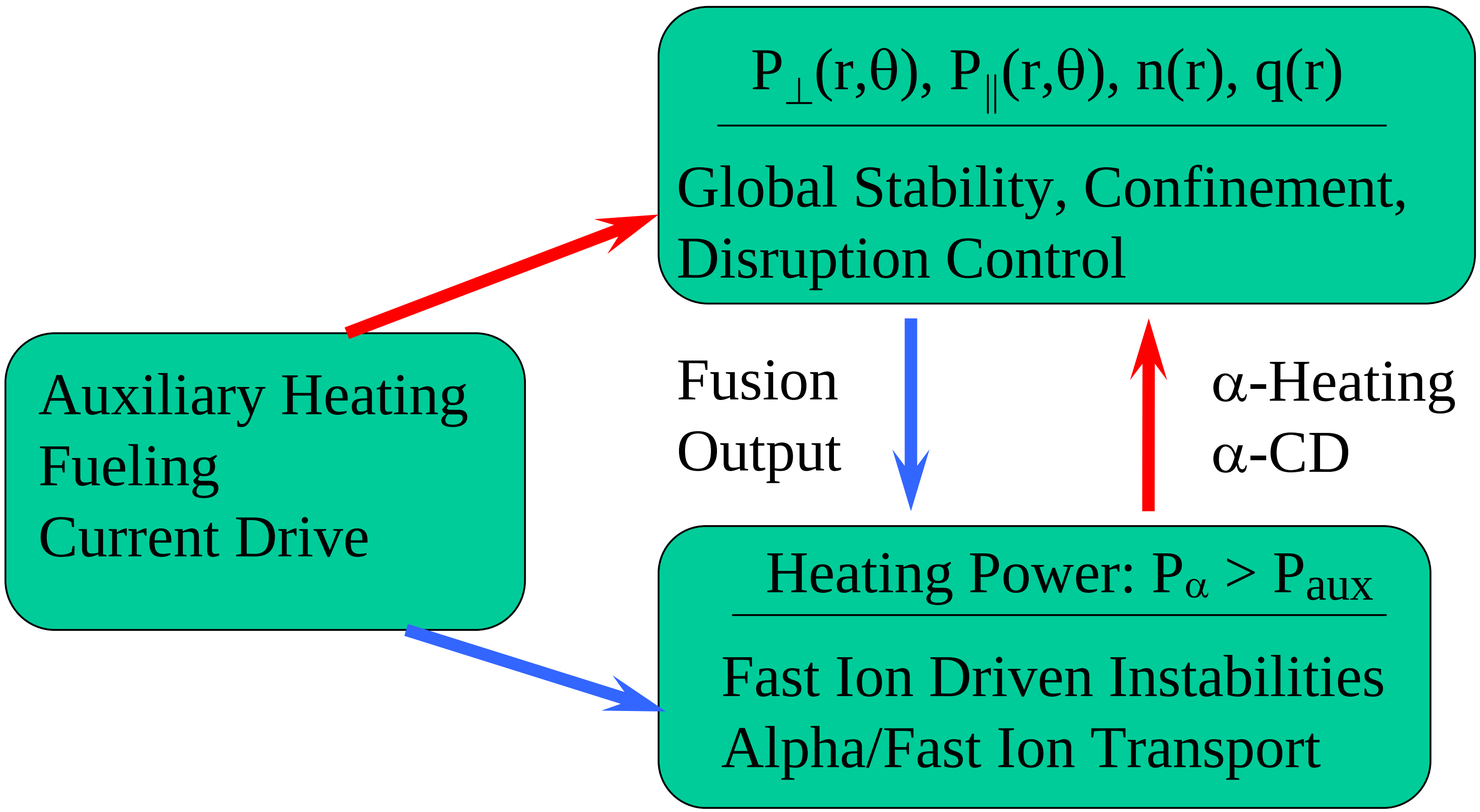
Plasma rotation is significant in determining TAE frequencies.

Plasma Rotation in NSTX



Integrated Modeling of Burning Plasmas

α interaction with thermal plasmas is a strongly nonlinear process.



Must develop efficient methods to control profiles for burn control!

➔ Need nonlinear kinetic-fluid simulation codes!

Limitations of Kinetic-MHD Model

- Negligible fast particle density
- $E_{||} = 0$
- No thermal particle kinetic effects (except in the perturbative version of linear NOVA-K code)

➔ Need a hybrid kinetic-fluid model that treats kinetic physics of both thermal and fast particles and retains single-fluid frame work to model burning plasma dynamics.

Thermal Particle Kinetic Effects

- Finite ion Larmor radius ($k_\perp \rho_i \sim O(1)$)
- Finite banana orbit width ($\Delta_b \sim k_\perp^{-1}$)
- Trapped particles
- Wave-particle resonances

➔

- finite parallel electric field $E_\| \neq 0$
- wave damping or drive
- Kinetic Alfven waves, KTAEs
- Radiation damping of TAEs and KTAEs
- stochastic ion heating by large amplitude Alfven waves
- boundary layer physics in kink and tearing modes
- current layer physics in magnetic reconnection
- stabilization of ballooning mode, kink modes

Kinetic-Fluid Model

- High-β multi-ion species plasmas
- Ordering: $\omega < \omega_{ci}$, $k_\perp \rho_i \sim O(1)$
- No ordering on n_h , n_c , P_c , P_h
- Single-fluid equations consisting of mass density and momentum equations, and generalized Ohm's law
- Closure of single-fluid equations by determining pressure tensor (including gyroviscosity) from particle distributions
- Particle dynamics governed by kinetic models such as gyrokinetic equations or full Vlasov equations
- Finite parallel electric field

[Cheng & Johnson, JGR, 1999]

Summary

- Kinetic effects are significant for MHD modes, e.g.,
  -- kinetic stabilization of ballooning modes by trapped electron dynamics and ion FLR
  -- destabilization of TAEs by fast ions
- Kinetic-MHD codes (linear codes: NOVA-K, NOVA-2, HINST; nonlinear codes: M3D-K, HYM) have been developed to study fast ion physics.
- A low frequency ($\omega < \omega_{ci}$) nonlinear kinetic-fluid model has been developed to include coupling between global modes and kinetic physics of both thermal and fast particles.
- Physics of wave-particle interaction and global geometrical effects are properly included in the kinetic-fluid model.
- Extension of kinetic-MHD codes to include thermal particle kinetic effects will be developed based on kinetic-fluid model.