# Equilibrium Reconstruction of Detailed Current Density Profile Structure from External and Internal Magnetic Measurements in VEST

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## Abstract

A detailed current density profile structure is obtained in VEST by full equilibrium reconstruction from external and internal magnetic measurements. Vertical component of the magnetic field is measured by magnetic Hall sensors from inside the plasma, in addition to the typical external magnetic diagnostics. Grad-Shafranov equation solver TEFIT is modified to incorporate internal magnetic, external magnetic and diamagnetic measurements. The reconstructed equilibrium parameters are compared to the discharge characteristics of VEST to check the consistency of the result. It is shown that with the internal magnetic probe data, the uncertainty of the current density profile by equilibrium reconstruction is reduced. From the reconstructed current density data, the transition from hollow to peaked profile is observed, which is a typical phenomena during a current ramp up in a tokamak. The current penetration phenomena will be discussed in more detail with the established equilibrium reconstruction procedure including the internal magnetic probe data in addition to the typical external magnetic diagnostics.

## Grad-Shafranov Equation

- Grad-Shafranov Equation: calculate the axisymmetric 2D current density distribution based on the MHD equilibrium assumption

$$\nabla p = \boldsymbol{J}\times\boldsymbol{B} \quad \nabla\times\boldsymbol{B} = \mu_0\boldsymbol{J} \quad \nabla\cdot\boldsymbol{B} = 0$$

$$\downarrow$$

$$\underbrace{\nabla\cdot(r^{-2}\nabla\psi)}_{B_p} = \underbrace{\mu_0 p'(\psi)}_{\nabla p} + \underbrace{\frac{F(\psi)F'(\psi)}{r^2}}_{B_\phi}$$

or, $\Delta^*\psi = -\mu_0 r J_\phi(\psi)$

**Nonlinear partial differential equation →** requires boundary condition and solved numerically (by SOR or MGM)

- Boundary condition is given by the Green's function solution of the Ampere's law

$$\Delta^*\psi_{bndry} = -\mu_0 r \underbrace{J_\phi(r,z)}_{J_{coil}+J^k_{plasma}}$$

- Current density is updated at by least square method: The response matrix is updated at each iteration step

- Green's function for solving Ampere's law

$$\Delta^*\psi_{bndry} = -\mu_0 r J_\phi(r,z)$$

$$\Delta^*\psi = -\mu_0 r J_\phi(\psi)$$

- Iteration 1: SOR for solving Grad-Shafranov equation

- Iteration 2: fixed-point iteration or Picard scheme

Initialize grid → Initialize $\psi_k$ → Calculate $J_k(\psi_k)$ → Calculate $\psi_{bndry_k}$ → Solve for $\psi_{k+1}$ → Check convergence $(\psi_{k+1}-\psi_k)/\psi_k$ → Post-process

Singular Value Decomposition

Update Response Matrix (Plasma Part)

- A method to decompose a matrix – applied to optimization problems
- A vector x that minimizes || A·x – b || is the component of V corresponding to the smallest diagonal entry of Σ, where A = UΣV* (factorized) by singular value decomposition

Vector x = $[\alpha_i\ \gamma_j\ I_{c_k}]$

where $p' = \sum_{i=0}^{N_P}\alpha_i\psi^i - \psi^{N_P+1}\sum_{i=0}^{N_P}\alpha_i$ and $F^{2\prime} = \sum_{j=0}^{N_F}\gamma_j\psi^j - \psi^{N_F+1}\sum_{j=0}^{N_F}\gamma_j$

and $I_{c_k}$ is the toroidal current in PF coils and wall segments

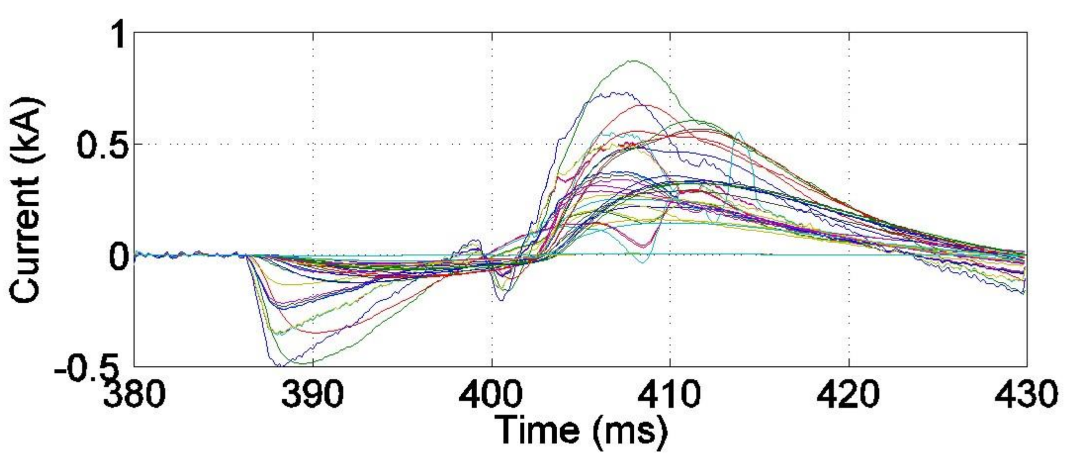
Vector b = $[M_l/\sigma_l]$ where $M_l$ is measured or given value and $\sigma_l$ is uncertainty

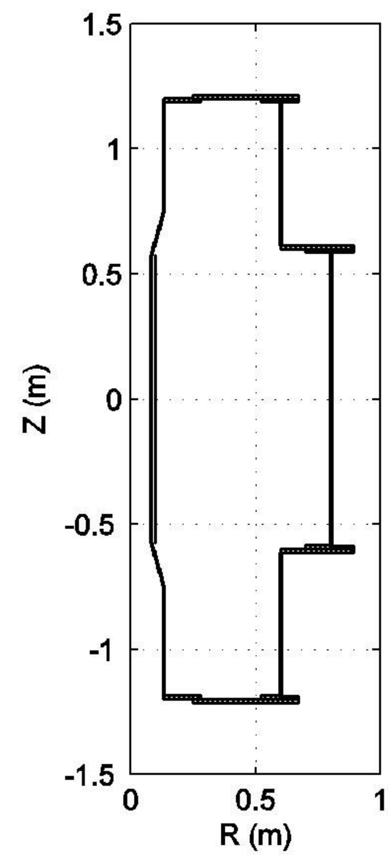
- The response matrix A is updated at each iteration, except for the part that links $I_{c_k}$ to the $M_l$, because $\psi$ is changing by each iteration

## Eddy Current Evaluation

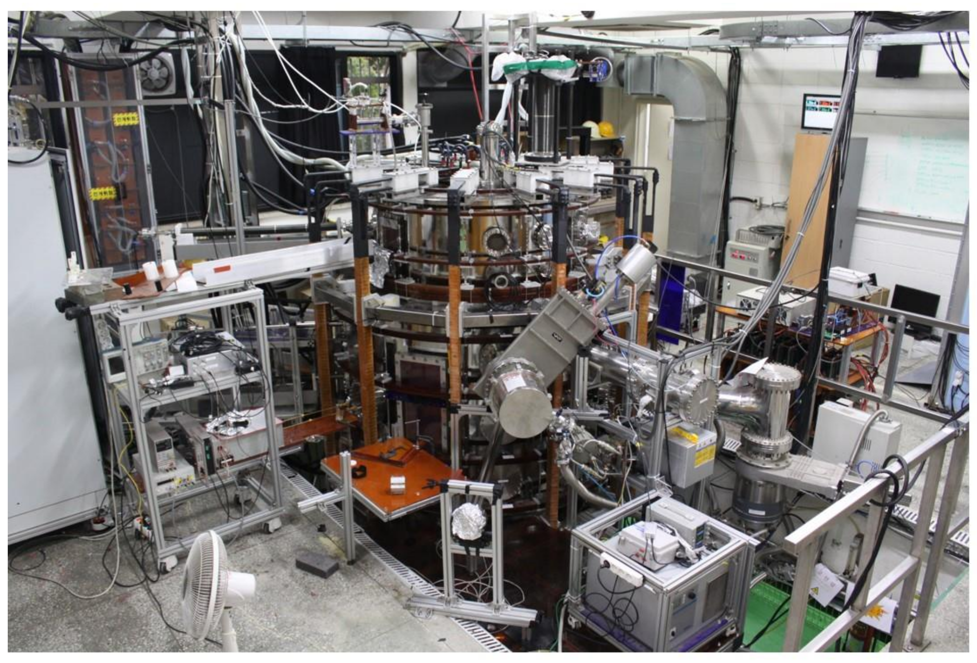
- Discretize walls into 368 elements
- Express plasma current with a moving filament
- Solve time dependent circuit equation (classical Runge-Kutta method)

$$V_k = R_k I_k + L_k\frac{dI_k}{dt} + M_{k,l}\frac{dI_k}{dt}$$



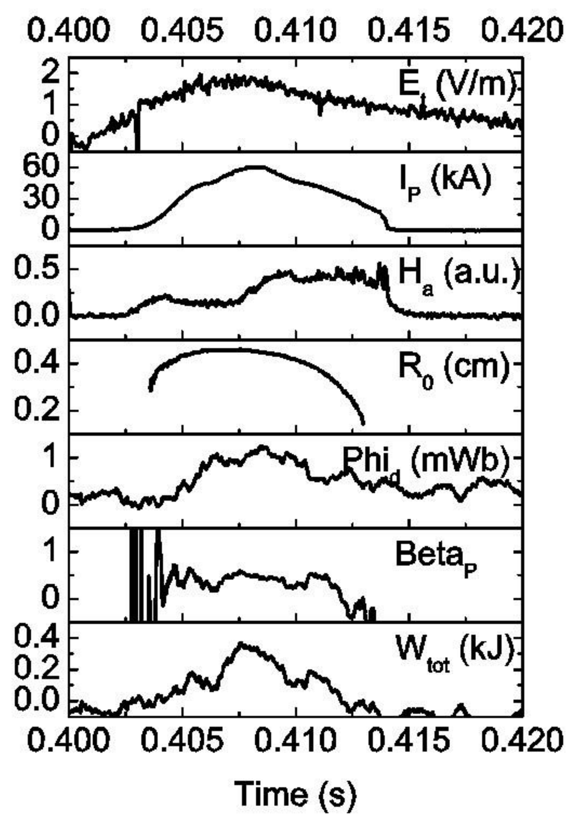



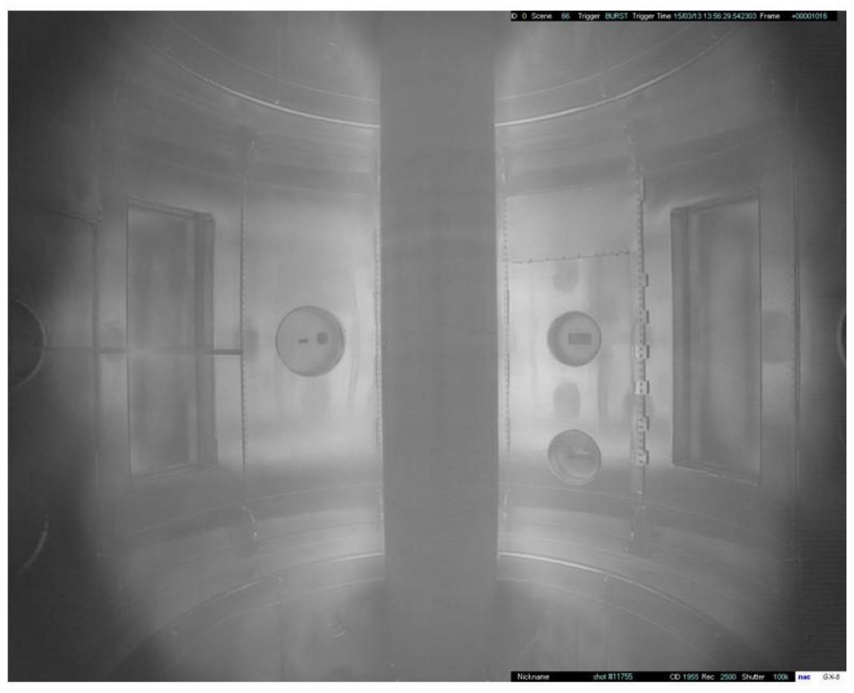

## VEST (Versatile Experiment Spherical Torus)



| | | |
|---|---|---|
| $B_{T0}$ | 0.1 | T |
| $I_P$ | < 70 | kA |
| $R_0$ | 0.4 | m |
| A | > 1.3 | |

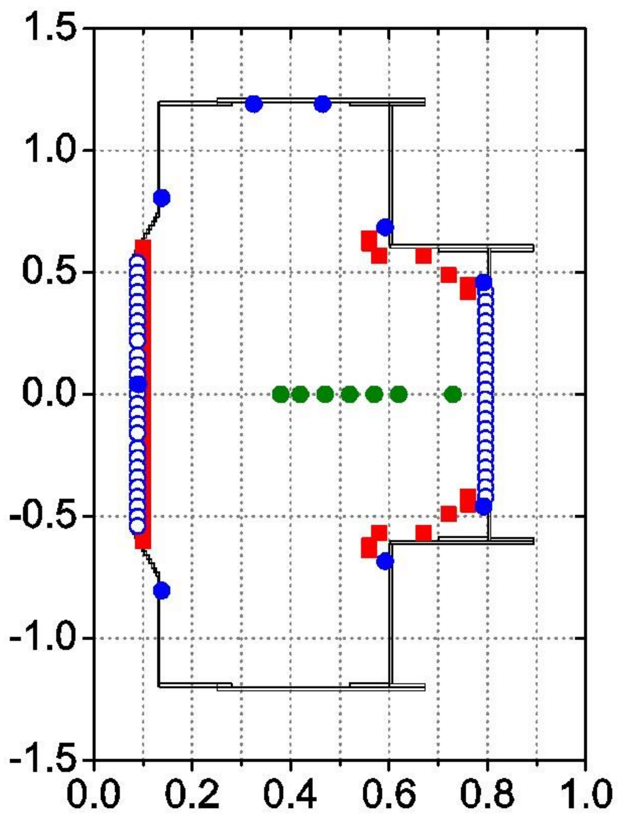
Magnetic diagnostics
$H_\alpha$ and $O_I$ emission
Triple/Mach probe
AXUV array
94 GHz Interferometry



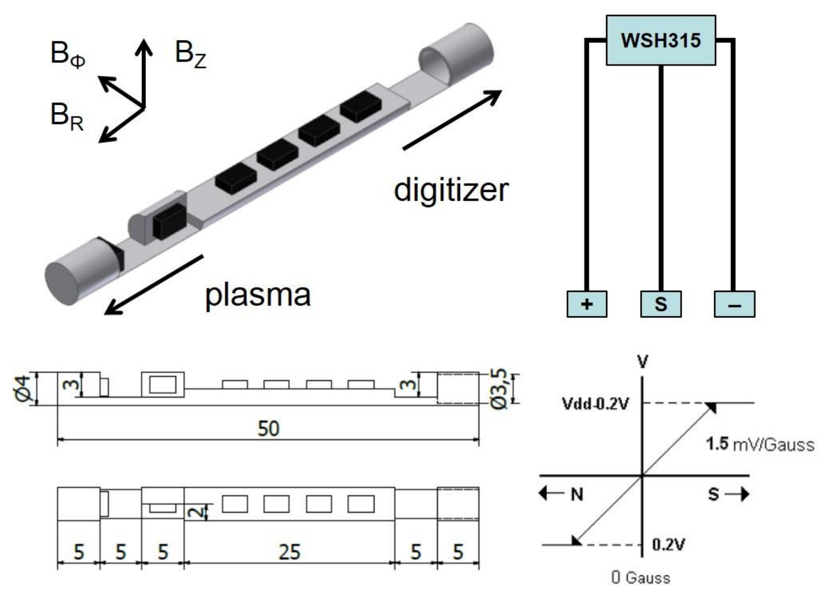
- Plasma ends coincide with an oscillation in magnetic probe signals: presumably disrupt by MHD instability

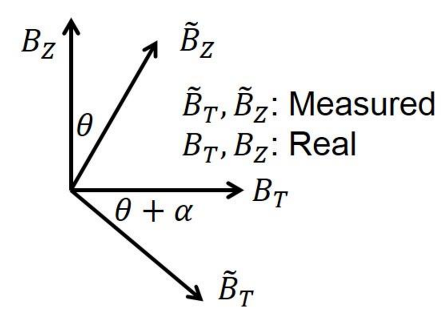
## Simulation Setting and Diagnostics




| | |
|---|---|
| Limiter (Filled Square) | 135 |
| Magnetic Probe (Open Circle) | 49 |
| Flux Loop (Filled Circle) | 9 |
| Internal Probe (Filled Circle) | 7 |

- 36 coils + 368 virtual coils: given
- 2+ coefficients of current density formulation: unknown
- 65 magnetic measurements: fitting target
- Plasma current and magnetic axis position: fitting target



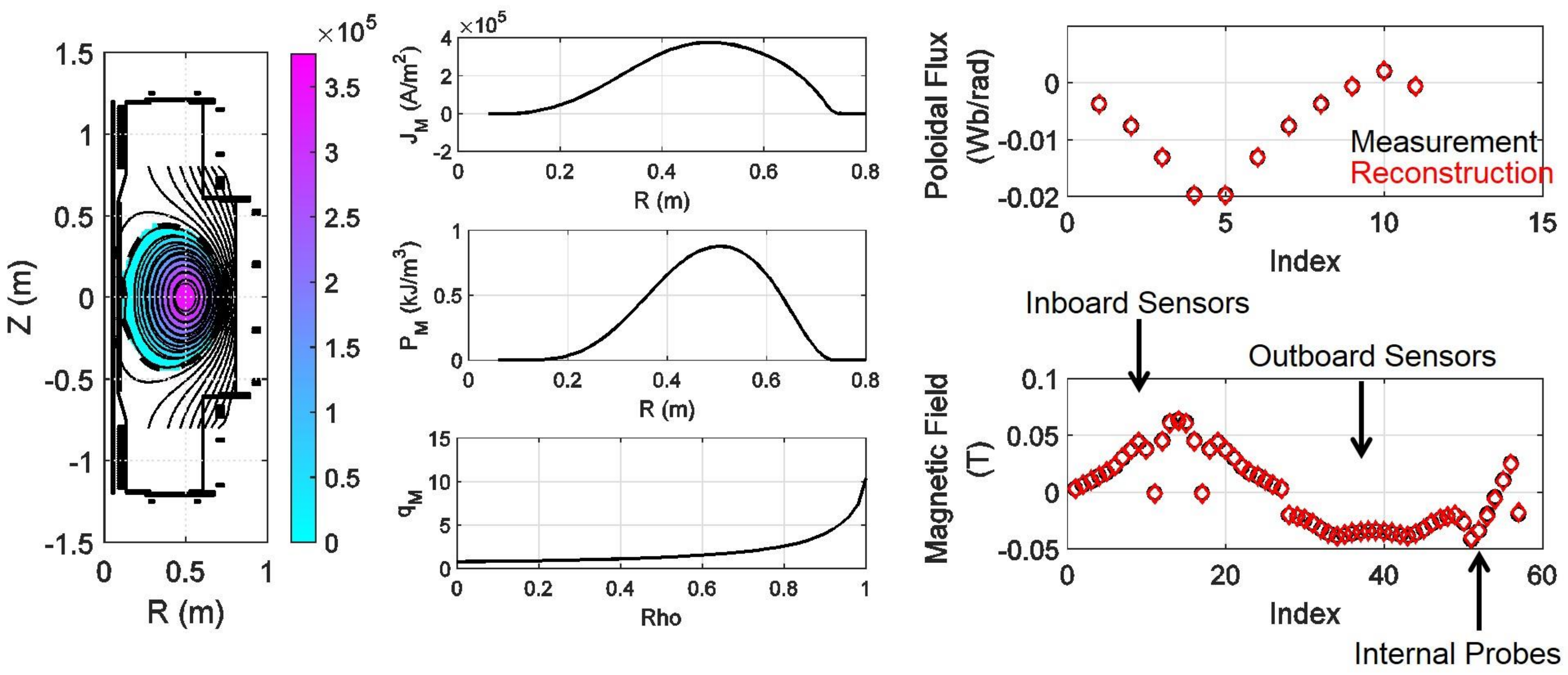




$$\begin{bmatrix}B_T\\B_Z\end{bmatrix} = \frac{1}{\cos\alpha}\begin{bmatrix}\cos\theta & \sin(\theta+\alpha)\\ -\sin\theta & \cos(\theta+\alpha)\end{bmatrix}\begin{bmatrix}\tilde{B}_T\\ \tilde{B}_Z\end{bmatrix}$$

$\theta = \sin^{-1}(\tilde{B}_Z/\tilde{B}_T)$ and
$\alpha = \sin^{-1}\left[(1/(X^2+1))(-Y+X\sqrt{X^2+1-Y^2})\right]$
where
$X \equiv \cot\theta$ and $Y \equiv \tilde{B}_T/\tilde{B}_Z$

- Internal magnetic probe measures from inside the plasma (alumina + s/s case)

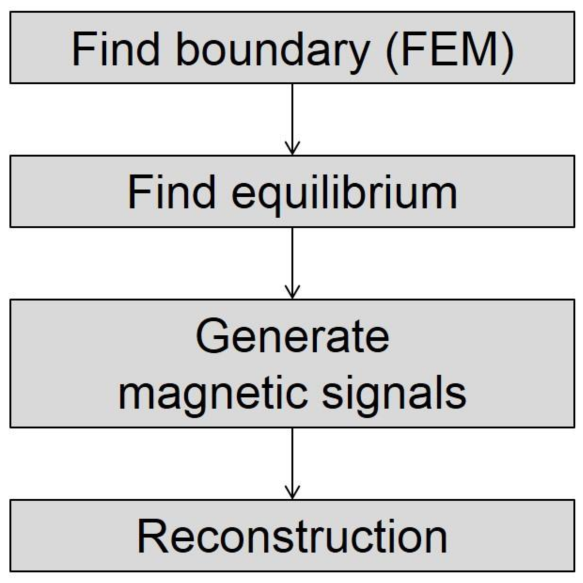
## Reconstruction of Generated Signals




$$J_\phi(\psi) = \lambda_0\left(\beta_0\frac{r}{r_0} + (1-\beta_0)\frac{r_0}{r}\right)(1-\psi_N^m)^n$$

- Current model: Create-L
- Peakedness factor m = 2, n = 2
- Equilibrium parameters from FEM assessment on a real VEST shot

| | | | | | |
|---|---|---|---|---|---|
| $I_P$ [kA] | 50 | $R_0$ [cm] | 43 | A [cm] | 33 |
| $\beta_P$ | 0.5 | κ | 1.6 | δ | 0.2 |




## Sensitivity Study

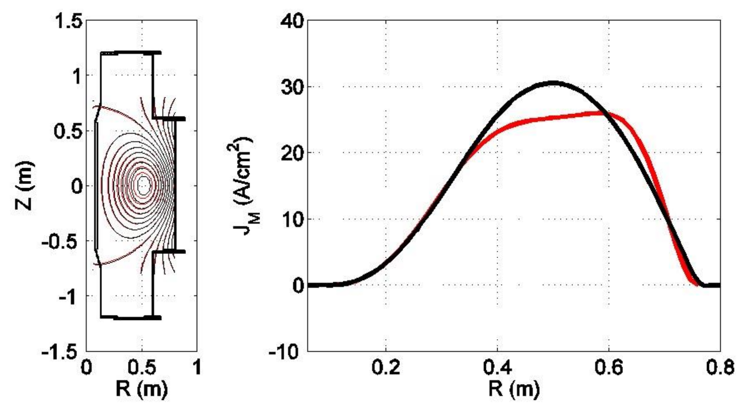
- 10% error random number generator used to produce noisy measurement data
- With internal magnetic field information, peaked profile global parameter is reconstructed by error within 10%

| | Exact | EXT1 | EXT2 | EXT3 | EXT4 | EXT5 | ERROR |
|---|---|---|---|---|---|---|---|
| Poloidal Beta | 0.5 | 0.37 | 0.43 | 0.36 | 0.43 | 0.35 | 0.30 |
| Internal Inductance | 1.05 | 1.27 | 1.19 | 1.31 | 1.19 | 1.31 | 0.25 |
| Toroidal Beta | 3.24 | 3.17 | 3.25 | 3.01 | 3.25 | 2.94 | 0.09 |
| Core Safety Factor | 1.02 | 0.71 | 0.8 | 0.73 | 0.79 | 0.73 | 0.30 |
| Edge Safety Factor | 15.74 | 10.67 | 14.85 | 14.61 | 14.8 | 14.53 | 0.32 |
| Volume | 1.37 | 1.26 | 1.32 | 1.31 | 1.31 | 1.3 | 0.08 |
| Geometric R | 0.43 | 0.44 | 0.43 | 0.43 | 0.43 | 0.43 | 0.02 |
| Geometric Z | 0 | 0 | 0 | 0 | 0 | 0 | 0.00 |
| Minor Radius | 0.33 | 0.32 | 0.33 | 0.33 | 0.33 | 0.33 | 0.03 |
| Elongation | 1.59 | 1.49 | 1.54 | 1.51 | 1.53 | 1.51 | 0.06 |
| Triangularity | 0.18 | 0.15 | 0.2 | 0.2 | 0.19 | 0.2 | 0.17 |
| Plasma Current | 49.7 | 49.8 | 49.8 | 49.8 | 49.8 | 49.8 | 0.00 |

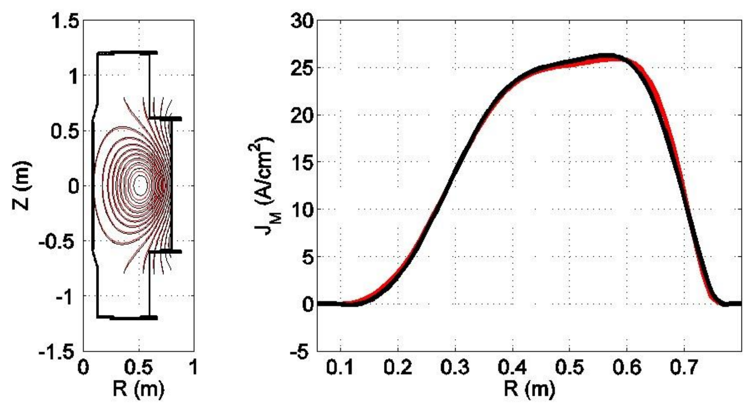
- External sensors only
- Major/minor radius, elongation and plasma volume are reliable

| | Exact | INT1 | INT2 | INT3 | INT4 | INT5 | ERROR |
|---|---|---|---|---|---|---|---|
| Poloidal Beta | 0.5 | 0.48 | 0.55 | 0.46 | 0.54 | 0.52 | 0.10 |
| Internal Inductance | 1.05 | 1.03 | 0.98 | 1.04 | 0.98 | 0.98 | 0.07 |
| Toroidal Beta | 3.24 | 3.11 | 3.35 | 3.03 | 3.34 | 3.24 | 0.06 |
| Core Safety Factor | 1.02 | 0.99 | 1.03 | 1 | 1 | 1.05 | 0.03 |
| Edge Safety Factor | 15.74 | 16.87 | 17.11 | 16.82 | 16.99 | 16.82 | 0.09 |
| Volume | 1.37 | 1.34 | 1.34 | 1.34 | 1.33 | 1.32 | 0.04 |
| Geometric R | 0.43 | 0.43 | 0.43 | 0.43 | 0.43 | 0.43 | 0.00 |
| Geometric Z | 0 | 0 | 0 | 0 | 0 | 0 | 0.00 |
| Minor Radius | 0.33 | 0.33 | 0.33 | 0.33 | 0.33 | 0.33 | 0.00 |
| Elongation | 1.59 | 1.54 | 1.56 | 1.54 | 1.55 | 1.55 | 0.03 |
| Triangularity | 0.18 | 0.23 | 0.23 | 0.23 | 0.23 | 0.23 | 0.28 |
| Plasma Current | 49.7 | 49.8 | 49.8 | 49.8 | 49.8 | 49.8 | 0.00 |

- With additional internal sensors
- All global parameters except for triangularity are reliable

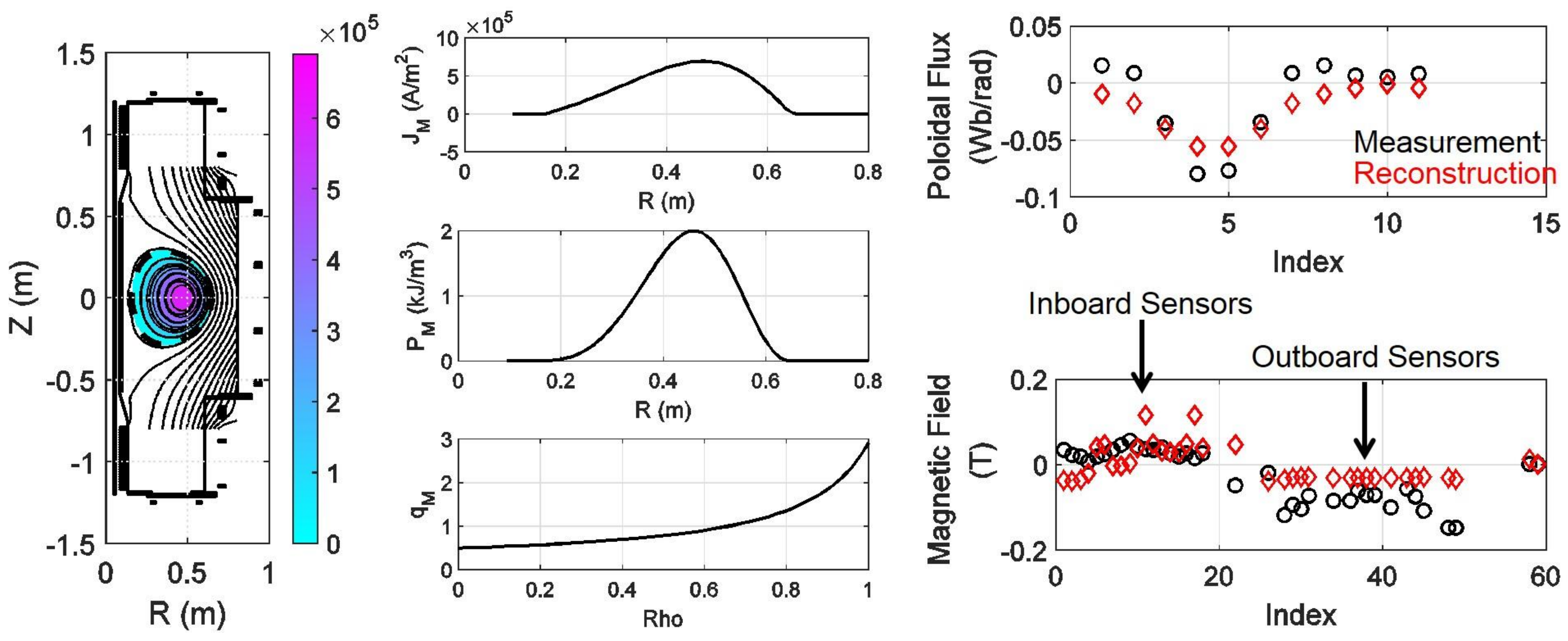



Reconstruction using only external magnetic measurements (black) compared to the exact solution (red): current profile is *not* in good agreement




Reconstruction using external *and internal* magnetic measurements (black) compared to the exact solution (red): current profile is in good agreement

## Reconstruction *without* Internal Probe



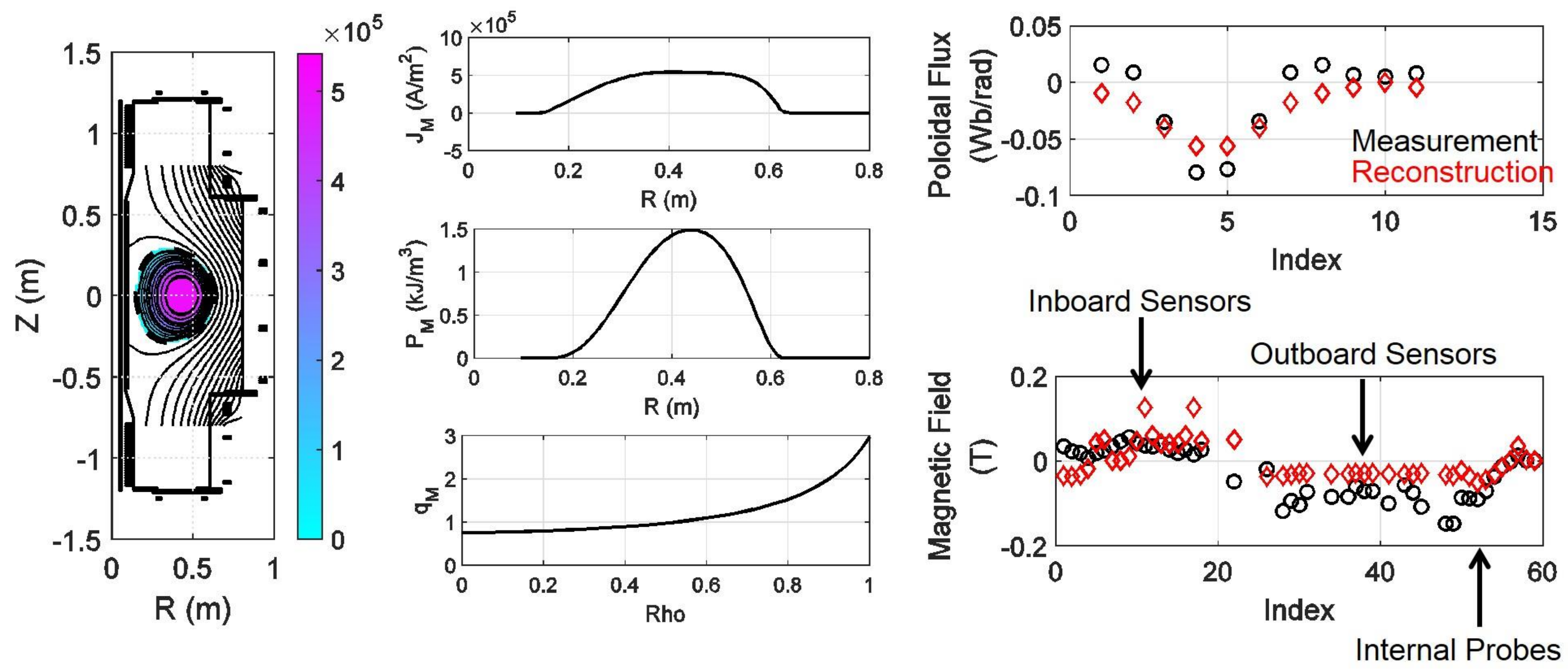

```
##==================================================================##
##       IDK-TEFIT (Tokamak Equilibrium FITTING) Ver-0.3            ##
##                                  - BY Y.M. JEON @2010-0801      ##
##==================================================================##
* READING INPUT FILES AND CONFIGURATIONS...

  n    AXSR    AXSZ        PSIA       PSIB     IS[kA]     ERRFIT    ERRLOOP
  1  0.3806 -0.0000  0.002466  0.000472  0.000027  7.472e-02  5.653e-03
  2  0.4507 -0.0000  0.006077  0.000373  0.000016  7.515e-02  1.848e-05
  3  0.4540 -0.0000  0.004712  0.000377  0.000016  7.517e-02  4.208e-06
  4  0.4585 -0.0000  0.004486  0.000375  0.000016  7.518e-02  8.078e-07
  5  0.4593 -0.0000  0.004440  0.000375  0.000016  7.519e-02  1.834e-07
  6  0.4595 -0.0000  0.004430  0.000375  0.000016  7.519e-02  4.050e-08
  7  0.4595 -0.0000  0.004428  0.000375  0.000016  7.519e-02  9.799e-09

  ELAPSED TIME        =   3.00 [Sec].
  POST PROCESSING ... (TAKES LONG TIME IN A CASE)

  TOTAL ELAPSED TIME =   3.00 [sec].
```

- Calculation took 3 sec. after 7 iterations

- At plasma current maximum
- Fields (radial and vertical) at magnetic axis is set to zero and given 2.5 weighting to fit the measurement (R = 0.46 m)
- Found q(a) ~ 3: maybe the plasma disrupts because of MHD instability
- Large error in the outboard probes

✓ Without the internal magnetic probe, $N_P > 2$ and $N_F > 2$ cannot be used.

## Reconstruction *with* Internal Probe




```
##==================================================================##
##       IDK-TEFIT (Tokamak Equilibrium FITTING) Ver-0.3            ##
##                                  - BY Y.M. JEON @2010-0801      ##
##==================================================================##
* READING INPUT FILES AND CONFIGURATIONS...

  n    AXSR    AXSZ        PSIA       PSIB     IS[kA]     ERRFIT    ERRLOOP
  1  0.3806 -0.0000  0.002466  0.000472  0.000029  7.344e-02  5.653e-03
  2  0.4551 -0.0000  0.003358  0.000412  0.000022  7.287e-02  6.708e-06
  3  0.4431 -0.0000  0.003638  0.000389  0.000020  7.278e-02  4.449e-06
  4  0.4394 -0.0000  0.003782  0.000385  0.000019  7.281e-02  8.861e-07
  5  0.4389 -0.0000  0.003809  0.000385  0.000019  7.281e-02  2.785e-07
  6  0.4391 -0.0000  0.003819  0.000385  0.000019  7.282e-02  5.535e-08
  7  0.4391 -0.0000  0.003823  0.000385  0.000019  7.282e-02  1.979e-08
  8  0.4391 -0.0000  0.003825  0.000385  0.000019  7.282e-02  9.607e-09

  ELAPSED TIME        =   3.00 [Sec].
  POST PROCESSING ... (TAKES LONG TIME IN A CASE)

  TOTAL ELAPSED TIME =   3.00 [sec].
```

- Calculation took 3 sec. after 8 iterations

- At plasma current maximum
- Internal probe signal is fitted with extra weighting of 10 → good agreement between real and reconstructed signal
- Broad current density profile structure is observed
- More accurate q(0) ~ 1

✓ With the internal magnetic probe, the detailed structure of $J_M(R)$ is reconstructed.

## Conclusion

- Evaluation of the wall eddy current is very important in the convergence of the solution.
- The code is not sensitive to the reconstruction to the uncertainty in the measurements, as tested by the phantom current distribution analysis.
- Including the internal magnetic probe signal, it is possible to reconstruct the current density profile and the core safety factor in more detail.
- It is suggested that in VEST, the low edge safety factor may be the reason of plasma disruption.
- With an added undetermined eddy current source near the outboard, it is expected that the code will be more accurate.

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