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NSTX-U

The effects of the HHFW wave-field on the evolution of fast ion / beam ion populations in NSTX plasma

N. Bertelli[1], *E. Valeo*[1],

M. Gorelenkova[1], *M. Podestà*[1], *and D. L. Green*[2]

[1] *PPPL*, [2] *ORNL*

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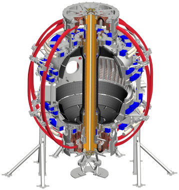
PPPL



Outline

- Motivation
- `TORIC v.5`: brief code description
- Non-Maxwellian extension of `TORIC v.5` in HHFW heating regime
  - Implementation of the non-Maxwellian dielectric tensor
  - `P2F` code: from a particles list to a continuum distribution function
  - Test: `TORIC` wave solution: particle list + `P2F` for a Maxw. case
- “RF-kick” operator in `NUBEAM`
- “Coupling” `TORIC+NUBEAM` in `TRANSP`
- Initial applications to NSTX
- Conclusions

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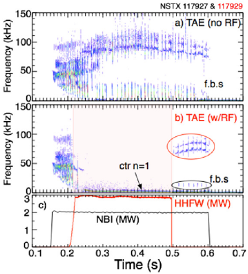
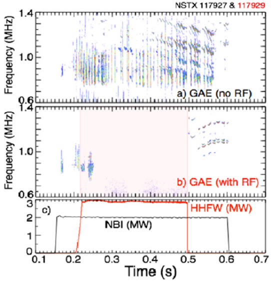
Motivation

- Experiments show that the interactions between fast waves and fast ions can be so strong to significantly modify the fast ion population from neutral beam injection (NBI)
  - The distribution function modifications will, generally, result in finite changes in the amount and spatial location of absorption
  - In NSTX, fast waves (FWs) can modify and, under certain conditions, even suppress the energetic particle driven instabilities, such as toroidal Alfvén eigenmodes (TAEs) and global Alfvén eigenmodes (GAEs) and fishbones





E.D. Fredrickson, et al,

Nucl. Fusion **55** (2015) 013012

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TORIC v.5 code

- The TORIC v.5 code solves the wave equation for the electric field $\mathbf{E}$:

$$\nabla \times \nabla \times \mathbf{E} - \frac{\omega^2}{c^2}\varepsilon \cdot \mathbf{E} = 4\pi i \frac{\omega}{c^2}\mathbf{J}^A$$

  - TORIC: IC minority regime
    - FLR corrections only up to the $\omega = 2\omega_{ci}$
  - TORIC-HHFW: High Harmonic Fast Wave regime
    - Full **hot-plasma** dielectric tensor employed
    - The $k_\perp^2$ value in the argument of the Bessel functions is obtained by solving the local dispersion relation for FWs

- TORIC v.5 is implemented in TRANSP

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$$\boldsymbol{\varepsilon} \equiv \mathbf{I} + \frac{4\pi i}{\omega}\boldsymbol{\sigma} = \mathbf{I} + \boldsymbol{\chi}$$

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Implemented the susceptibility for a hot plasma with an arbitrary distribution function (Eq. 48 in Stix's book page 255)

Local coordinate frame $(\hat{\mathbf{x}}, \hat{\mathbf{y}}, \hat{\mathbf{z}})$ with $\hat{\mathbf{z}} = \hat{\mathbf{b}}$ and $\mathbf{k} \cdot \hat{\mathbf{y}} = 0$ (Stix)

$$\begin{aligned}\boldsymbol{\chi}_{\mathrm{s}} &= \frac{\omega_{\mathrm{ps}}^2}{\omega} \int_0^{+\infty} 2\pi v_\perp \mathrm{d}v_\perp \int_{-\infty}^{+\infty} \mathrm{d}v_\parallel \hat{\mathbf{z}}\hat{\mathbf{z}} \frac{v_\parallel^2}{\omega} \left(\frac{1}{v_\parallel} \frac{\partial f}{\partial v_\parallel} - \frac{1}{v_\perp} \frac{\partial f}{\partial v_\perp} \right)_{\mathrm{s}} + \\ &+ \frac{\omega_{\mathrm{ps}}^2}{\omega} \int_0^{+\infty} 2\pi v_\perp \mathrm{d}v_\perp \int_{-\infty}^{+\infty} \mathrm{d}v_\parallel \sum_{n=-\infty}^{+\infty} \left[\frac{v_\perp U}{\omega - k_\parallel v_\parallel - n\Omega_{\mathrm{cs}}} \mathbf{T}_n \right]\end{aligned}$$

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\end{aligned}
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where

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U \equiv \frac{\partial f}{\partial v_\perp} + \frac{k_\parallel}{\omega} \left(v_\perp \frac{\partial f}{\partial v_\parallel} - v_\parallel \frac{\partial f}{\partial v_\perp} \right) \qquad \text{and}
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$$

and

$$
\mathbf{T}_n = \begin{pmatrix} \frac{n^2 J_n^2(z)}{z^2} & \frac{i n J_n(z) J_n'(z)}{z} & \frac{n J_n^2(z) v_\parallel}{z v_\perp} \\ -\frac{i n J_n(z) J_n'(z)}{z} & (J_n'(z))^2 & -\frac{i J_n(z) J_n'(z) v_\parallel}{v_\perp} \\ \frac{n J_n^2(z) v_\parallel}{z v_\perp} & \frac{i J_n(z) J_n'(z) v_\parallel}{v_\perp} & \frac{J_n^2(z) v_\parallel^2}{v_\perp^2} \end{pmatrix}, \quad z \equiv \frac{k_\perp v_\perp}{\Omega_{\mathrm{cs}}}
$$

Numerical evaluation of χ needed for arbitrary distribution function: χ is pre-computed

- Integrals in the expression for χ are computed numerous times in `TORIC-HHFW` so **an efficient evaluation is essential**
- Precomputation of χ:
  - A set of N_ψ files is constructed, each containing the principal values and residues of χ for a single species on a uniform $(v_\parallel, B/B_{\min}, N_\perp)$ mesh, for a specified flux surface ψ_j
  - The distribution, $f(v_\parallel, v_\perp)$, is specified in functional form at the minimum field strength point $B(\theta) = B_{\min}$ on ψ_j
  - An interpolator returns the components of χ
- We can prescribe
  - an analytical distribution function: bi-Maxwellian, slowing down, etc.
  - a numerical distribution function: for instance from a Monte Carlo particle code `NUBEAM` or a Fokker-Planck code `CQL3D`
    - need a "smooth" distribution function

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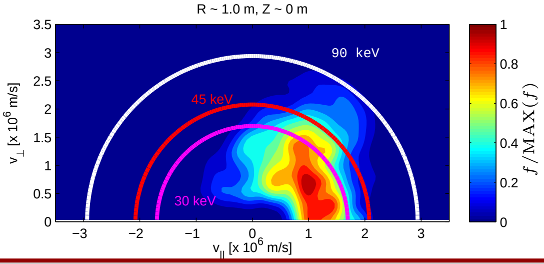
`P2F` code: from a particles list to a distribution function

- `P2F` code was developed by D. L. Green (ORNL)
- `P2F` takes a particle list and creates a 4D $(R, z; v_\parallel, v_\perp)$ distribution function for use in a continuum code like `TORIC`
- Tested `P2F` code starting with a particles list representing a Maxwellian:
  - Excellent agreement between the input kinetic profiles and the corresponding ones obtained from the distribution generated by `P2F` code



NSTX shot 141711

$P_{\rm NBI} = 2$ MW

$E_{\rm NBI,inj,max} = 90$ keV

$I_{\rm P} = 900$ kA

particles number = 53115

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TORIC + non-Maxw. + P2F code: test on Maxwellian case

Procedure:

1. generate particle list representing a Maxwellian
2. run P2F to obtain a distribution function, f
3. pre-compute χ with f above
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$n_e(\rho = 0) = 2.5 \times 10^{13}$ cm^{-3}

$n_e(\rho = 1) = 2.5 \times 10^{12}$ cm^{-3}

$T_e(\rho = 0) = 1$ keV; $T_e(\rho = 1) = 0.1$ keV

$n_{FI}(\rho = 0) = 2.0 \times 10^{12}$ cm^{-3}

$n_{FI}(\rho = 1) = 2.0 \times 10^{11}$ cm^{-3}

$T_{FI}(\rho = 1) = 20$ keV; $T_e(\rho = 1) = 5$ keV

Parabolic profiles for n_e, T_e, and n_{FI}

$T_{FI}(\rho) = (T_{FI,0} - T_{FI,1}) \left(1 - \rho^2\right)^5 + T_{FI,1}$

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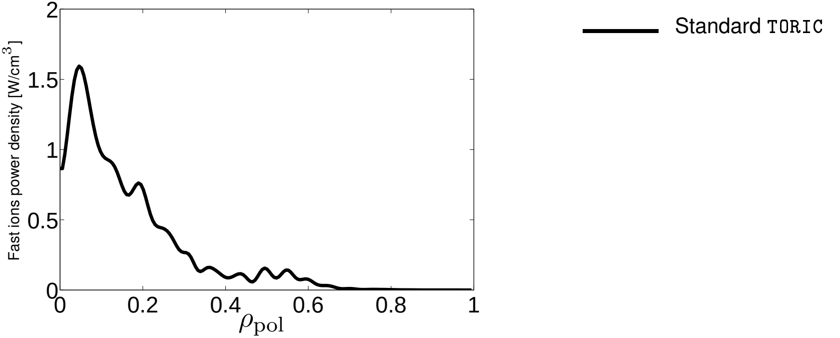

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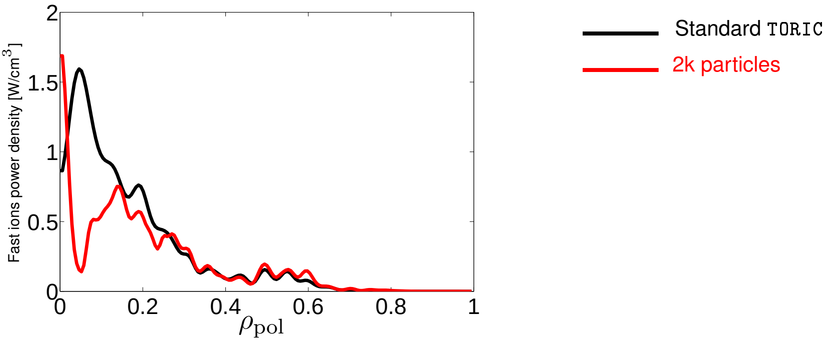

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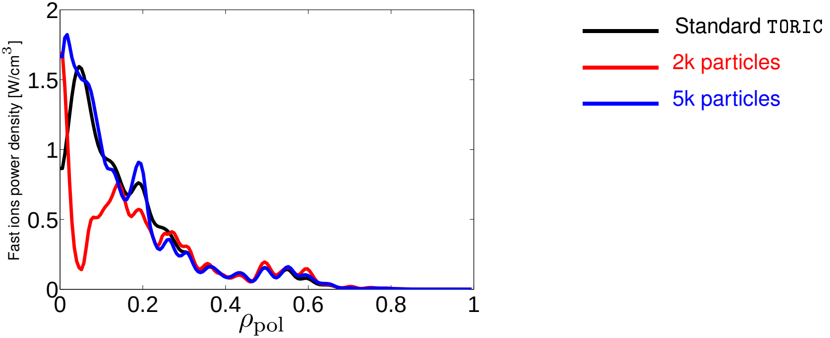

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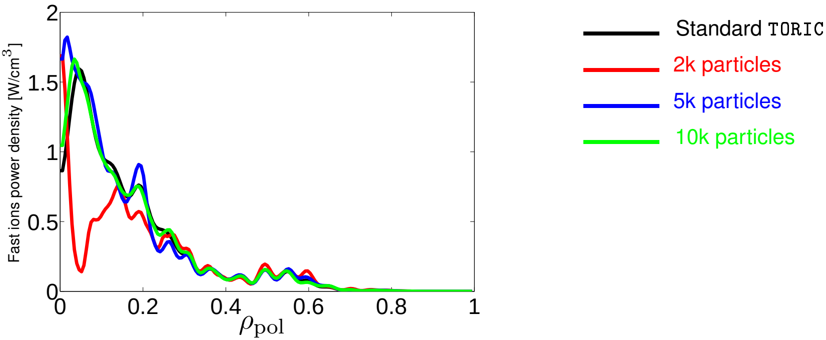

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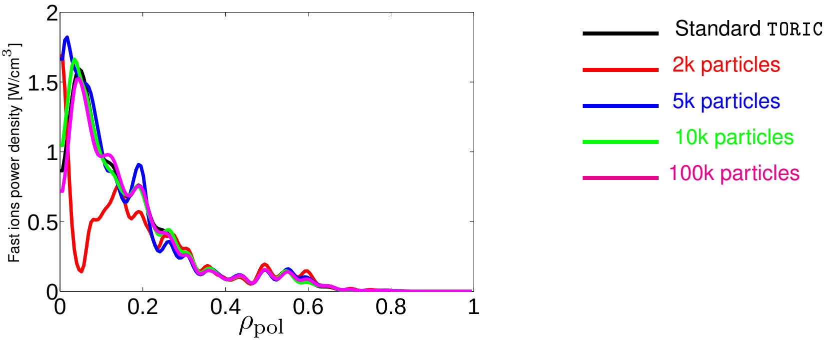

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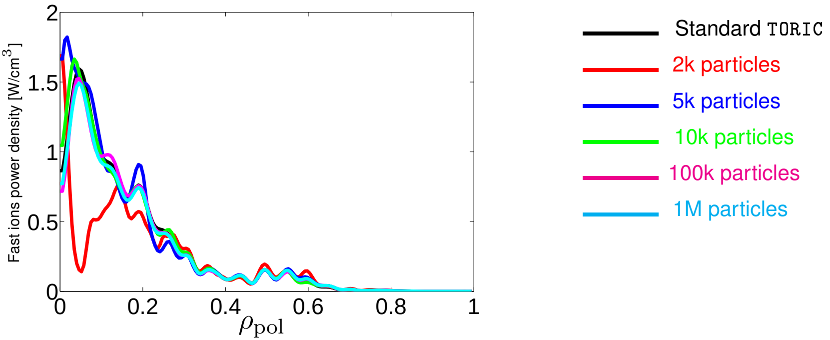

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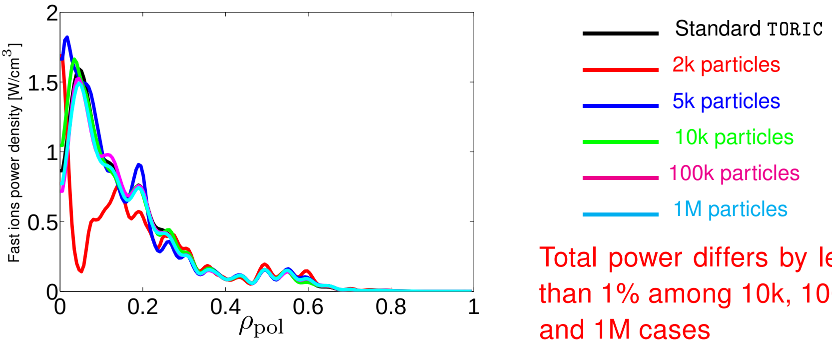

Total power differs by less than 1% among 10k, 100k, and 1M cases

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“RF-kick” operator in NUBEAM

- implemented in NUBEAM [J-H. Kwon et al, APD-DPP 2006 & 2007] based on

  Kennel and Engelmann, PoF **9**, 2377 (1966), Stix, Nucl. Fus. **15** , 737 (1975), Xu and Rosenbluth, PoF B **3**, 627 (1991)

- New $v'_\parallel$ & $v'_\perp$ of particle velocity from the old velocities $(v_\parallel, v_\perp, v)$:

$$v'_\parallel = v_\parallel(1-\nu^{\rm rf}_{\parallel s}\Delta t) + 2\sqrt{3}(R_1-0.5)\sqrt{\nu^{\rm rf}_\parallel v^2 \Delta t}$$

$$v'^2_\perp = v^2_\perp - \nu^{\rm rf}_{\perp,s} v^2 \Delta t + 2\sqrt{3}(R_2-0.5)\times v^2\sqrt{\left[\nu^{\rm rf}_\perp - \frac{(\nu^{\rm rf}_{\parallel\perp})^2}{\nu^{\rm rf}_\parallel}\right]\Delta t} + 2\sqrt{3}(R_1-0.5)\frac{\nu^{\rm rf}_{\parallel\perp}}{\nu^{\rm rf}_\parallel} v\sqrt{\nu^{\rm rf}_\parallel v^2\Delta t}$$

where $\nu^{\rm rf}_\perp, \nu^{\rm rf}_\parallel, \nu^{\rm rf}_{\parallel\perp}$ indicate the $\perp$ and $\parallel$ velocity RF diffusion processes.

$$D_{\rm rf} \propto |E_+|^2 J^2_{n-1}\left(\frac{k_\perp v_\perp}{\omega_{\rm ci}}\right)$$

- RF wave field information from TORIC: E_+, and $k_\perp$ for each n_ϕ

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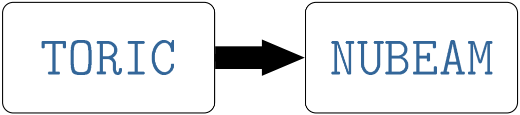
- TORIC v.5: brief code description
- Non-Maxwellian extension of TORIC v.5 in HHFW heating regime
  - Implementation of the non-Maxwellian dielectric tensor
  - P2F code: from a particles list to a continuum distribution function
  - Test: TORIC wave solution: particle list + P2F for a Maxw. case
- “RF-kick” operator in NUBEAM
- “Coupling” TORIC+NUBEAM in TRANSP
- Initial applications to NSTX
- Conclusions

Current "coupling" TORIC+NUBEAM in TRANSP

TORIC

Current "coupling" TORIC+NUBEAM in TRANSP

TORIC → NUBEAM

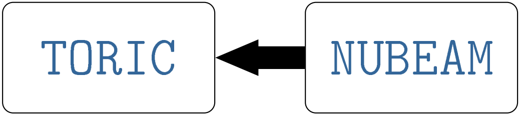


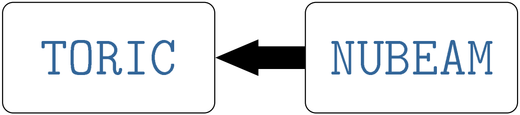
E wave field, $k_\perp$, and n_ϕ for the RF-kick operator

Current “coupling” TORIC+NUBEAM in TRANSP

NUBEAM

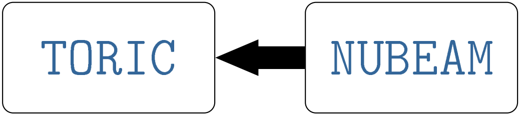
Current "coupling" TORIC+NUBEAM in TRANSP





$\mathcal{E}$ = flux surface average of the total beam ions energy density

$n_{\rm FI}$ = flux surface average of the density profile of the beam ions



$\mathcal{E}$ = flux surface average of the total beam ions energy density

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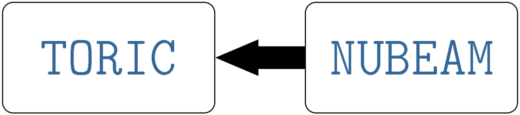
$$T_{\mathrm{FI}} = \frac{2}{3}\frac{\mathcal{E}}{n_{\mathrm{FI}}} \quad \text{(effective temperature)}$$

Current “coupling” TORIC+NUBEAM in TRANSP



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$$T_{\mathrm{FI}} = \frac{2}{3}\frac{\mathcal{E}}{n_{\mathrm{FI}}} \quad \text{(effective temperature)}$$

In TRANSP: a re-normalization factor in E field is introduced to enforce

$$P_{\mathrm{FI,NUBEAM}} \sim P_{\mathrm{FI,TORIC}}$$

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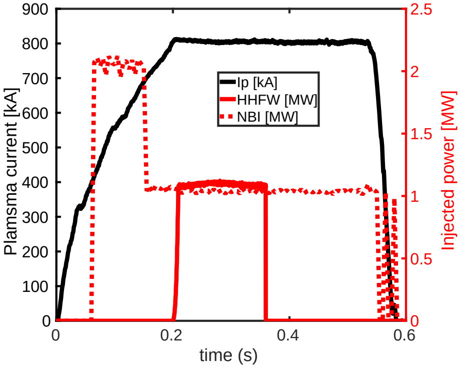
NSTX case

Main parameters:

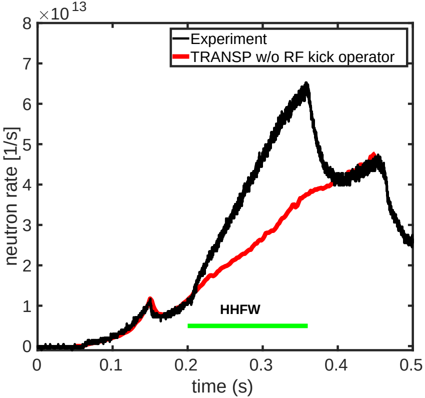
- NSTX discharge # 128733
- $B_{\rm T} = 0.53$ T
- $P_{\rm HHFW} = 1$ MW
- $P_{\rm NBI} = 1$ MW during HHFW
- $E_{\rm NBI,inj,max} = 65$ keV
- $I_{\rm p,flatop} \sim 800$ kA
- 90 degree antenna phasing (= 8 m^{-1})
- antenna frequency = 30 MHz
- from 3rd to 11th deuterium cyclotron resonances for NSTX



Neutron rate rises during the HHFW heating

Neutron rate: discharge 128733

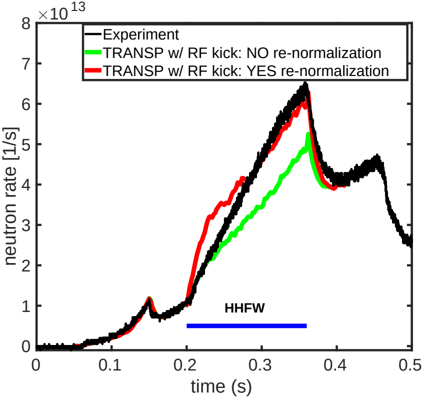
- Neutron rate is dominated by beam-plasma reactions
  - significant fraction of high-energy fast ions are accelerated by the RF heating
- TRANSP (without RF kick operator) completely fails in predicting the rise in the neutron rate
- RF full wave simulations commonly show significant absorbed power to beam ions



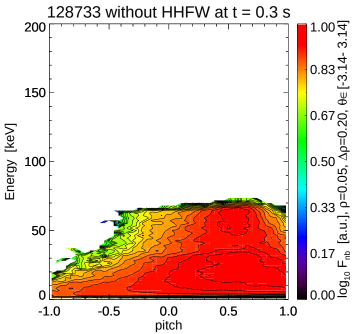
TRANSPP runs w/ & w/o re-normalization factor

- The predicted neutron rate without a re-normalization factor tends to underestimate the measured neutron rate except for the very beginning of the HHFW application
- The predicted neutron rate assuming a re-normalization factor tends to be closer to the measured one
  - However, it appears an overestimate of the neutron rate for about the first half of the HHFW application

Neutron rate: discharge 128733



Very high energy tail in the NUBEAM fast ion distribution function when HHFW applied



- NBI injection energy = 65 keV
- with HHFW: few fast ions can be accelerated to 200 keV !!
- mainly acceleration in the perpendicular direction

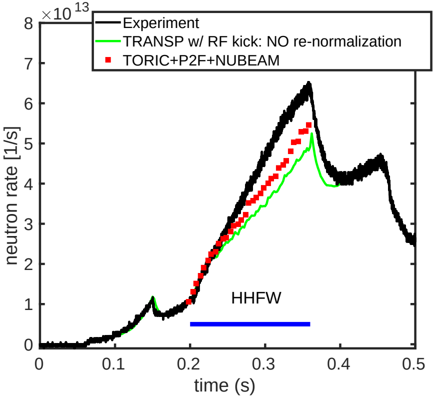
First & preliminary test of "RF-kick" operator in NUBEAM with TORIC+P2F+NUBEAM

- TORIC+P2F $\Longleftrightarrow$ NUBEAM+"RF-kick"
- Reasonable agreement when consistency between TORIC & NUBEAM
- **However**, it appears a clear underestimate of the neutron rate
- very similar results obtained with a bi-Maxwellian

$$T_{\mathrm{FI},\|} = 2\mathcal{E}_{\|}/n_{\mathrm{FI}},$$
$$T_{\mathrm{FI},\perp} = \mathcal{E}_{\perp}/n_{\mathrm{FI}},$$

- Work in progress

Neutron rate: discharge 128733



Conclusions

- A generalization of the full wave `TORIC v.5` code in the high harmonic and minority heating regimes has been implemented to include species with arbitrary velocity distribution functions
- Implementation of the full hot dielectric tensor reproduces the analytic Maxwellian case
- Non-Maxwellian extension of `TORIC` in HHFW regime reproduces previous simulations with both a specified functional form of the distribution functions and a particle list
  - Excellent agreement of the 2D electric field and in terms of absorbed power
- All results above are published in N. Bertelli et al. *Nuclear Fusion* **57**, 056035 (2017).
- First attempts to apply `TORIC` generalization with a `NUBEAM` particle list
  - preliminary results with arbitrary distribution functions appears significantly different than Maxw. case results
  - still additional tests/checks needed

Thank you

TORIC code: additional info

- Spectral ansatz $\mathbf{E}(\mathbf{r},t) = \sum_{m,n} \mathbf{E}^{mn}(\psi) e^{i(m\theta + n\phi - \omega t)}$

  $m \rightarrow$ poloidal mode number; $n \rightarrow$ toroidal mode number

- For each toroidal component one has to solve a (formally infinite) system of coupled ordinary differential equations for the physical components of $E^{mn}(\psi)$, written in the local field-aligned orthogonal basis vectors.

- The Spectral Ansatz transforms the θ-integral of the constitutive relation into a convolution over poloidal modes.

- Due to the toroidal axisymmetry, the wave equations are solved separately for each toroidal Fourier component.

- A spectral decomposition defines an accurate representation of the "local" parallel wave-vector $k_{\parallel}^{m} = (m\nabla\theta + n\nabla\phi) \cdot \hat{\mathbf{b}}$

- The ψ variation is represented by Hermite cubic finite elements

- Principal author M. Brambilla (IPP Garching, Germany)

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The susceptibility for a hot plasma with a Maxwellian distribution function can be evaluated analitically

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$$\chi_\mathrm{s} = \left[\hat{\mathbf{z}}\hat{\mathbf{z}} \frac{2\omega_\mathrm{p}^2}{\omega k_\parallel v_\mathrm{th}^2} \langle v_\parallel \rangle + \frac{\omega_\mathrm{p}^2}{\omega} \sum_{n=-\infty}^{+\infty} e^{-\lambda} \mathbf{Y}_n(\lambda) \right]_s$$

where

$$\mathbf{Y}_n = \begin{pmatrix} \frac{n^2 I_n}{\lambda} A_n & -in(I_n - I_n') A_n & \frac{k_\perp}{\omega_c} \frac{n I_n}{\lambda} B_n \\ in(I_n - I_n') A_n & \left(\frac{n^2}{\lambda} I_n + 2\lambda I_n - 2\lambda I_n' \right) A_n & \frac{i k_\perp}{\omega_c} (I_n - I_n') B_n \\ \frac{k_\perp}{\omega_c} \frac{n I_n}{\lambda} B_n & -\frac{i k_\perp}{\omega_c} (I_n - I_n') B_n & \frac{2(\omega - n\omega_c)}{k_\parallel v_\mathrm{th}^2} I_n B_n \end{pmatrix}$$

$$A_n = \frac{1}{k_\parallel v_\mathrm{th}} Z_0(\zeta_n), \quad B_n = \frac{1}{k_\parallel} (1 + \zeta_n Z_0(\zeta_n)), \quad Z_0(\zeta_n) \equiv \text{plasma dispersion func.}$$

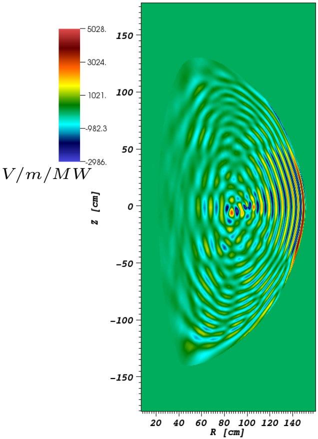
$$\zeta_n \equiv \frac{\omega - n\omega_c}{k_\parallel v_\mathrm{th}}, \quad \lambda \equiv \frac{k_\perp^2 v_\mathrm{th}^2}{2\Omega_c^2}$$

Excellent agreement between numerical and analytical evaluation of the 2D HHFW fields

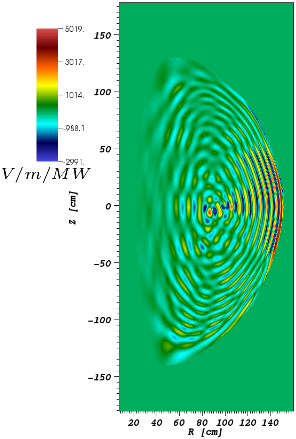
TORIC resolution: $n_{\rm mod} = 31$, $n_{\rm elm} = 200$

Resolution used for χ: $N_{v_{\|}} = 100$ and $N_{v_{\perp}} = 50$

Maxw. analytical: Re(E_{-})



Maxw. numerical: Re(E_{-})



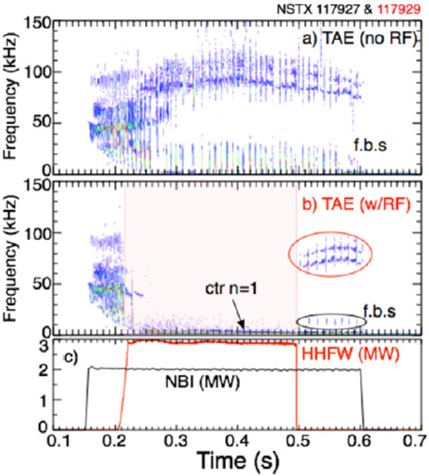
Excellent agreement in terms of absorbed power

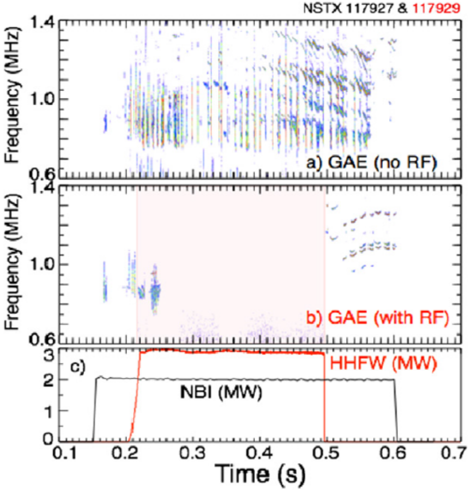
TORIC resolution: $n_{\mathrm{mod}} = 31$, $n_{\mathrm{elm}} = 200$
Resolution used for χ: $N_{v_\parallel} = 100$ and $N_{v_\perp} = 50$

Absorbed fraction	Maxw. analytical	Maxw. numerical
D	0.22 %	0.22 %
D-NBI	73.88 %	73.58 %
Electrons	25.90 %	26.21 %

Additional applications have been done with bi-Maxwellian and slowing down distributions (not shown here)

NSTX shot 117929 from Fredrickson et al. NF 2015





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Finite elements to use to compute the resonant integrals

- We need to evaluate integrals of the form

$$I_k = \int \mathrm{d}v \frac{C(v)}{v - v_k}.$$

- Since I_k is a smooth function of v_k , evaluate on a uniform mesh $v_k = k\Delta v$, and interpolate
- Express smooth integrand $C(v)$ in terms of (linear) finite elements $C(v) = \sum_j c_j T_j$, with T_j centered at v_j

Then

$$I_k = \sum_j \int \mathrm{d}v \frac{c_j T_j}{v - v_k} = \sum_j c_j K_{j-k} = \sum_j c_{j+k} K_j$$

where the kernel is given by

$$K_j = \int_{-1}^{1} \mathrm{d}v \frac{1 - |v|}{v + j\Delta v} = \begin{cases} \ln\left(\frac{j+1}{j-1}\right) - j \ln\left(\frac{j^2}{j^2-1}\right), & |j| > 1, \\ \pm \ln 4, & j = \pm 1, \\ i\pi, & j = 0. \end{cases}$$