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FROM: C NEUMEYER
SUBJECT: UPDATE OF NSTX INFLUENCE MATRICIES AND EM LOADS

## Reference:

[1] NSTX-CALC-13-020, "PF Coil Axial and Radial Force Calculation", R1, 5/3/4
This memo presents results from an analysis performed to generate influence matrices for calculation of in-plane loads on the OH and PF coils as well as the out-of-plane loads on the TF coil inner bundle elements. It supercedes the prior revision of the reference calculation, which only addressed the PF coils. In addition to the inclusion of the TF, it also includes the revised PF1a coil geometry.

The finite element analysis program FEMLAB was used to model the PF coils using the 2-d axi-symmetric magnetics mode. Sub-domains were created to represent regions of the TF coil so that fields and forces could be calculated based on interaction with the OH and PF. However, the TF conductors do not effect the in-plane field calculations, which are generated only by the out-of-plane currents in OH and PF. Figure 1 shows an example field plot.


Figure 1 - FEMLAB Model Including Zoom View of Region in Lower Half Plane ( $\mathrm{B}_{\mathrm{r}}$ and flux shown for the case with rated current in each coil, except PF4)

Characteristics of the PF and TF coils in each half plane are given in Table 1 below.
TABLE 1: PF Coil Summary

| Coil | R (center) | $\Delta \mathrm{R}$ | Z (center) | $\Delta \mathrm{Z}$ | Turns |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | (in) | (in) | (in) | (in) |  |
| OH | 5.2088 | 1.7335 | 41.7490 | 83.4980 | 482 |
| PF1a | 7.2403 | 1.6265 | 62.6215 | 9.1820 | 20 |
| PF1b | 11.9768 | 3.3055 | 71.8570 | 7.5030 | 32 |
| PF2a | 31.4634 | 6.4060 | 76.1225 | 2.6760 | 14 |
| PF2b | 31.4634 | 6.4060 | 72.9385 | 2.6760 | 14 |
| PF3a | 58.8370 | 7.3400 | 64.3114 | 2.6760 | 15 |
| PF3b | 58.8370 | 7.3400 | 61.1274 | 2.6760 | 15 |
| PF4b | 70.6540 | 3.6040 | 31.7800 | 2.6760 | 8 |
| PF4c | 71.1210 | 4.5380 | 34.9640 | 2.6760 | 9 |
| PF5a | 78.5280 | 5.3500 | 25.6840 | 2.6980 | 12 |
| PF5b | 78.5280 | 5.3500 | 22.7440 | 2.6980 | 12 |
| PFAB1 | 16.9530 | 0.8565 | 69.1550 | 2.6725 | 48 |
| PFAB2 | 24.8750 | 0.8565 | 75.8210 | 2.6725 | 48 |
| TF Inner | 1.1090 | 2.2180 | 55.7500 | 111.5000 | 12 |
| TF Inner Flag | 13.8590 | 20.2820 | 109.0000 | 5.0000 | 12 |
| TF Inner Flag Stub | 2.9680 | 1.5000 | 109.0000 | 5.0000 | 12 |
| TF Outer | 3.0750 | 1.5460 | 49.6875 | 99.3750 | 24 |
| TF Outer Flag | 13.9240 | 20.1520 | 96.8750 | 5.0000 | 24 |
|  |  |  |  |  |  |
|  |  |  |  |  |  |
| Coil | R (center) | $\Delta \mathrm{R}$ | Z (center) | $\Delta \mathrm{Z}$ | Turns |
|  | (cm) | (cm) | (cm) | (cm) |  |
| OH | 13.2302 | 4.4031 | 106.0425 | 212.0849 | 482 |
| PF1a | 18.3902 | 4.1313 | 159.0586 | 23.3223 | 20 |
| PF1b | 30.4209 | 8.3960 | 182.5168 | 19.0576 | 32 |
| PF2a | 79.9170 | 16.2712 | 193.3510 | 6.7970 | 14 |
| PF2b | 79.9170 | 16.2712 | 185.2637 | 6.7970 | 14 |
| PF3a | 149.4460 | 18.6436 | 163.3510 | 6.7970 | 15 |
| PF3b | 149.4460 | 18.6436 | 155.2637 | 6.7970 | 15 |
| PF4b | 179.4612 | 9.1542 | 80.7212 | 6.7970 | 8 |
| PF4c | 180.6473 | 11.5265 | 88.8086 | 6.7970 | 9 |
| PF5a | 199.4611 | 13.5890 | 65.2374 | 6.8529 | 12 |
| PF5b | 199.4611 | 13.5890 | 57.7698 | 6.8529 | 12 |
| PFAB1 | 43.0606 | 2.1755 | 175.6537 | 6.7882 | 48 |
| PFAB2 | 63.1825 | 2.1755 | 192.5853 | 6.7882 | 48 |
| TF Inner | 2.8169 | 5.6337 | 141.6050 | 283.2100 | 12 |
| TF Inner Flag | 35.2019 | 51.5163 | 276.8600 | 12.7000 | 12 |
| TF Inner Flag Stub | 7.5387 | 3.8100 | 276.8600 | 12.7000 | 12 |
| TF Outer | 7.8105 | 3.9268 | 126.2063 | 252.4125 | 24 |
| TF Outer Flag | 35.3670 | 51.1861 | 246.0625 | 12.7000 | 24 |

Since the PF1b coil is located in the lower half plane only, the calculations for TF were performed in the lower half plane only, since that represents the worst case. Radial extent
of the flag elements given above is noted to extend out to the mipoint of the flex links at approx. 24"

For the force coefficients on the PF and OH coils an excitation current was imposed one at a time in each of the PF and OH coils, and the total radial $\left(\mathrm{JxB}_{\mathrm{v}}\right)$ and vertical $\left(\mathrm{JxB}_{\mathrm{r}}\right)$ force on each of the other coils, per unit of current, was calculated. This involves the integration of the field over the cross section of the coil, multiplied by the circumference and divided by the area of the coil...

$$
C_{x y}=\frac{2 \pi r_{x}}{\Delta r_{x} \Delta z_{x}} \iint B_{y} d r d z
$$

where $C_{x y}$ is the coefficient of force on coil " $x$ " due to the current in coil " $y$ ". The resultant influence matricies are given in Tables 2 and 3.

TABLE 2: Radial Force Coefficients (lbf/kA^2)

| Fr | OH | PF1aU | PF1aL | PF1b | PF2U | PF2L | PF3U | PF3L | PF4U | PF4L | PF5U | PF5L |
| :--- | :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| OH | 24620 | 1069 | 1069 | 1681 | 972 | 972 | 1017 | 1017 | 660 | 660 | 896 | 896 |
| PF1aU | -19 | 129 | 0 | 1 | 95 | 1 | 67 | 5 | 24 | 6 | 28 | 11 |
| PF1aL | -19 | 1 | 129 | 179 | 2 | 95 | 5 | 67 | 6 | 24 | 11 | 28 |
| PF1b | -76 | 1 | 10 | 545 | 4 | 374 | 12 | 192 | 15 | 61 | 27 | 70 |
| PF2U | -19 | -6 | 1 | 2 | 444 | 6 | 443 | 19 | 112 | 25 | 129 | 47 |
| PF2L | -19 | 1 | -6 | -73 | 6 | 444 | 19 | 443 | 25 | 112 | 47 | 129 |
| PF3U | -17 | -3 | 1 | 3 | -113 | 9 | 575 | 33 | 222 | 43 | 266 | 85 |
| PF3L | -17 | 1 | -3 | -18 | 9 | -113 | 33 | 575 | 43 | 222 | 85 | 266 |
| PF4U | -11 | 0 | 1 | 2 | -4 | 7 | 14 | 28 | 189 | 38 | 612 | 80 |
| PF4L | -11 | 1 | 0 | -1 | 7 | -4 | 28 | 14 | 38 | 189 | 80 | 612 |
| PF5U | -17 | 0 | 1 | 2 | -3 | 8 | 7 | 36 | -300 | 46 | 392 | 105 |
| PF5L | -17 | 1 | 0 | -1 | 8 | -3 | 36 | 7 | 46 | -300 | 105 | 392 |

TABLE 3: Vertical Force Coefficients (lbf/kA^2)

| Fz | OH | PF1aU | PF1aL | PF1b | PF2U | PF2L | PF3U | PF3L | PF4U | PF4L | PF5U | PF5L |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| OH | 0 | 6 | -6 | -72 | 54 | -54 | 27 | -28 | 7 | -7 | 6 | -6 |
| PF1aU | -6 | 0 | 0 | 0 | 11 | 0 | 0 | 0 | -1 | 0 | -2 | -1 |
| PF1aL | 6 | 0 | 0 | -88 | 0 | -11 | 0 | 0 | 0 | 1 | 1 | 1 |
| PF1b | 72 | 0 | 90 | 0 | 1 | -20 | 2 | 10 | 2 | 7 | 3 | 8 |
| PF2U | -55 | -11 | 0 | -1 | 0 | -2 | -99 | -7 | -39 | -9 | -41 | -16 |
| PF2L | 55 | 0 | 11 | 21 | 2 | 0 | 7 | 99 | 9 | 39 | 16 | 41 |
| PF3U | -28 | -1 | -1 | -2 | 99 | -7 | 0 | -26 | -225 | -36 | -214 | -66 |
| PF3L | 28 | 1 | 1 | -9 | 7 | -99 | 26 | 0 | 36 | 225 | 66 | 214 |
| PF4U | -7 | 1 | -1 | -2 | 39 | -9 | 228 | -36 | 0 | -52 | -533 | -100 |
| PF4L | 7 | 1 | -1 | -6 | 9 | -39 | 36 | -228 | 52 | 0 | 100 | 533 |
| PF5U | -6 | 2 | 0 | -3 | 42 | -15 | 214 | -65 | 530 | -99 | 0 | 200 |
| PF5L | 6 | 0 | -2 | -8 | 15 | -42 | 65 | -214 | 99 | -530 | 200 | 0 |

The above results are noted to differ from the prior revision of the reference calculation as a result of the changed geometry of PF1a. Using these results the force on coil " $x$ " due to its current and the currents in the other " $y$ " coils is is calculated as follows..

$$
F_{x}=\sum_{y} C_{x y} I_{x} I_{y}
$$

where the currents are in kA and the resultant force in lbf.
For the TF coil, the forces and moments (taken w.r.t. machine axis) of interest are:

$$
\begin{aligned}
& F_{\text {inner }}=\text { total force generated on an inner flag due to } J_{T F} \times B_{v} \\
& F_{\text {outer }}=\text { total force generated on an outer flag due to } J_{T F} \times B_{V} \\
& M_{\text {inner }}=\text { total moment generated on inner flag due to } J_{T F} \times B_{V} \\
& M_{\text {outer }}=\text { total moment generated on outer flag due to } J_{T F} \times B_{V} \\
& R_{\text {eff_inner }}=\text { effective radius of } F_{\text {inner }} \\
& R_{\text {eff_outer }}=\text { effective radius of } F_{\text {outer }}
\end{aligned}
$$

$$
\mathrm{M}_{\mathrm{inner} \text { bundle }}^{-}=\text {moment on inner turn in bundle due to } \mathrm{J}_{\mathrm{TF}} \times \mathrm{B}_{\mathrm{r}} \text { and reacted by hub }
$$

$$
\mathrm{M}_{\text {outer bundle }}=\text { moment on outer turn in bundle due to } \mathrm{J}_{\mathrm{TF}} \times \mathrm{B}_{\mathrm{r}} \text { and reacted by hub }
$$

$$
\mathrm{M}_{\mathrm{bundle}}=\text { total moment generated on } 12 \text { inner and } 24 \text { outer turns and reacted by hub }
$$

$$
\mathrm{M}_{\text {flag }}=\text { total torsional moment generated on } 12 \text { inner and } 24 \text { outer flags } \mathrm{J}_{\mathrm{TF}} \times B_{\mathrm{v}}
$$

$$
\mathrm{M}_{\text {net }}=\text { net moment of bundle and flags transmitted through vacuum vessel }
$$

Equations for the influence matrix coefficients are as follows...
Flags:

$$
\begin{aligned}
& C_{F_{-} \text {inner,outer }}=w \iint J_{T F} B_{v} d r d z \quad \text { where } \mathrm{w} \text { is the width of the flag (1") } \\
& C_{M_{-} \text {inner,outer }}=w \iint J_{T F} B_{v} r d r d z \\
& R_{\text {eff }- \text { inner,outer }}=\frac{C_{M_{-} \text {inner,outer }}}{C_{F_{-} \text {inner,outer }}}
\end{aligned}
$$

Bundle:

$$
\begin{aligned}
& C_{\text {Mbundle_inner }=2 \pi \iint J_{T F} B_{r} r^{2} \frac{z}{z_{\max }} d r d z+w \iint J_{T F} B_{r} r \frac{z}{z_{\max }} d r d z}^{C_{\text {Mbundle_outer }}=2 \pi \iint J_{T F} B_{r} r^{2} \frac{z}{z_{\max }} d r d z}
\end{aligned}
$$

where $z_{\max }$ is the turn height in a half plane. The $\mathrm{z} / \mathrm{z}_{\max }$ term is used to approximate the amount of torsional load taken out at the hub end of the bundle as if it was a fixed
boundary. The remainder is taken by the bundle twist. Note also that the second term in the inner calculation accounts for the flag stub.

The resultant matrix is given in Table 4.
TABLE 4: TF Influence Coefficients (forces: $\mathrm{lbf} / \mathrm{kA}^{2}$, moments: $\mathrm{lbf}-\mathrm{in} / \mathrm{kA}^{2}$, radii: inches)
Note: Moments taken about NSTX machine axis

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| :--- | :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
|  | OH | PF1aU | PF1aL | PF1b | PF2U | PF2L | PF3U | PF3L | PF4U | PF4L | PF5U | PF5L |
| F_Inner | 0.222 | 0.003 | 0.024 | 0.175 | 0.011 | 0.652 | 0.037 | 0.655 | 0.049 | 0.195 | 0.092 | 0.240 |
| F_Outer | 0.471 | 0.004 | 0.048 | 0.382 | 0.015 | 1.178 | 0.048 | 0.890 | 0.063 | 0.260 | 0.115 | 0.310 |
| M_Inner | 2.7 | 0.0 | 0.3 | 2.1 | 0.1 | 8.5 | 0.5 | 8.9 | 0.7 | 2.7 | 1.2 | 3.3 |
| M_Outer | 5.1 | 0.1 | 0.6 | 4.3 | 0.2 | 15.7 | 0.7 | 12.2 | 0.9 | 3.6 | 1.6 | 4.2 |
| Reff_Inner | 12.2 | 13.3 | 12.7 | 12.3 | 13.5 | 13.1 | 13.6 | 13.5 | 13.6 | 13.7 | 13.5 | 13.6 |
| Reff_Outer | 10.7 | 13.1 | 12.1 | 11.4 | 13.5 | 13.3 | 13.7 | 13.8 | 13.7 | 13.7 | 13.9 | 13.7 |
| CM_Bundle | -188 | -0.2 | -8 | -10 | -0.5 | 0.1 | -1 | -1 | -1 | -2 | -2 | -3 |
| CM_Flag | 154 | 2 | 17 | 130 | 7 | 478 | 22 | 400 | 29 | 117 | 53 | 141 |
| ZM_Net | -34 | 2 | 10 | 120 | 6 | 478 | 21 | 399 | 27 | 115 | 51 | 138 |

Maximum values for the TF forces and moments are given in Table 5. These values result from all coils being at maximum current with the same polarity, and are equivalent to prior published results, except for M_bundle. Prior results reported the total torque integrated along the bundle without accounting for the load sharing according to the $\mathrm{z} / \mathrm{z}_{\text {max }}$ term. The calculation methodology given here, under maximum possible conditions, finds the total torque magnitude to be $36 \mathrm{ft}-\mathrm{klbf}$ and, with the inclusion of the $\mathrm{z} / \mathrm{z}_{\text {max }}$ term, $30 \mathrm{ft}-\mathrm{klbf}$ loading to the hub/spline/vacuum vessel load path.

TABLE 5: TF Forces and Moments for Maximum Possible Conditions

| F_Inner_TF_Flag | 3077 | lbf |
| :--- | ---: | :--- |
| F_Outer_TF_Flag | 5077 | lbf |
| M_Inner_TF_Flag | 40 | in-klbf |
| M_Outer_TF_Flag | 65 | in-klbf |
| Reff_Inner_TF_Flag | 13 | in |
| Reff_Outer_TF_Flag | 13 | in |
| SM_TF_Bundle | -30 | ft-klbf |
| DM_TF_Flag | 171 | ft-klbf |
| MM_TF_Net | 141 | ft-klbf |

Maximum values for the TF and PF forces under realistic maximum conditions are given in Table 6. These forces result from the coils being in worst case current combination within the range indicated in Table 7, which is constrained by the polarity considerations associated with normal plasma operation. Specific worst case current combinations were derived for each case and are given in Table 8. Note that PF4 energization is not included; this is a special case to be treated separately. Cases shown are limited to those relevant to the limiting structural support clamp designs, i.e. forces away from the midplane, except for PF5, whose struts must react the forces toward the midplane as well.

TABLE 6: TF and PF Forces and Moments for Realisitic Maximum Conditions

| F_Inner_TF_Flag | 1848 | lbf |
| :---: | :---: | :---: |
| F_Outer_TF_Flag | 2765 | lbf |
| M_Inner_TF_Flag | 25 | in-klbf |
| M_Outer_TF_Flag | 36 | in-klbf |
| Reff_Inner_TF_Flag | 13 | in |
| Reff_Outer_TF_Flag | 13 | in |
| \M_TF_Bundle | -28 | ft-klbf |
| \M_TF_Flag | 96 | ft-klbf |
| 之M_TF_Net | 78 | ft-klbf |
| Fz_OH | 35 | klbf |
| Fz_PF1a (max toward MP) | 11 | klbf |
| Fz_PF1b (max toward MP) | 82 | klbf |
| Fz_PF2 (max away from MP) | 92 | klbf |
| Fz_PF3 (max away from MP) | 156 | klbf |
| Fz_PF5 (max away from MP) | 144 | klbf |
| Fz_PF5 (max toward MP) | 157 | klbf |

TABLE 7: Realistic Maximum Current Range

|  | Min | Max |
| :--- | :--- | ---: |
| Ioh | -24.0 | 24.0 |
| Ipf1a | -24.0 | 5.0 |
| Ipf1b | -20.0 | 0.0 |
| Ipf2 | -20.0 | 0.0 |
| Ipf3 | -20.0 | 20.0 |
| Ipf4 | 0.0 | 0.0 |
| Ipf5 | 0.0 | 20.0 |
| Itf | 0.0 | 71.2 |

TABLE 8: Worst Case Currents (kA) for Various Limiting Conditions

|  | OH <br> Launch | PF1a <br> Away | PF1b Away | PF2 <br> Away | PF3 <br> Away | PF5 <br> Away | PF5 <br> Toward | TF (all except M_Net) | TF <br> (M_Net) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Ioh | 24 | 24 | -24 | 24 | 24 | -24 | 24 | 24 | -24 |
| Ipf1a | -24 | 5 | -24 | 5 | 5 | 5 | -24 | 5 | 5 |
| Ipf1b | -20 | -20 | -20 | 0 | -20 | -20 | -20 | 0 | 0 |
| Ipf2 | -20 | -20 | 0 | -20 | -20 | 0 | -20 | 0 | 0 |
| Ipf3 | -20 | 20 | -20 | 20 | -20 | 20 | -20 | 20 | 20 |
| Ipf4 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Ipf5 | 0 | 20 | 0 | 20 | 20 | 20 | 20 | 20 | 20 |
| Itf | 71 | 71 | 71 | 71 | 71 | 71 | 71 | 71 | 71 |

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