Current Profile Reconstruction using X-ray Imaging on the PEGASUS Toroidal Experiment

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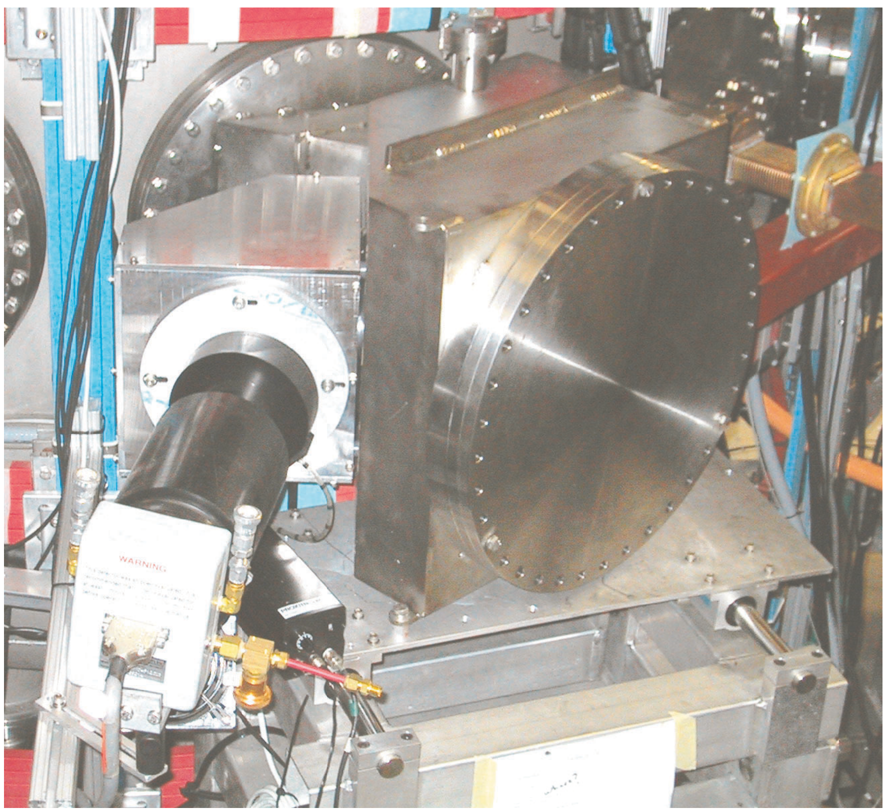

Monday Physics Meeting

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Thesis Statements

- Internal plasma profiles are necessary for equilibrium and stability analysis
  - $j(\psi)$, $p(\psi)$, $q(\psi)$

- External magnetic measurements are not sufficient for accurate profile reconstruction
  - Profiles are "integrally indistinguishable" to external magnetics
  - Artificial profile restriction can force dependence

- Tangential soft X-ray imaging allows accurate profile reconstruction for shaped toroidal plasmas
  - Intensity contours contain flux surface shape information
  - Shape information used to constrain equilibrium code

- Reconstructed Pegasus q profile demonstrates broad region of low shear
  - q profiles consistent with observed MHD activity

Pegasus Machine Overview

<u>Achieved Parameters</u>

$A = 1.12 - 1.3$

$R = 0.2 - 0.45$ m

$I_p = 0.16$ MA

$RB_T \leq 0.03$ T-m

$\kappa = 1.4 - 3.7$

$\Delta t_{pulse} = 0.01 - 0.03$ s

$\langle n_e \rangle = 1 - 5 \times 10^{19}$ m^{-3}

$\beta_T \leq 20\%$

$(\beta_t \equiv 2\mu_0 \langle p \rangle / B_{T0,vac}{}^2)$

- Pegasus is an Extremely Low Aspect Ratio Toroidal (ELART) experiment

- High toroidal beta, $\langle \beta_T \rangle$, can be accessed with ohmic-only operation
  - Auxiliary RF heating will help explore beta limits

- Flexible shaping allows access to high elongation and triangularity
  - Present vertical field index restricts κ to ~1.5

P' and GG' Profile Parameterizations used to Calculate $j_\phi(R,z)$

$$j_\phi(R,z) = R\frac{dP}{d\psi} + \frac{\mu_0}{R}G\frac{dG}{d\psi}$$

- Power law functional form has 2 free parameters each for P' and GG'

$$F(\psi_N) = F_0(1 - \psi_N)^{\alpha-1}$$

- Polynomial function allows varying number of free parameters

$$F(\psi_N) = F_0 + \sum_{i=1}^{n} F_i \psi_N^{i-1} - \psi_N^n \sum_{i=1}^{n} F_i$$

- Spline function based on knot positions and values

$$F(\psi_N) = AF_i + BF_{i+1} + CF_i'' + DF_{i+1}''$$

  - A,B,C,D calculated from position in interval
  - Second derivatives calculated from knot values
  - Natural splines used; second derivative = 0 at boundaries

Profile Parameters Calculated using Fitting Algorithms

- Linear least squares fitting uses SVD to solve measurement reponse matrix
  - Response matrix generated from partial derivative of measurements with respect to each parameter
  - Parameters updated using relaxation parameter for stability
  - “Picard” iterations interleave fitting solutions with Grad-Shafranov solutions for faster convergence

- Levenberg-Marquardt nonlinear fitting algorithm searches parameter space solution
  - Uses gradient search routine to minimize χ^2
  - Near minimum, uses quadratic approximation for direct fit
  - Requires careful evaluation with regard to local minima

- Both algorithms require measurements to constrain fitting routines
  - Measurement values from diagnostics
  - Current in external field coils, vacuum vessel walls
  - Equilibrium restrictions can be used (e.g. limits on q)

External Measurements are Insufficient to Constrain Profile Reconstruction

- External magnetic diagnostics measure integral quantities
  - Only allows determination of plasma boundary
  - External magnetic field depends only weakly on profile
  - Profiles can exist which are integrally indiscernible

- Restriction of allowed profiles can force dependence
  - Reduction of free parameters
  - Solution only allowed from small family of profiles
  - “Calibration” of profile functions using intermittent internal measurements may work for similar discharges

- Internal measurements necessary for accurate and free profile reconstruction
  - Spline parameterization decouples internal from boundary
  - MSE is standard internal measurement
  - Flux surface shape can also constrain internal profiles

Knowledge of the Flux Surface Shape can Specify the Current Profile

- If the flux surfaces are defined as F(R,Z) = *constant*, then $\psi=\psi(F)$ and the G-S equation can be rewritten*:

$$\frac{d^2\psi}{dF^2}|\nabla F|^2 + \frac{d\psi}{dF}\Delta^* F = -\mu_0 R^2 \frac{dp}{d\psi} - g\frac{dg}{d\psi}$$

- All of the unknowns (ψ'', ψ', p', gg') are constant on a flux surface, and Δ^*F and $|\nabla F|^2$ vary over a flux surface

- By convolving this equation with a known function whose flux surface average is zero, one defines ψ(F) in terms of known quantities:

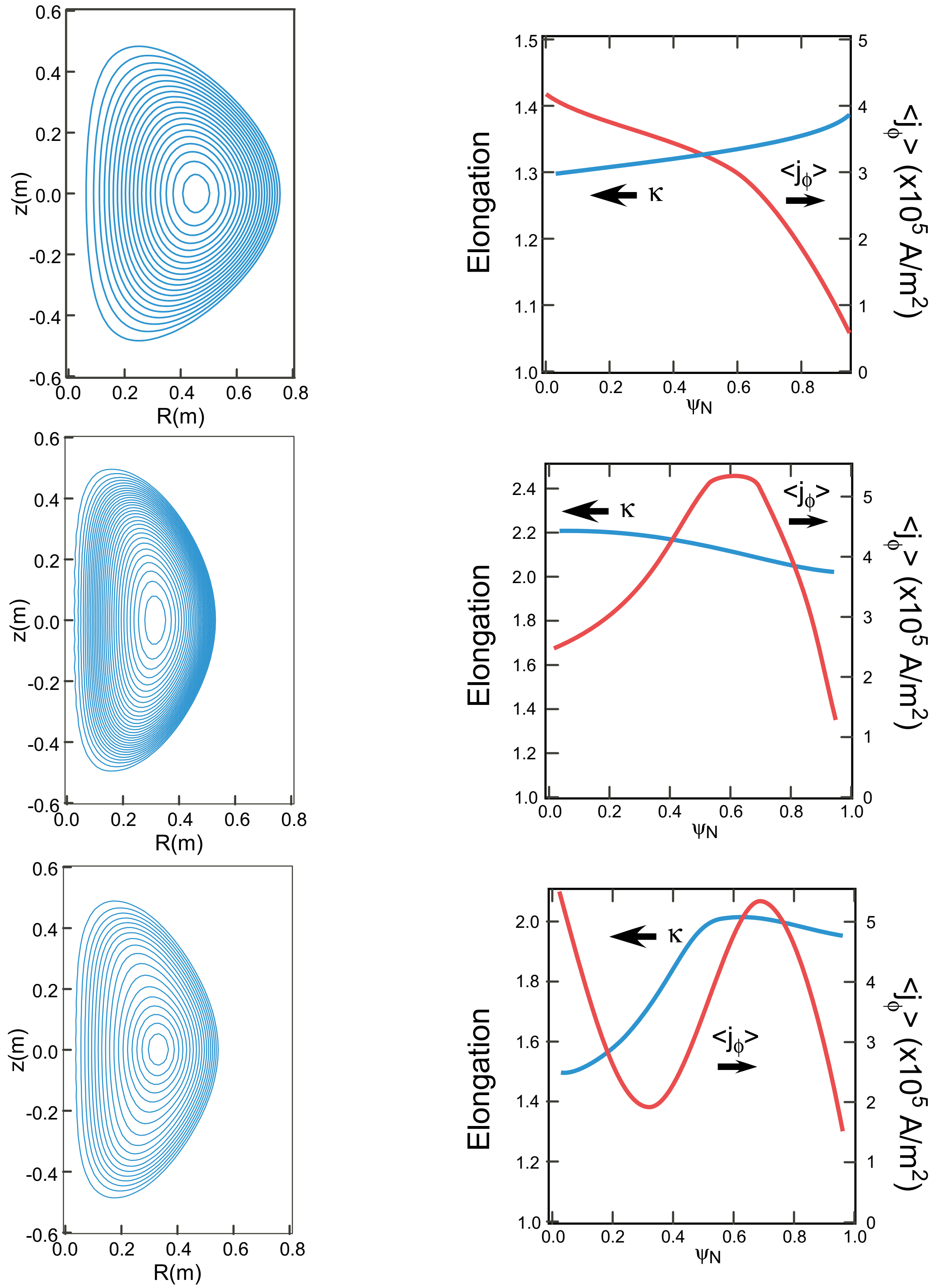
$$-\frac{d^2\psi}{dF^2}\left(\frac{d\psi}{dF}\right)^{-1} = \lambda(F)$$

- λ(F) depends on the flux surface variation of ∇F, which vanishes in the large aspect-ratio, circular flux surface limit
  - Solution for ψ determines j(R,z) through G-S equation
  - Shaped plasmas ideal for measurement technique

***Christiansen, J.P., Taylor, J.B., Nucl. Fusion 22 (1982) 111**

Current Profile and Flux Surface Elongation have a Strong Relationship

I_p, plasma height held constant

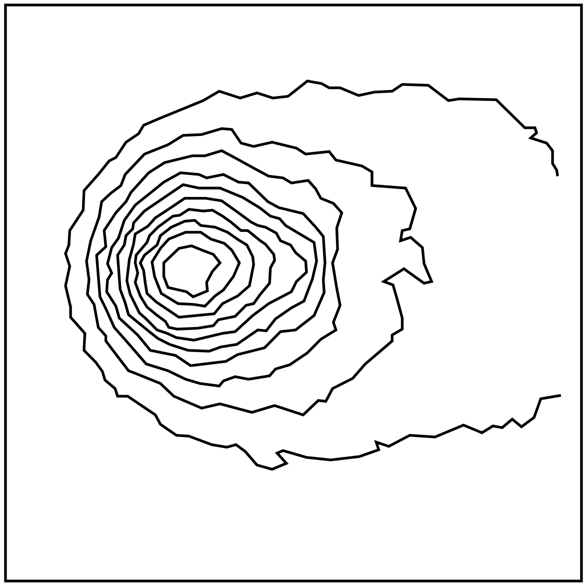


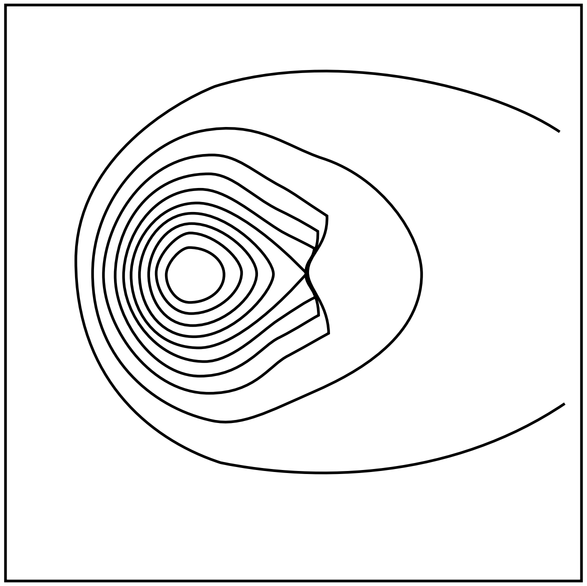
Two Methods of Tangential X-ray Imaging Have been Evaluated

- Vertically spaced tangentially-viewing horizontal arrays of X-ray diodes
  - Obtain intensity profiles at several vertical locations
  - Intensity profiles can be directly Abel inverted
  - Emissivity profiles identify coordinates of equal flux
  - Sub-sampled flux surface used to constrain reconstruction
  - Precise calibration necessary
  - Hardware and electronics implementation challenging

- 2-D tangentially-viewing soft X-ray pinhole camera
  - Obtain 2-D tangential intensity image
  - More compact, less complex imaging system
  - Forward modeling used to compare image to equilibrium projection
  - Analysis more complex and numerically intensive
  - Line integrated measurements subject to information loss

A 2-D Tangential Soft X-ray Pinhole Camera was used on PBX for Equilibrium Reconstruction



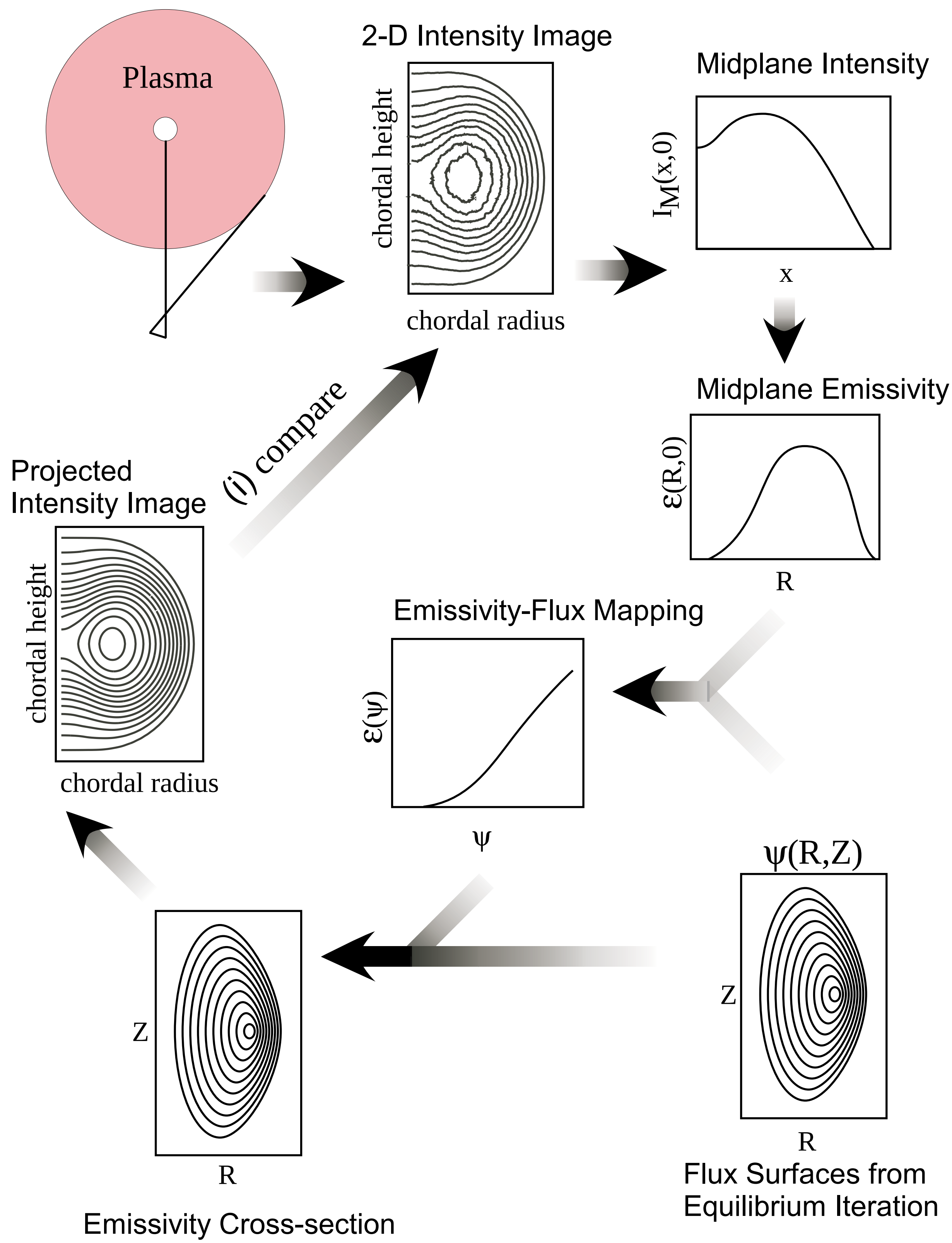


- Flux surface shape information contained in intensity contours
  - Forward modeling avoided direct 2-D inversion noise sensitivity

- Restricted profiles used for equilibrium reconstruction
  - q_0 manually scanned to find best fit of equilibrium projection to measured image

***Powell, E.T., et. al., Nucl. Fusion 33 (1993) 1493**

Forward Modeling is Integrated into Equilibrium Reconstruction Code

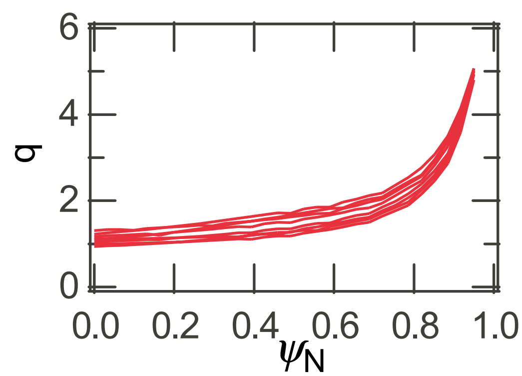


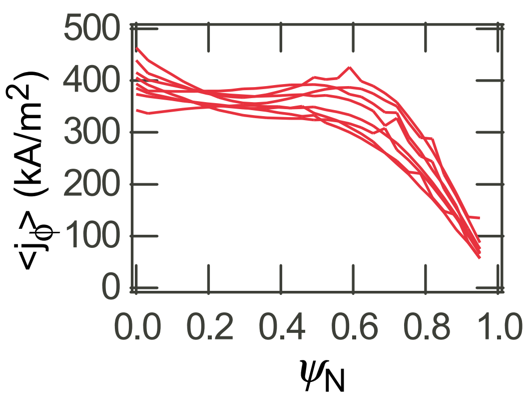
Several Imaging Constraints were Evaluated for Noise Sensitivity

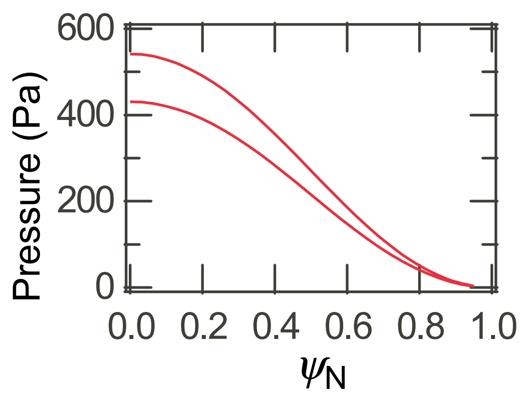
- 1-D tangentially viewing linear arrays
  - Code attempts to match flux for each set of specified coordinates
- Ellipticity of emissivity (flux surface) contours
  - Modeling of technique used on JET for profile reconstruction
- Emissivity contour path constraint
  - Code minimizes RMS distance between model and fitted emissivity contour paths
- Tangential intensity image residual (PBX method)
  - Minimization of RMS difference between model image and fitted image
- Intensity contour path constraint
  - Code minimizes RMS distance between model intensity contours and fitted projection contours

Monte Carlo Modeling Used to Determine Sensitivity to Noise and Initial Parameters

- Nonlinear fitting requires initial parameter guess
  - Parameters varied ~10% of model value



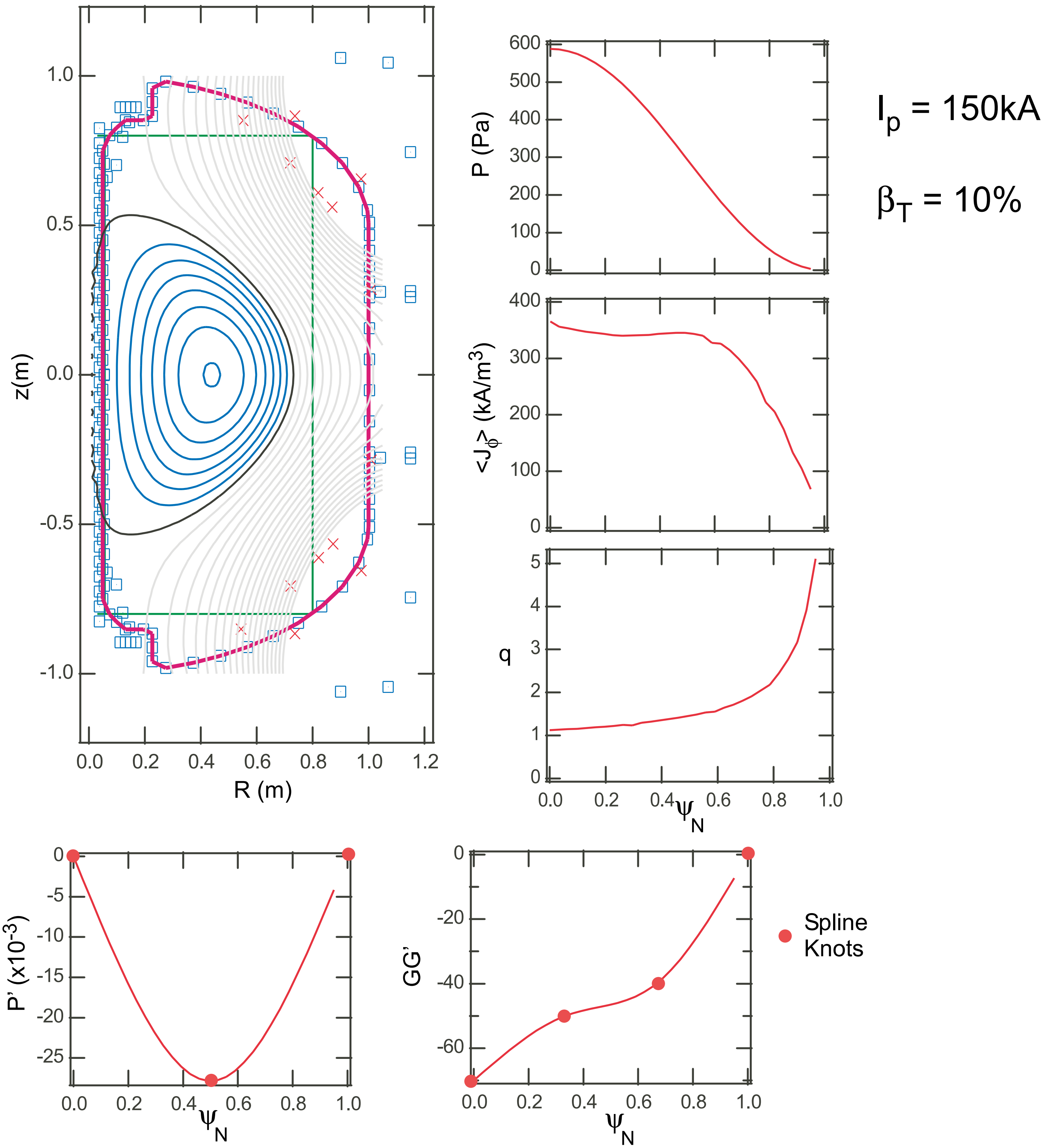




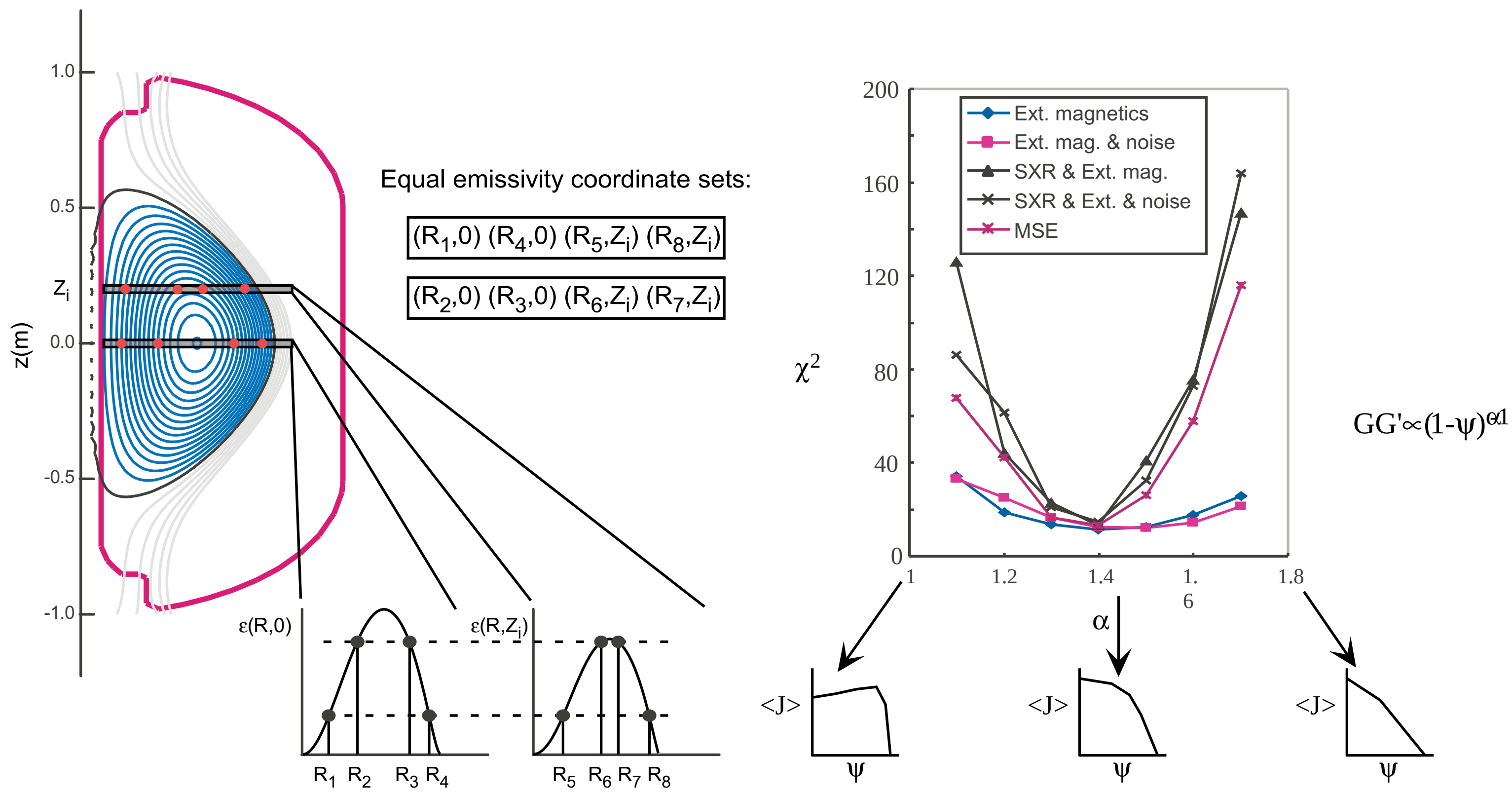
- Variation of initial guess and added noise demonstrates robustness of reconstruction
  - Noise added to external measurements and imaging constraint

- Parameters and profiles collected for each Monte Carlo iteration for statistical analysis

Canonical Pegasus Equilibrium Used for Constraint Sensitivity Modeling



1-D Tangentially Viewing Linear Array Constraint Demonstrated Profile Sensitivity

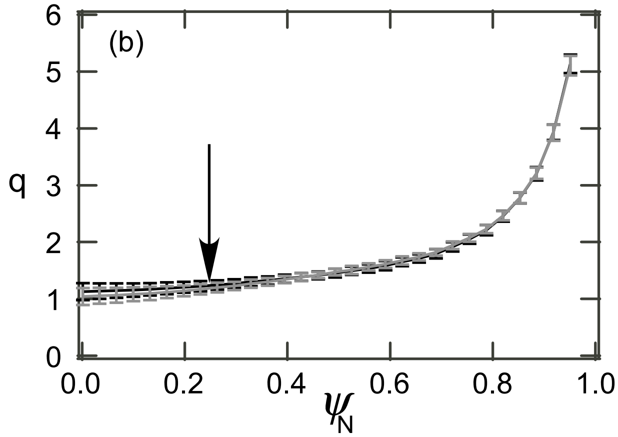


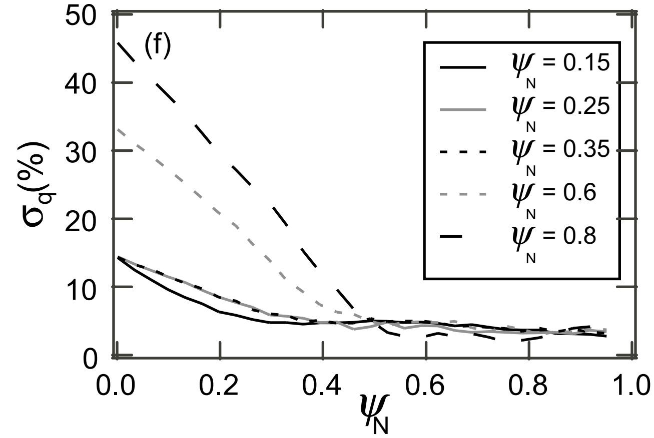
- Several sets of arrays needed for sensitivity, N ~ 5

- 1-D constraint demonstrated profile sensitivity similar to MSE measurement constraint

- External-magnetics-only reconstructions showed poor sensitivity to current profile

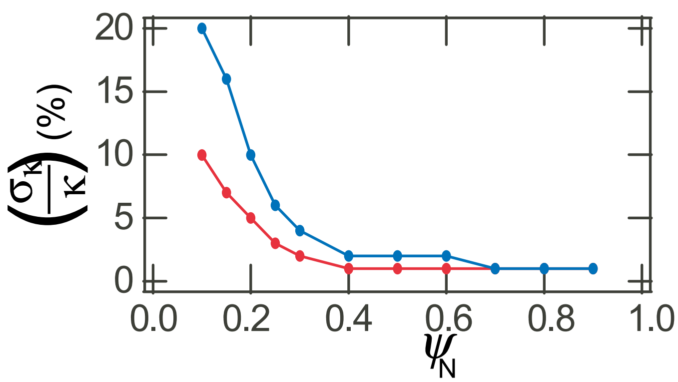
Emissivity Elongation Constraint Demonstrates q Profile Deviation < 10%



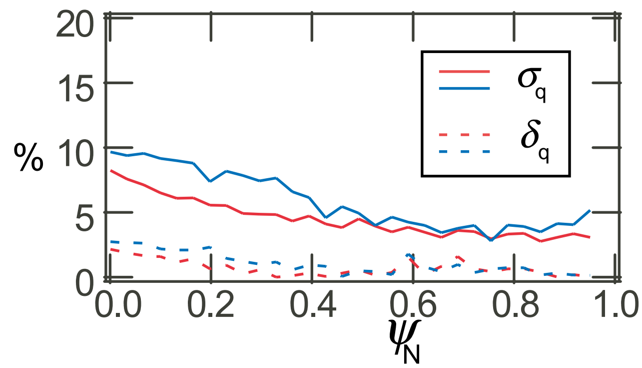


- Single point elongation measurements constrain q profile for $\psi_{N,meas} < 0.35$

Imposed elongation noise



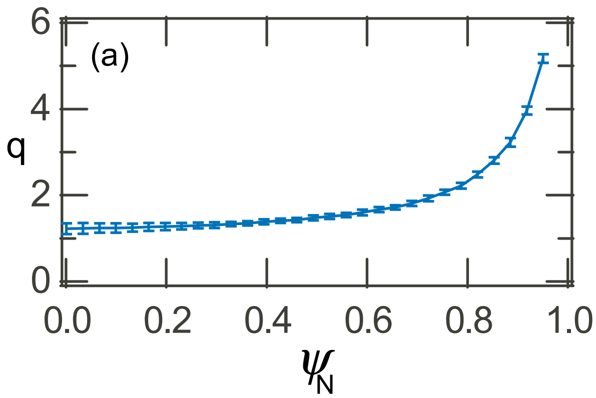
q profile deviation

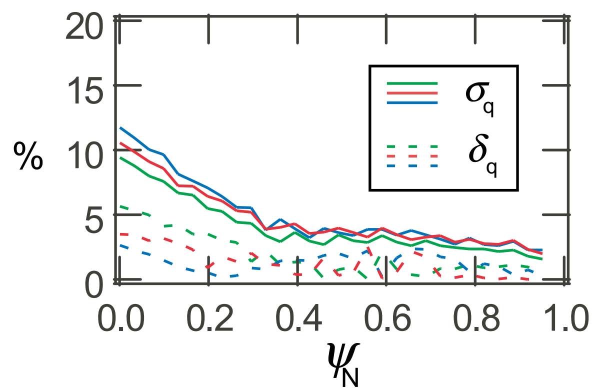


- Constant emissivity image noise translates to increasing noise on κ measurement as ψ_N -> 0

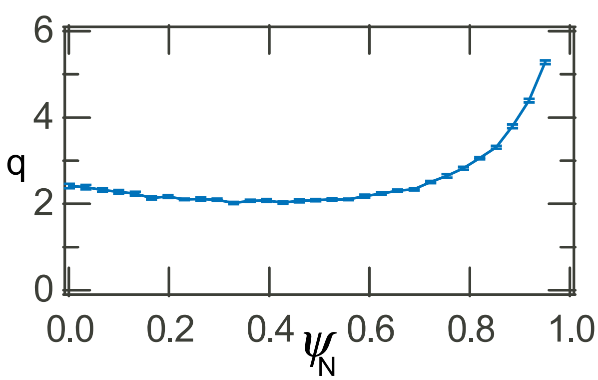
- Multipoint ellipticity constraint reconstructs q profile with < 10% error

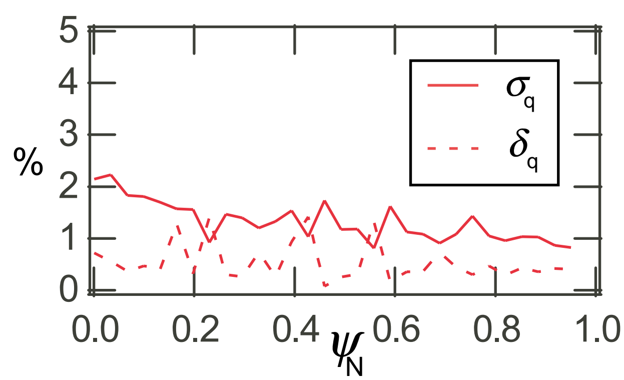
Emissivity Contour Constraint Comparable to Elongation Constraint





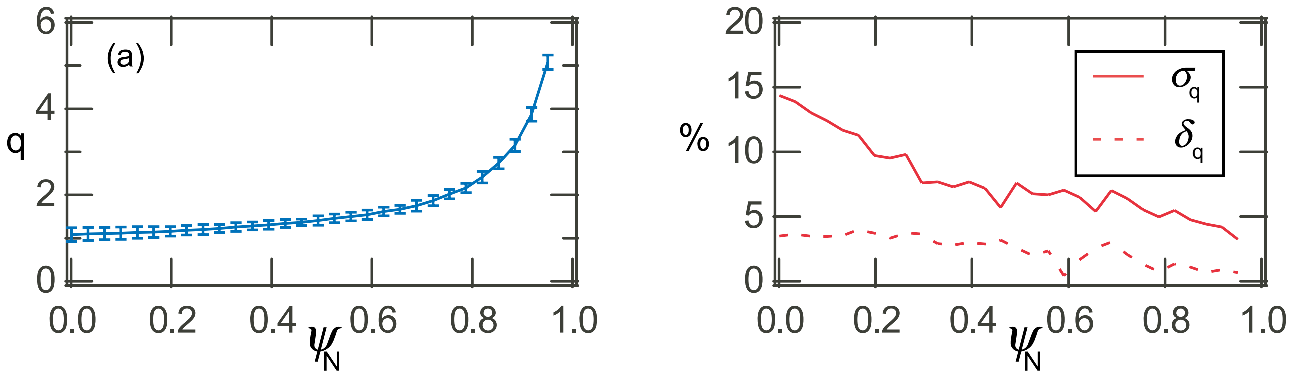
- 2% image noise constrains q profile to ~< 10%
- Peaked (green) and broad (blue) intensity profiles demonstrate, respectively, better and worse constraint



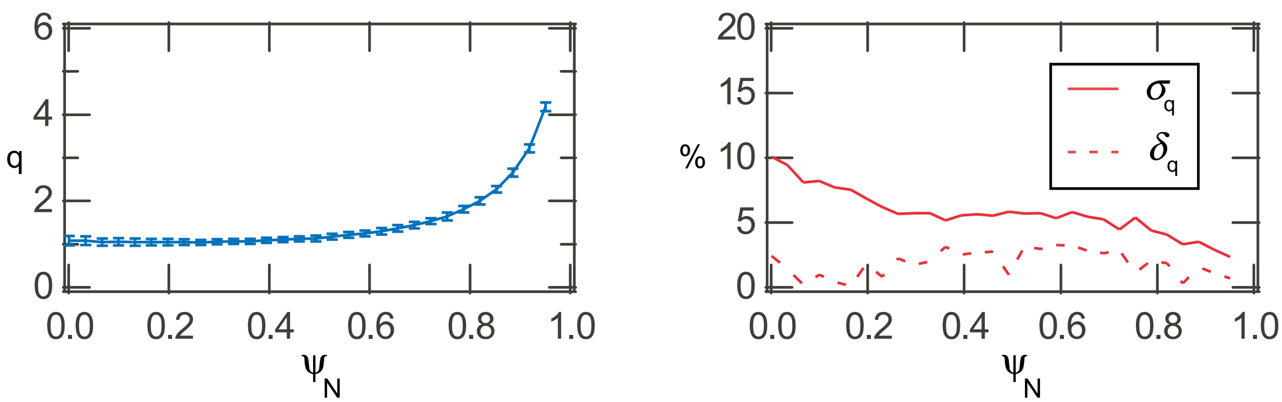


- Hollow current profiles show better sensitivity to emissivity contour constraint
  - 2% image noise constrains q profile to ~< 2%

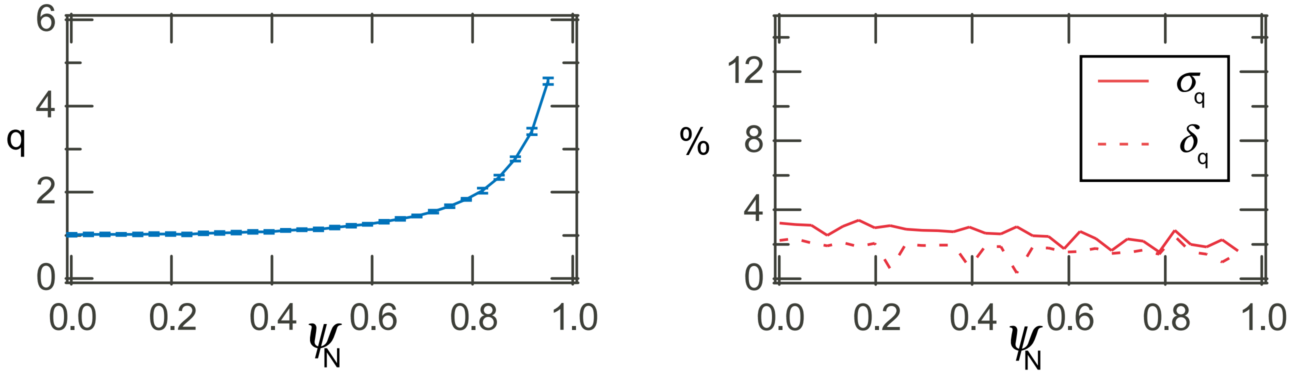
Intensity Contour Constraint Demonstrates Profile Sensitivity for Image Noise ~ 1%



- Reconstruction slightly less sensitive than emissivity contour constraint



- High β_T (~25%) reconstruction shows better sensitivity



- Reduction of reconstruction free parameters corresponds to very tight constraint of q profile

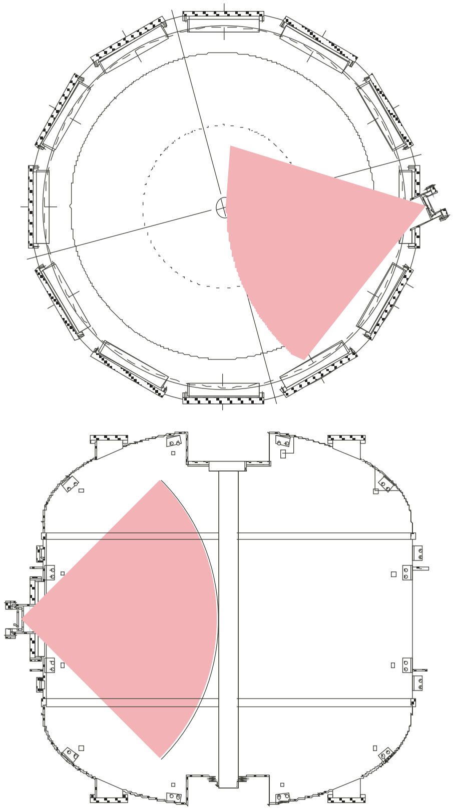
Both Emissivity and Intensity Contours Constrain Profile Reconstruction

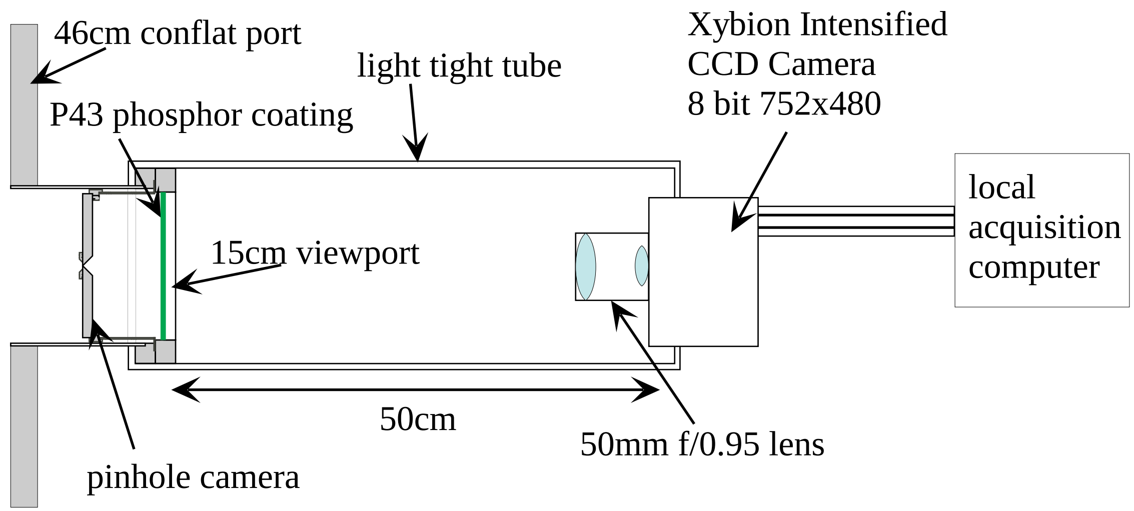
- Emissivity constraints slightly more sensitive than intensity constraints
  - Profiles well constrained with 2% image noise
  - 2-D emissivity image must come from intensity inversion
  - 2% emissivity noise may require << 1% intensity noise for accurate inversion

- Intensity residual constraint demonstrates poor reconstruction sensitivity
  - Smooth image corresponds to small RMS deviation
  - Tangential intensity images inherently smooth

- Intensity contour constraint demonstrates good constraint for image noise ~< 1%
  - SNR achievable with present imaging technology
  - High performance, high β_T plasma show better sensitivity

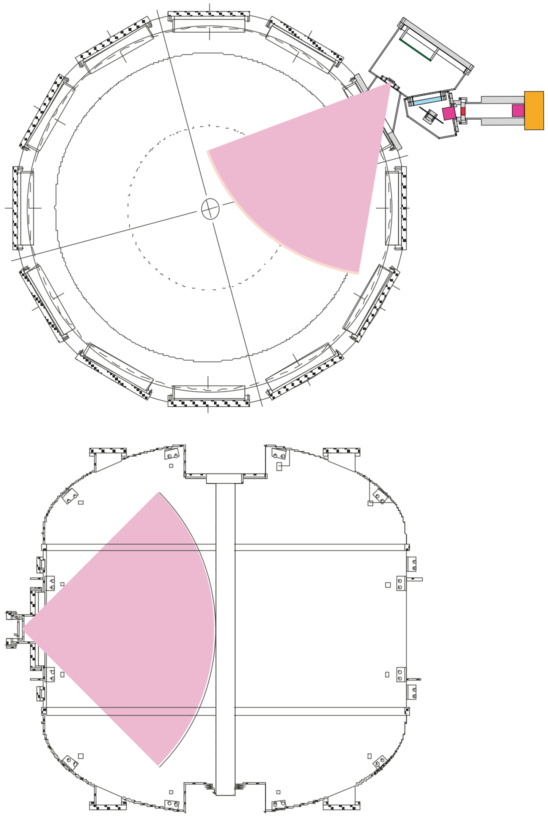
Prototype Soft X-ray Pinhole Camera Imaging System Schematic and Machine Field of View

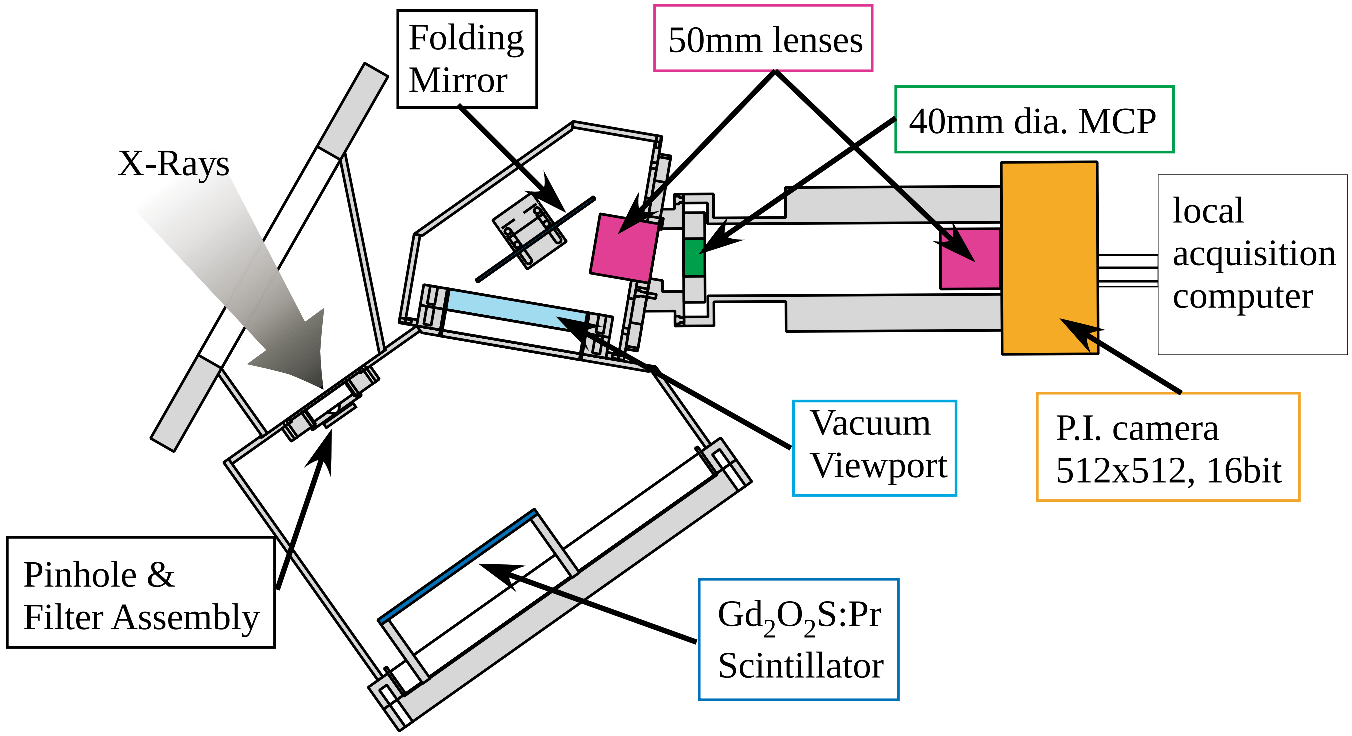




- System used a 0.05cm dia. pinhole with a 0.1µm beryllium filter
- High efficiency P43 (Gd_2O_2S:Tb) phosphor converts X-rays to visible light

First Generation Soft X-ray Imaging System Hardware Schematic and Machine FOV

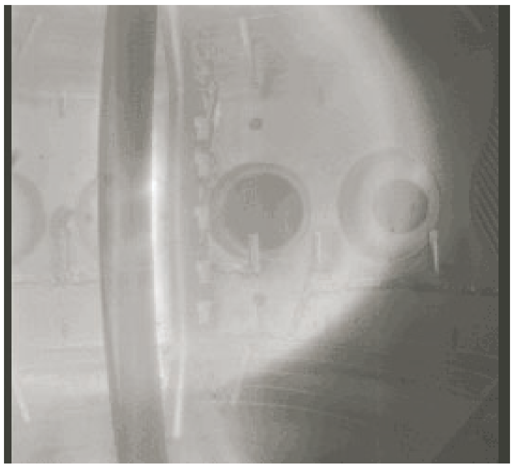




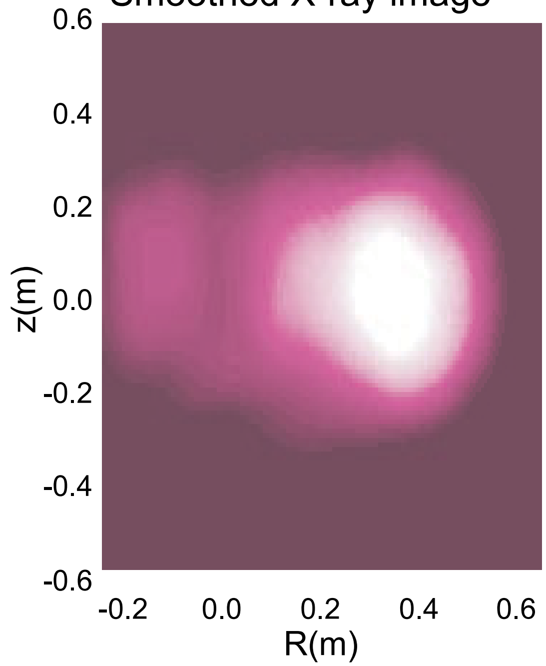
- Reflective phosphor imaging improves signal x2-5
- Gd_2O_2S:Pr phosphor used for faster time response (<100µs)

First Generation Soft X-ray Imaging System Used for Equilibrium Reconstruction Shot #14729

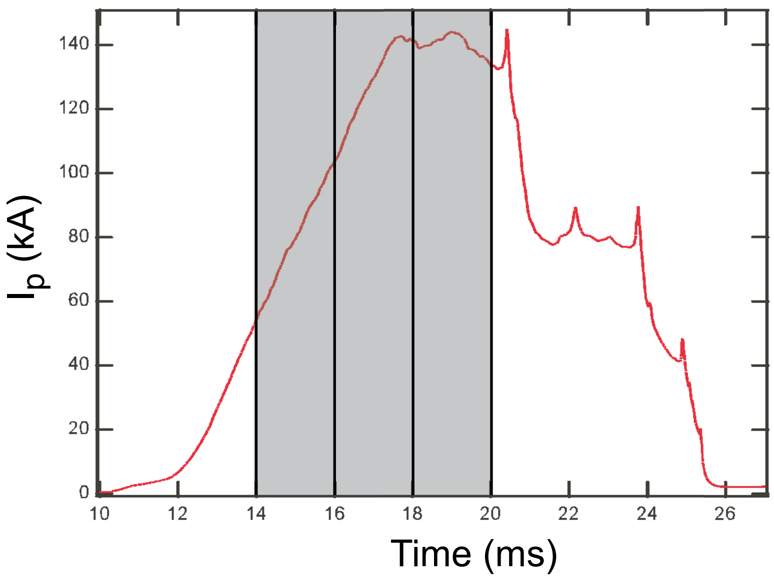
Dalsa camera image



Smoothed X-ray image

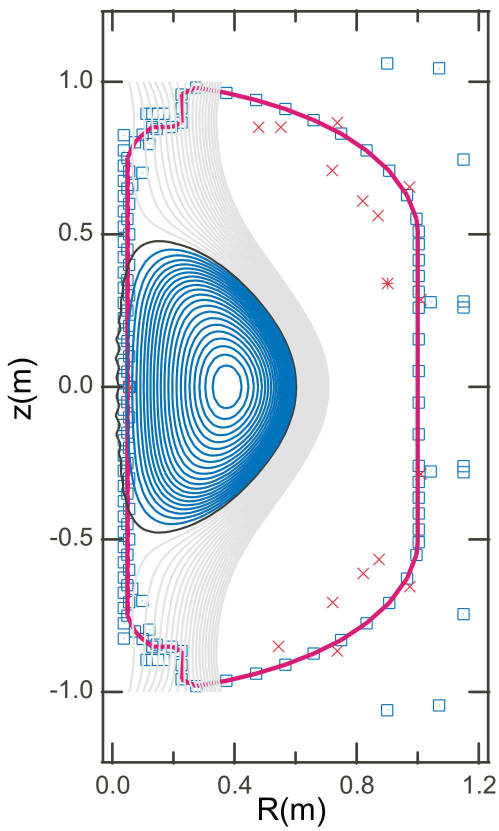


X-ray image exposure from 18-20ms



- X-ray signal low until final exposure time slice
- Multishot averaging used to boost SNR of X-ray image
- Shot-to-shot positional uncertainty complicated reconstruction
  - Images were vertically centered to minimize smearing artifacts

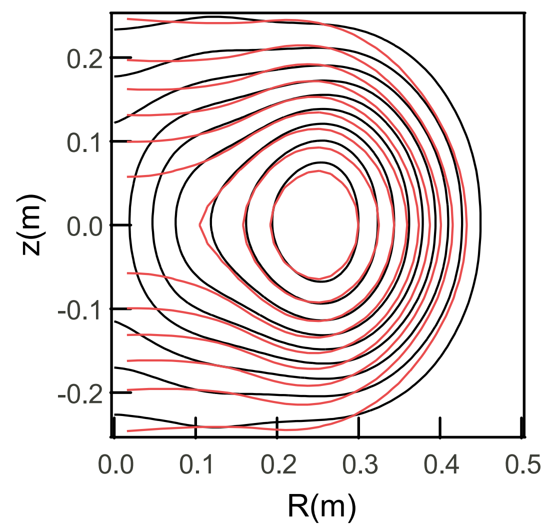
Shot #14729 Reconstruction Using Image Contour Constraint

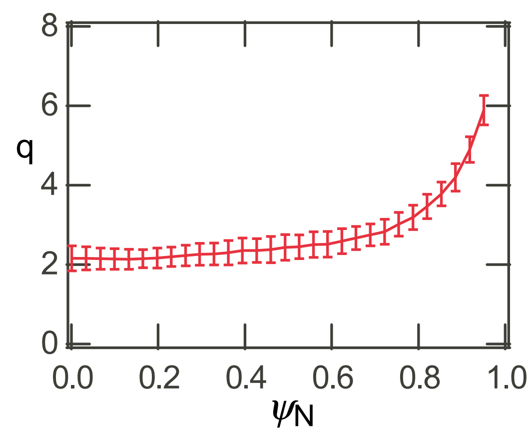


Reconstructed Parameters

I_p	140 kA
β_T	0.12
ℓ_i	0.44
q_0	2.2
A	1.2
κ	1.5

Measured Itensity/
Fitted Intensity Overlay

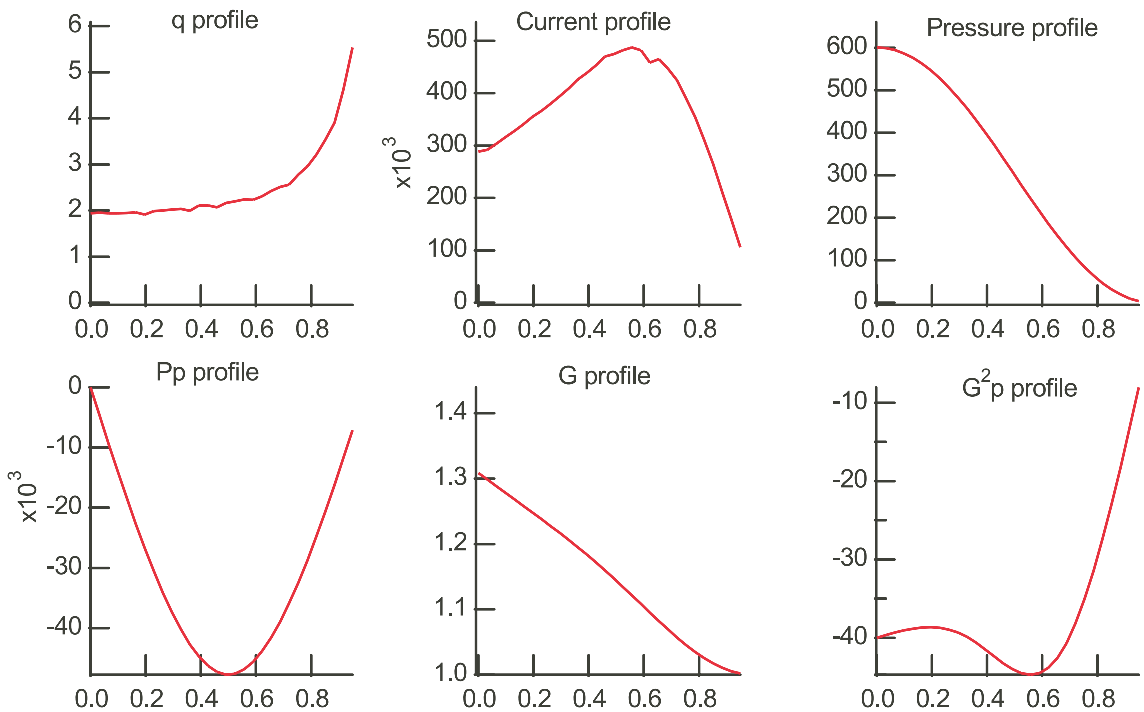




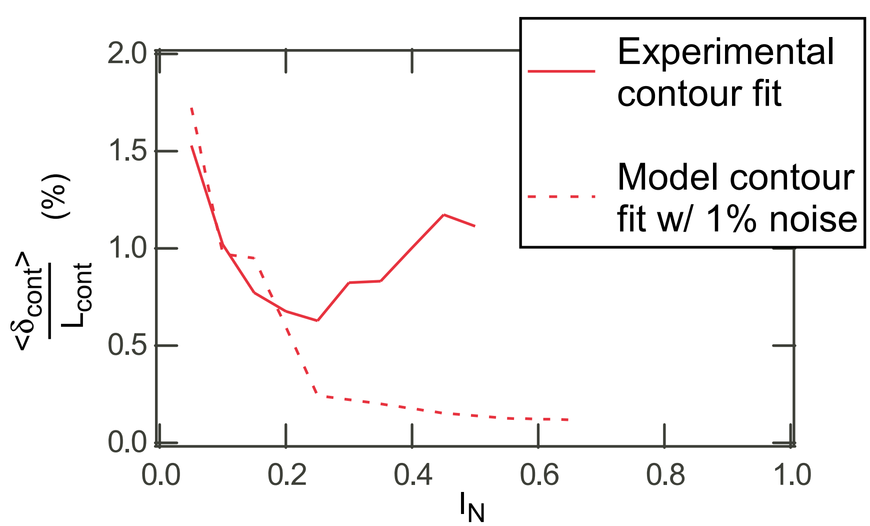
- $q_0 > 2$ consistent with lack of large coherent MHD activity

- Pegasus discharge often contains coherent 2/1 or 3/2 MHD mode activity

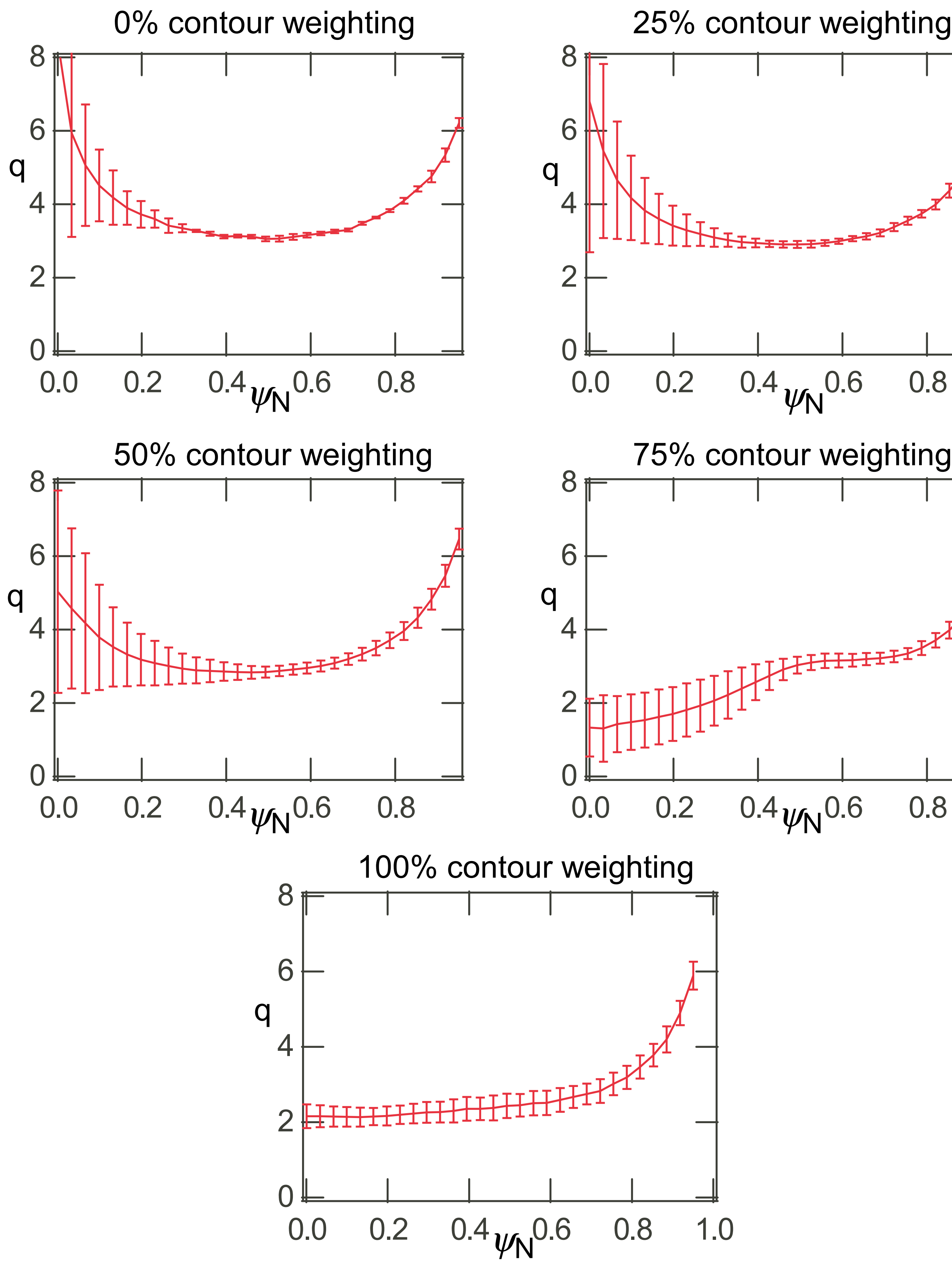
Hollow Current Profile Consistent with Fast Current Ramp



- Normalized contour fit metric consistent with image noise ~2%

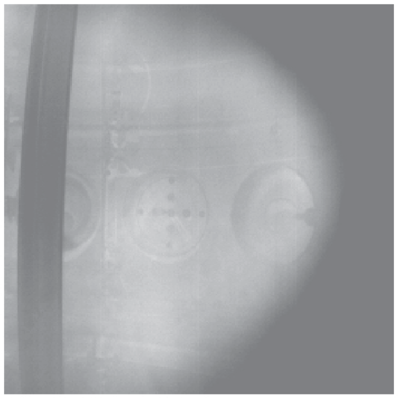


Increased Weighting of Intensity Contour Constrains Central q

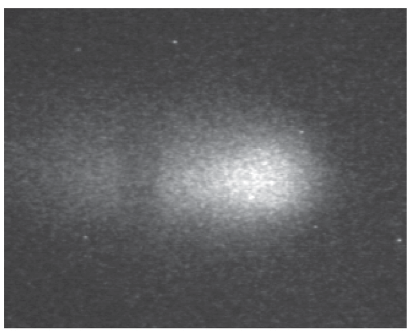


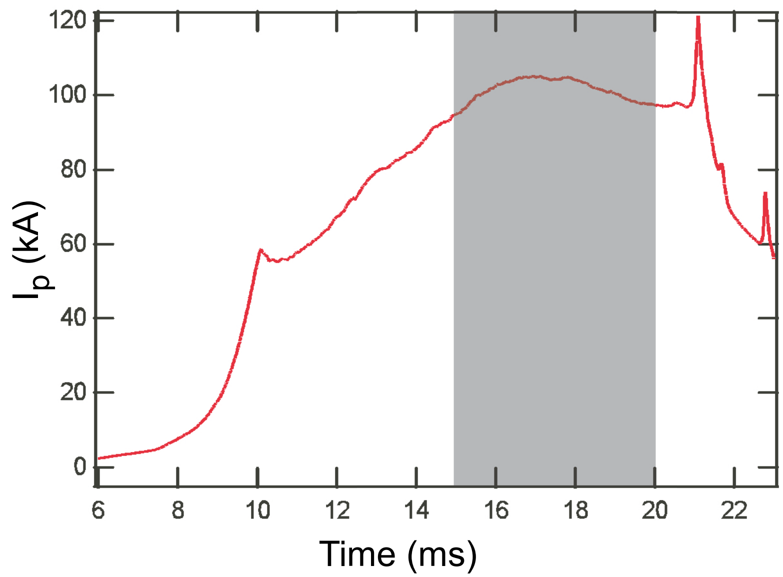
Prototype Generation Soft X-ray Imaging System Used for Equilibrium Reconstruction Shot #9639

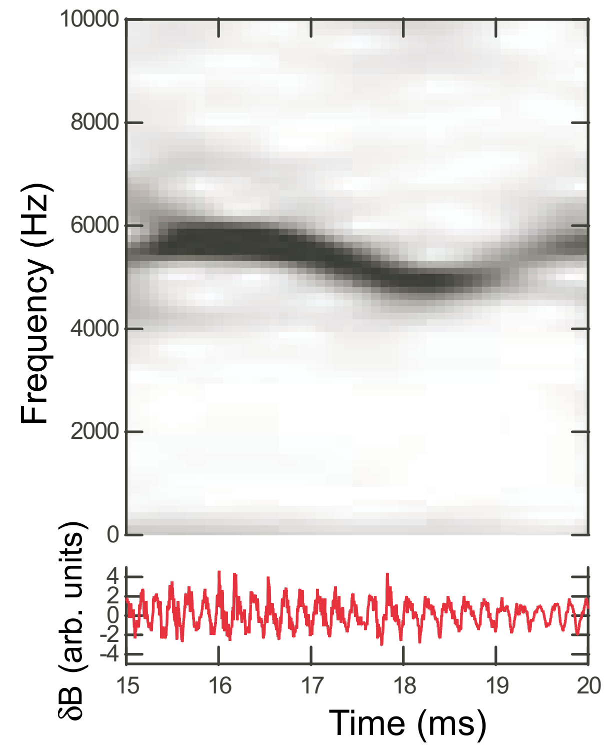
Dalsa image



X-ray image

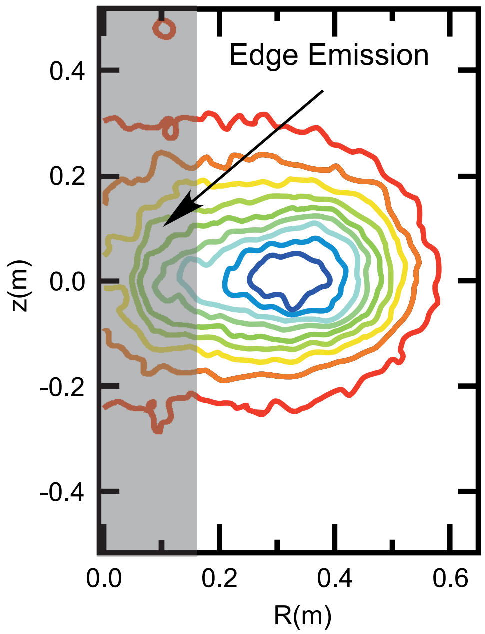


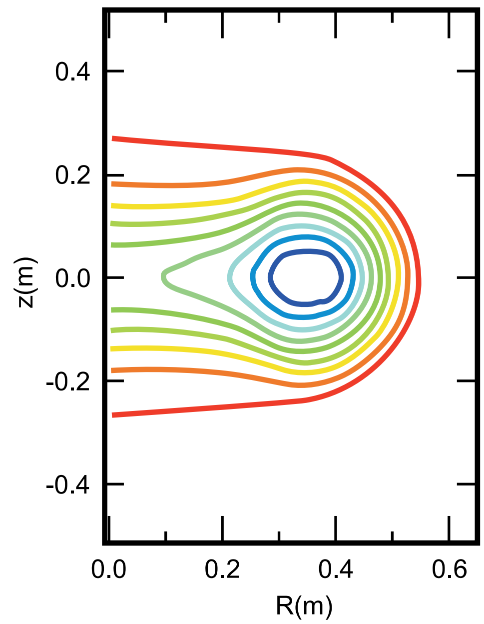




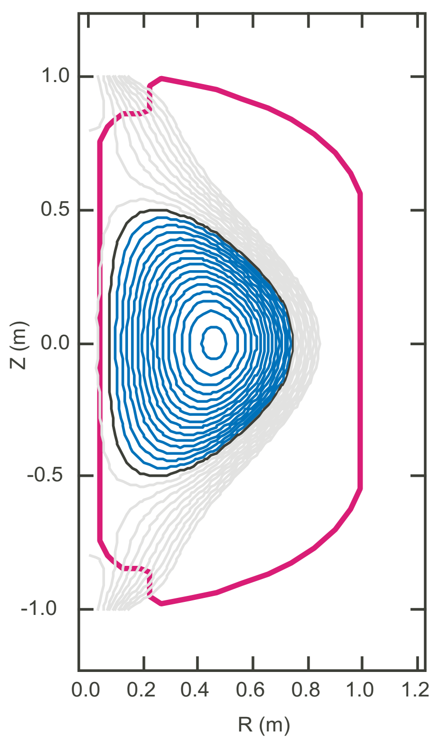
- X-ray image acquired over 5ms plasma current flattop
- Large scale coherent 2/1 mode MHD activity seen at 5-6 kHz

Image Residual Used with Power Law Parameterization

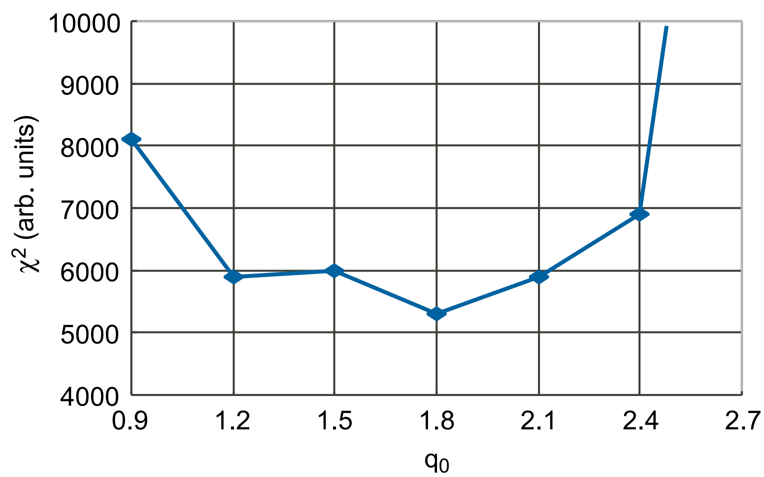




- Inner intensity image (R<0.2m) ignored to avoid edge impurity signal

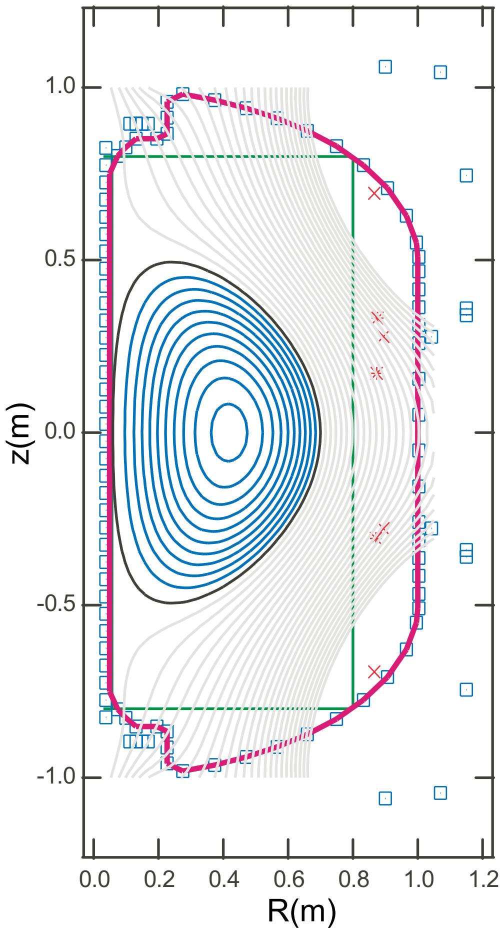


I_p	87 kA	q_0	1.8
β_T	0.06	A	1.16
ℓ_i	0.36	κ	1.4

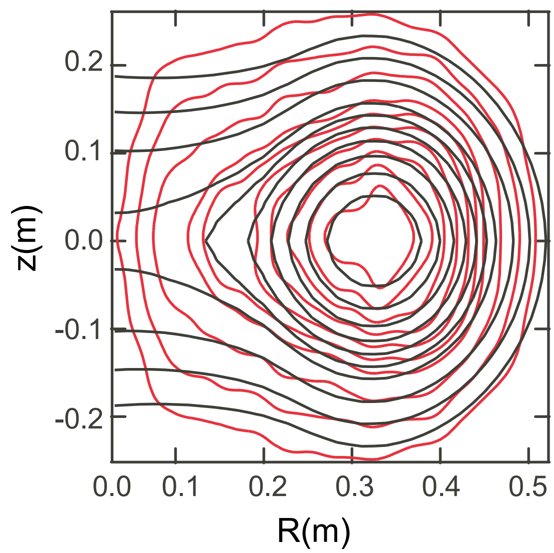


- Manual q profile scan verified best fit at q_0 = 1.8

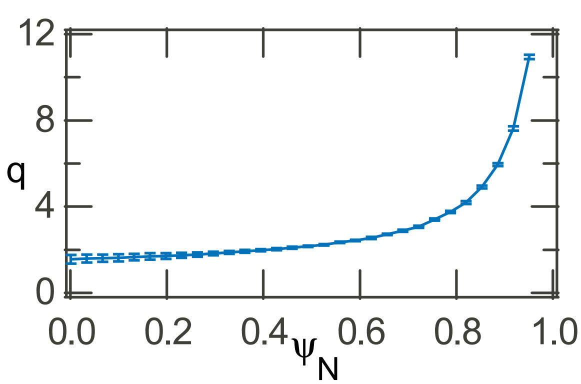
Image Contour Constraint Used with Spline Parameterization

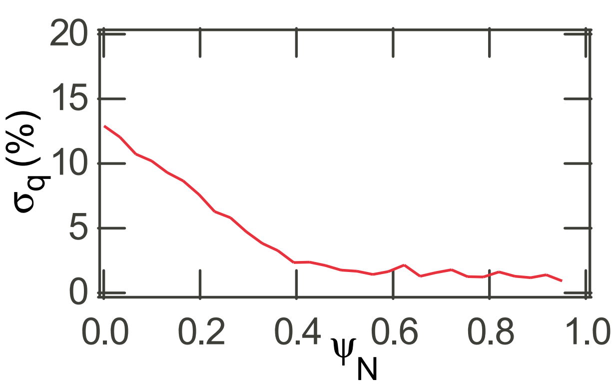


I_p	93 kA
β_T	0.03
ℓ_i	0.65
q_0	1.6
A	1.17
κ	1.4



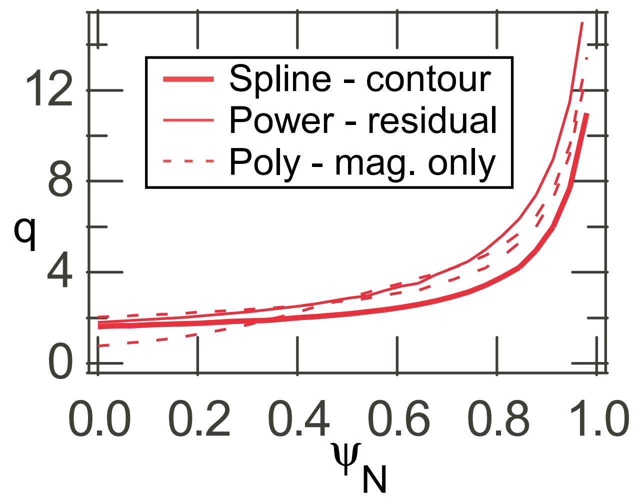
- q profile variation < 13%

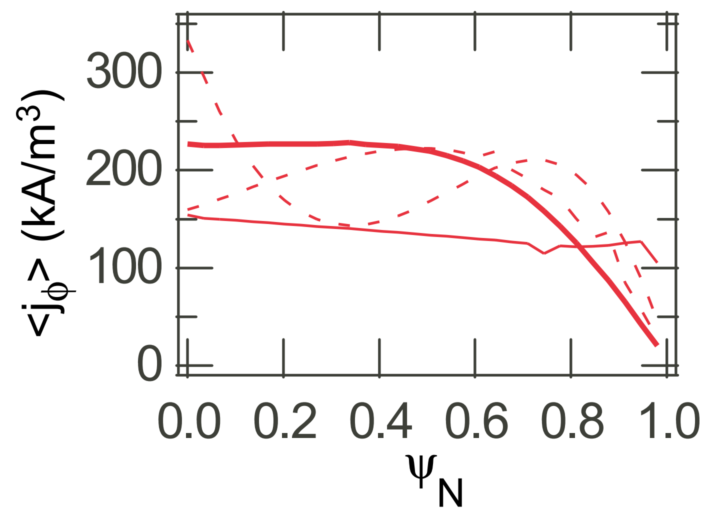




- Contour fit metric corresponds to ~4% image noise

Reconstruction Comparisons Show Some Differences





- Profiles vary significantly between reconstructions
  - Image contour constraint with spline parameterization shows flat current profile at plasma center consistent with large scale island

	Power	Spline	Poly
I_p	87 kA	93 kA	92kA
β_T	0.06	0.03 +/- 0.02	0.05
l_i	0.36	0.65 +/- 0.03	0.46
q_0	1.8	1.6 +/- 0.2	2.0
q_{95}	11	7.8 +/- 0.1	9
A	1.16	1.17 +/- 0.004	1.17
κ	1.4	1.4	1.4

- Reduced magnetics diagnostic set may account for differences

Summary of Experimental Results

- Tangential image contour measurements constrain q profile reconstruction to < 15%
  - Contour fit metric corresponds to image noise 2-4%
- q profile reconstructions are consistent with presence and absence of large scale MHD
  - Flat current profile in 9639 consistent with large island
- Reconstructions with magnetics only measurements show no ability to constrain profiles
  - Accurate magnetics still necessary to avoid reconstruction conflicts
- Higher SNR will tighten image constraint
  - Should approach model constraint with q_0 deviation at 10%

Potential Upgrades

- Direct illumination X-ray imaging system
  - X-ray exposure of CCD sensor with pixel mask for exposure control and multiple timepoint capability
  - Will have few 100 times better sensitivity than phosphor system
- Code parallelization will speed up total reconstruction time
  - Use Beowulf cluster for economical multiprocessing
  - LM nonlinear fitting trivial to parallelize
  - Can use dense solution grid for better match to stability codes
- Tangential imaging system can be used on other fusion experiments
  - Strongly shaped plasmas sensitive to flux surface constraint
  - Validate on well-diagnosed advanced tokamak (DIII-D, NSTX?)
  - Investigate advanced operational regimes (e.g. reversed shear)
  - Assist experiments which lack q profile diagnostics (NSTX)

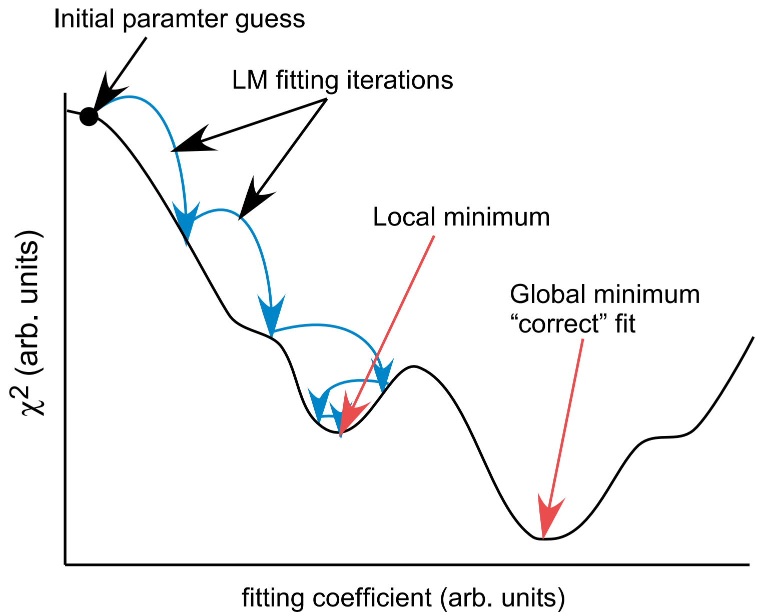
Conclusions

- Internal profiles are crucial for proper physics understanding of plasmas
- External magnetic diagnostics alone will not constrain the plasma profiles accurately
- Flux surface shape information provides a good constraint on profile reconstruction
- Tangential X-ray intensity images will provide adequate constraint with image noise ~< 1%
- Pegasus q profile reconstruction verifies low-A features: broad flat q profile with high edge shear
  - Constrained profiles consistent with observed MHD activity
- Future upgrades will improve system for Pegasus and potentially other interested research programs

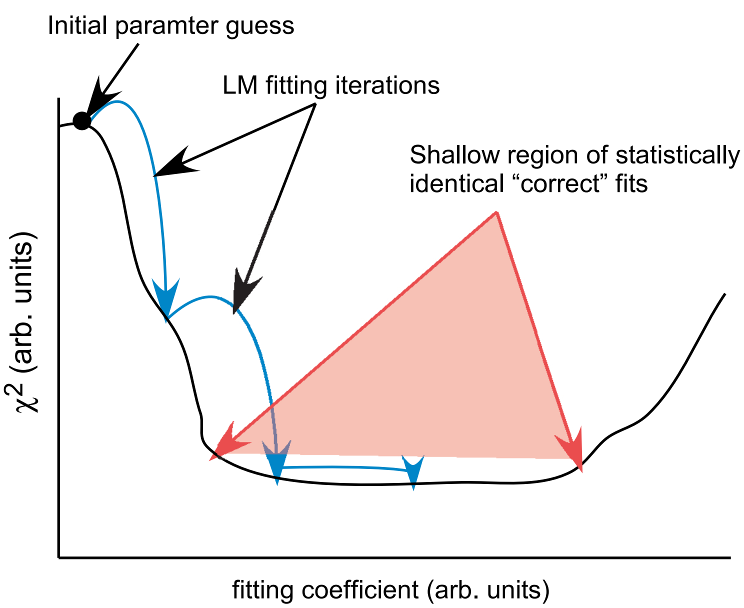
X-ray Emissivity Poloidal Asymmetries have been Observed on Other Machines

- Alcator C-MOD observes up-down asymmetries near transport barrier in H-mode plasmas
  - Impurity concentration poloidal asymmetry driven by collisional friction with bulk deuterium ions
- JET has large outboard impurity peaking during NBI
  - Unbalanced NBI drives large toroidal rotation
  - Centrifugal force enhances radial diffusion of heavy ions
- JET has large inboard impurity peaking during RF heating
  - Heating absorbed by hydrogen minority ions
  - Hydrogen ions collect on outboard, creating large electric field which drives impurities towards the inboard side
- PBX identified operational solution to problem
- Pegasus will generally avoid these regimes of operation
  - No neutral beam current drive
  - RF heats bulk electron population

Monte Carlo Analysis can Identify and Compensate for “Algorithm Trapping”

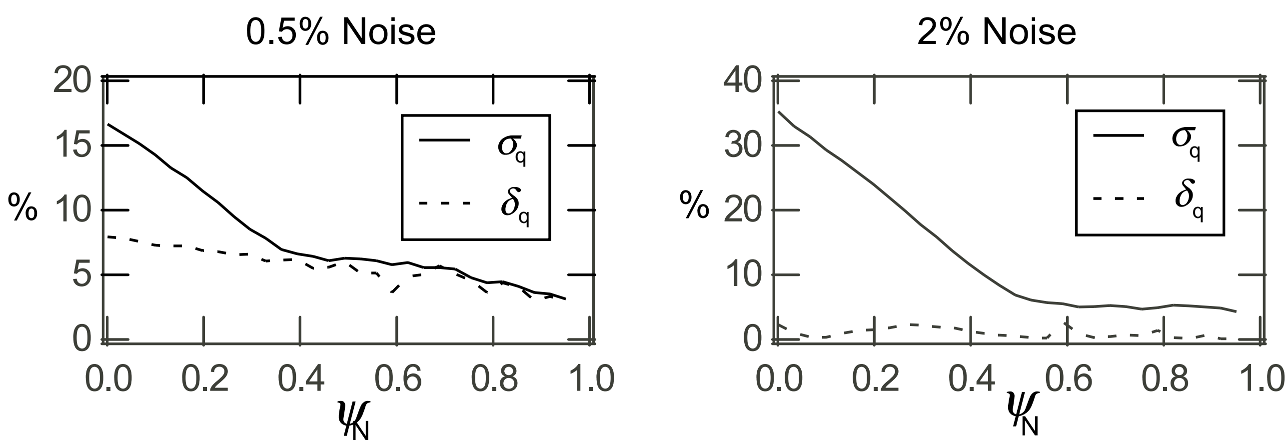


- Nonlinear fit can become trapped at local minimum

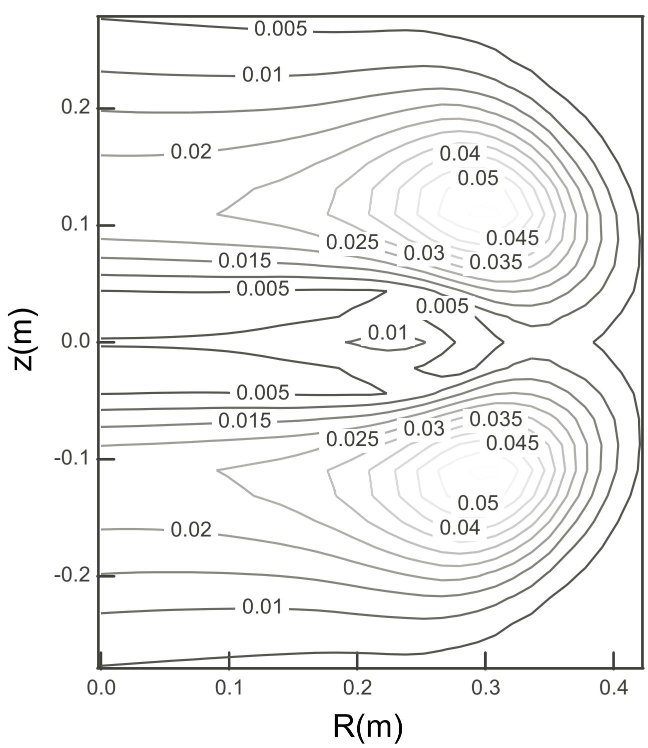


- Shallow chi-square space indicates poor sensitivity
- Analysis of χ^2 statistics can differentiate the two cases

Intensity Residual Constraint Shows Poor Reconstruction Sensitivity



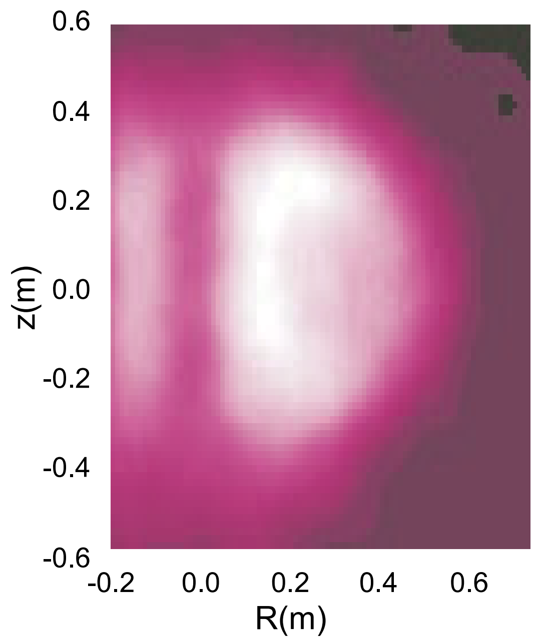
- q profile deviation > 30% with 2% image noise

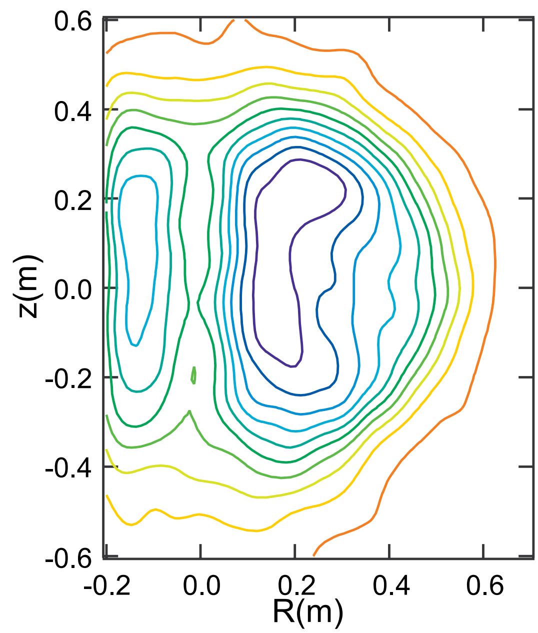


RMS image difference between $q_0 = 1.1$ and $q_0 = 2.0$ projected equilibrium

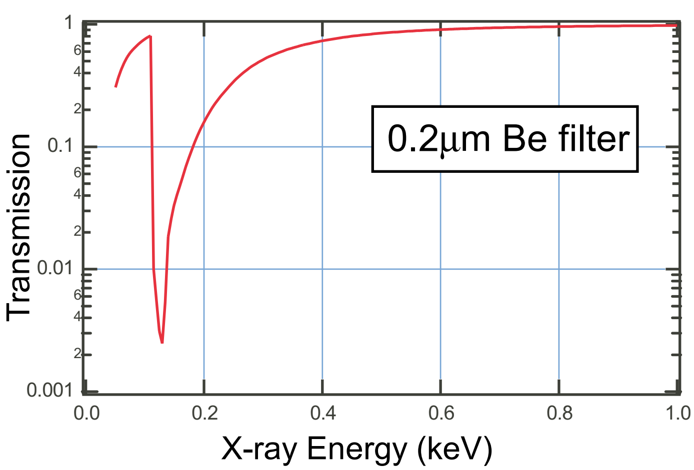
- Image residual method insensitive to changes in q_0
  - Central residual less than imposed noise
  - Smoothness of images causes profile insensitivity

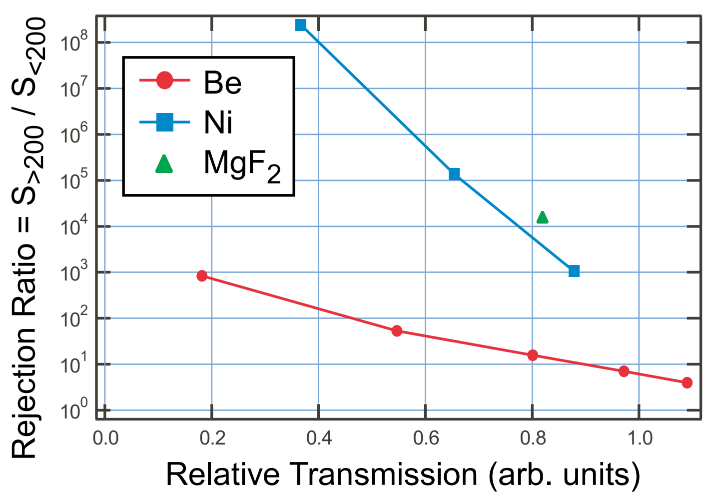
X-ray Filters Optimized to Reject Low Energy Emission from Edge





- Oxygen line radiation from limiter impurity influx pollutes tangential intensity image

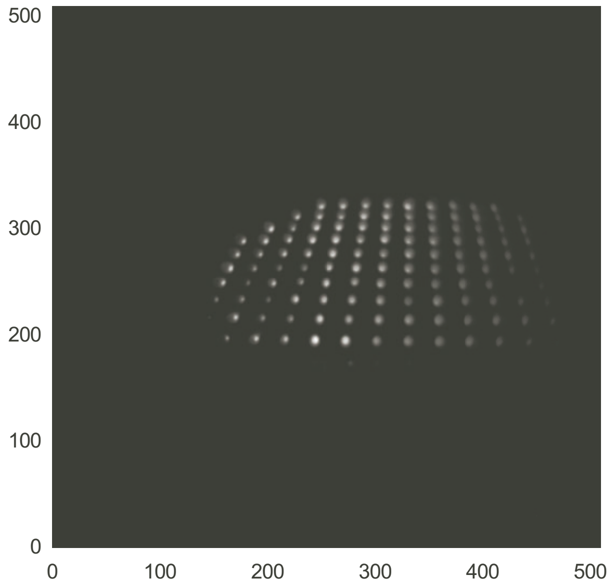


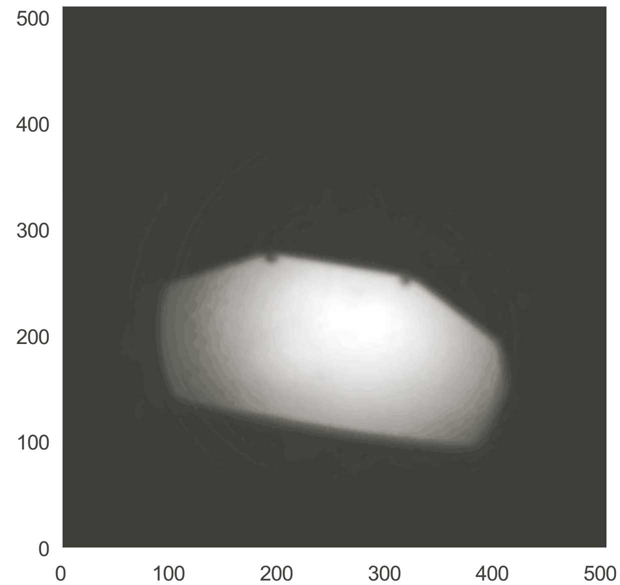


- Filter optimization code developed for optimal throughput and rejection ratio design

- Optimization should be investigated for specific plasma conditions to avoid spectral distortion

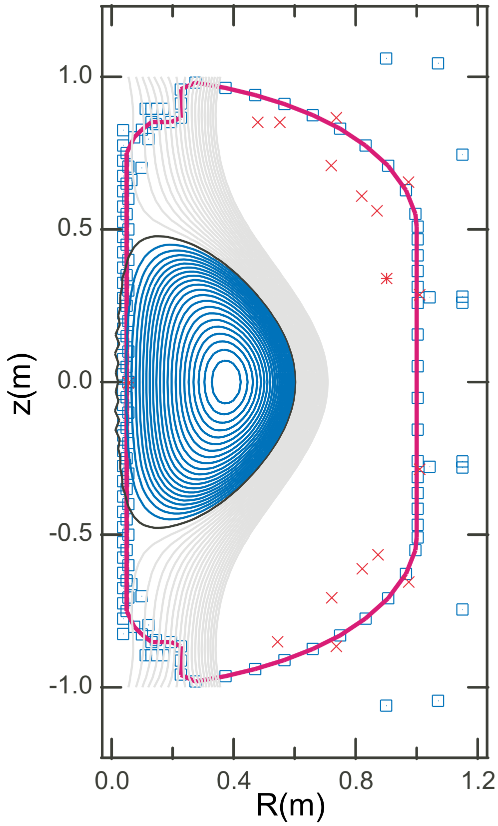
Imaging System Calibration Performed with Visible Light and X-ray Sources





- Backlit grid plate provided geometric transformation
- White field from visible light and Fe-55 X-ray calibration source used for intensity scaling
- Calibrated integrating sphere used for absolute measurement of imaging system visible light response
  - MCP image intensifier gain factor of x2 higher than specs
- Fe-55 source used for absolute X-ray response
  - Phosphor conversion efficiency factor of x2 lower than specs
- Lower X-ray emission from Pegasus than expected
  - T_e could be lower than estimate used in modeling
  - Cryopump may have lowered plasma impurity content

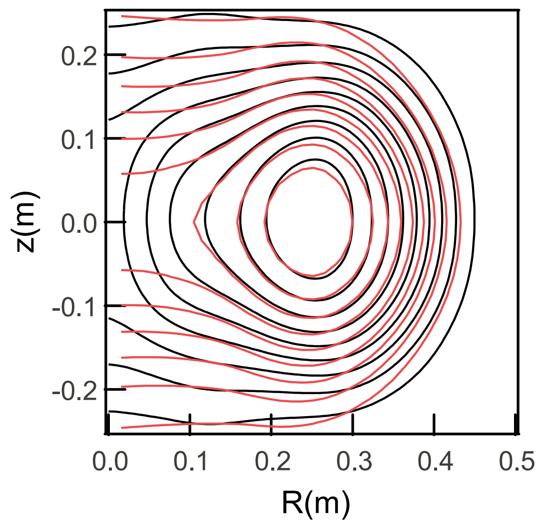
Shot #14729 Reconstruction Using Image Contour Constraint



Reconstructed Parameters

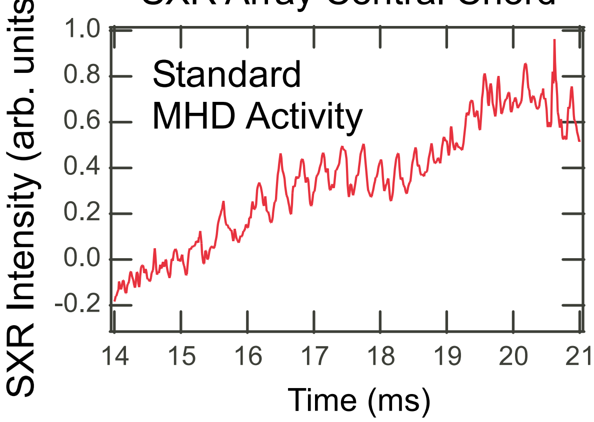
I_p	140 kA
β_T	0.12
ℓ_i	0.44
q_0	2.2
A	1.2
κ	1.5

Measured Itensity/
Fitted Intensity Overlay

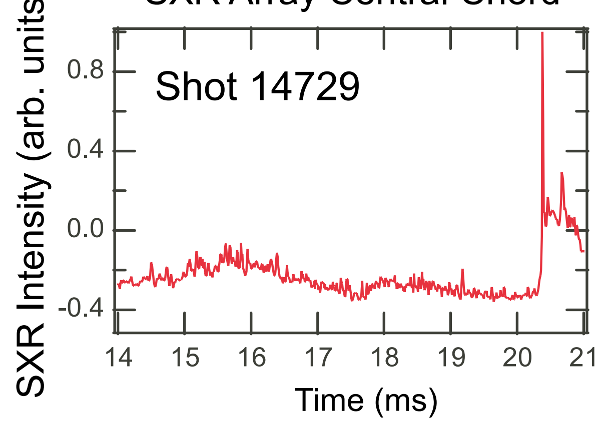


- q_0 > 2 consistent with lack of large coherent MHD activity

SXR Array Central Chord

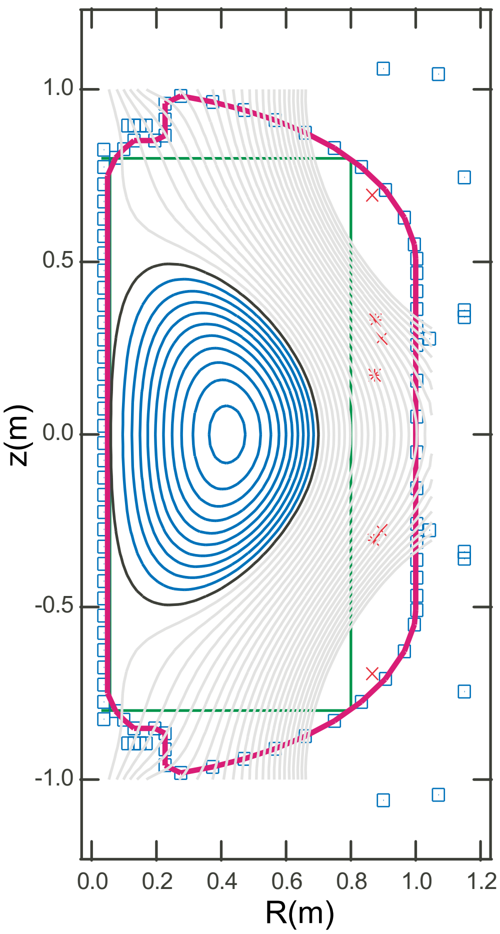


SXR Array Central Chord

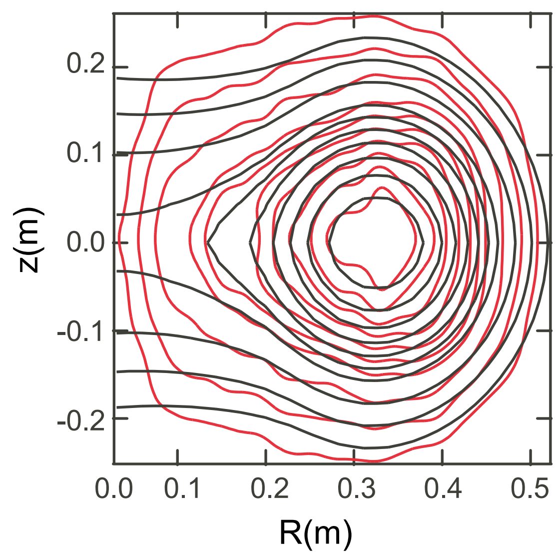


- Pegasus discharge often contains coherent 2/1 or 3/2 MHD mode activity

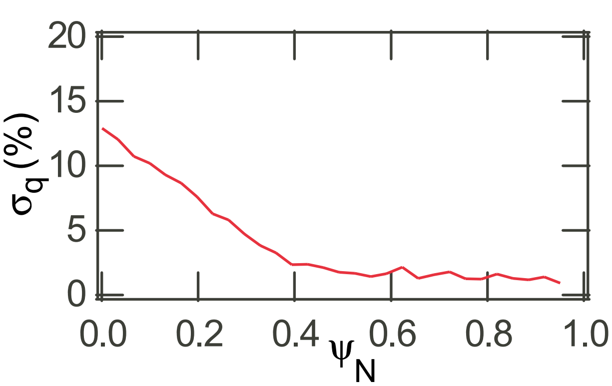
Image Contour Constraint Used with Spline Parameterization

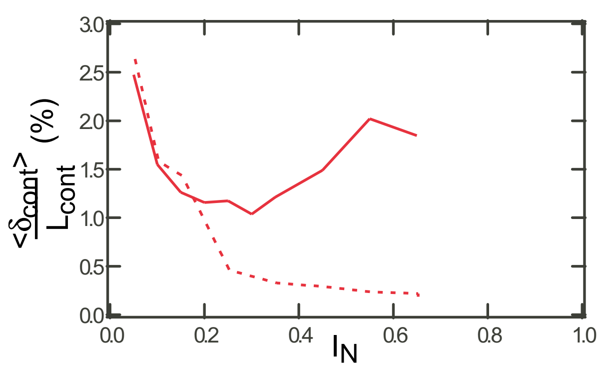


I_p	93 kA
β_T	0.03
ℓ_i	0.65
q_0	1.6
A	1.17
κ	1.4



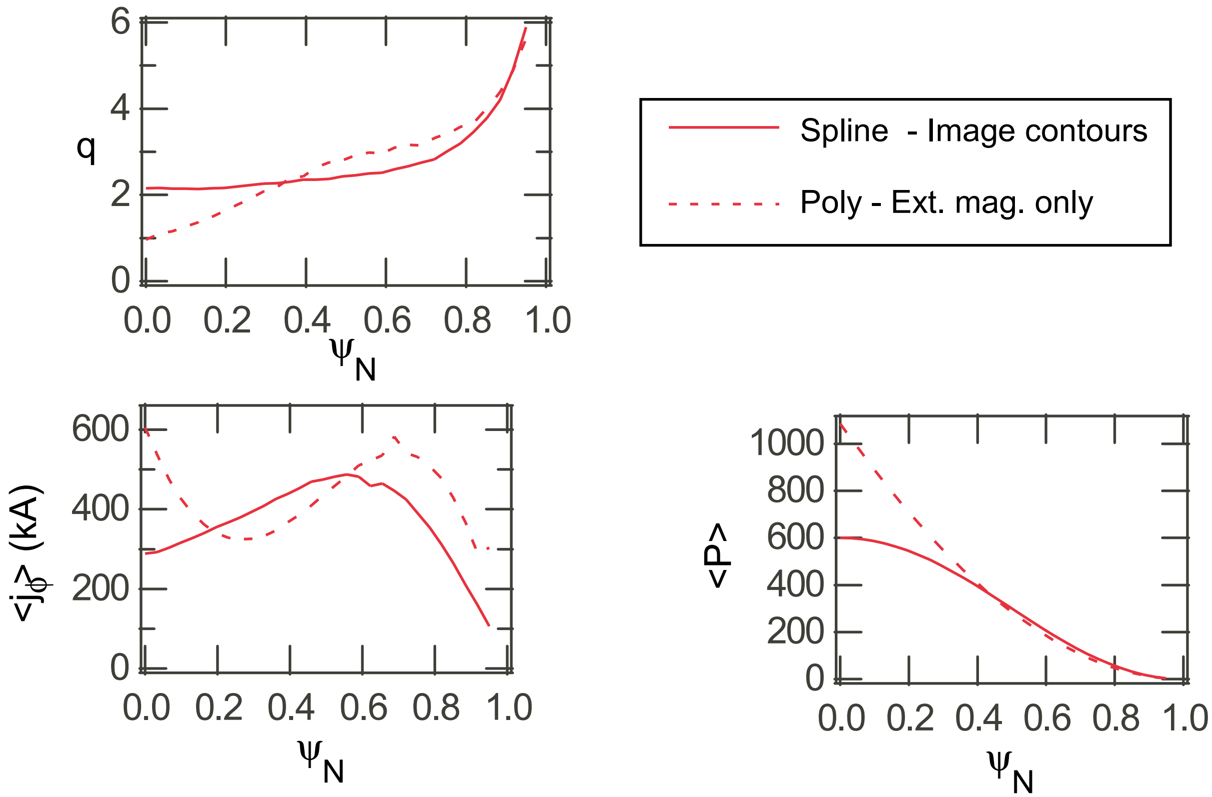
- q profile variation < 13%





- Contour fit metric corresponds to ~4% image noise

Reconstruction with External Magnetics Only Provides Markedly Different Profiles



Parameter Comparison

	Spline	Ext. Mag.
I_p	135kA	144kA
β_T	0.1 +/- 0.02	0.08
l_i	0.43 +/- 0.06	0.34
q_0	2.2 +/- 0.3	1.0
q_{95}	5.9 +/- 0.4	5.6
A	1.21 +/- 0.02	1.24
κ	1.5	1.6

- Reconstructions generally agree on bulk plasma parameters
- “Tension” between reconstructions increase uncertainty in q profile