Applications of Feedback control theory on plasma problems

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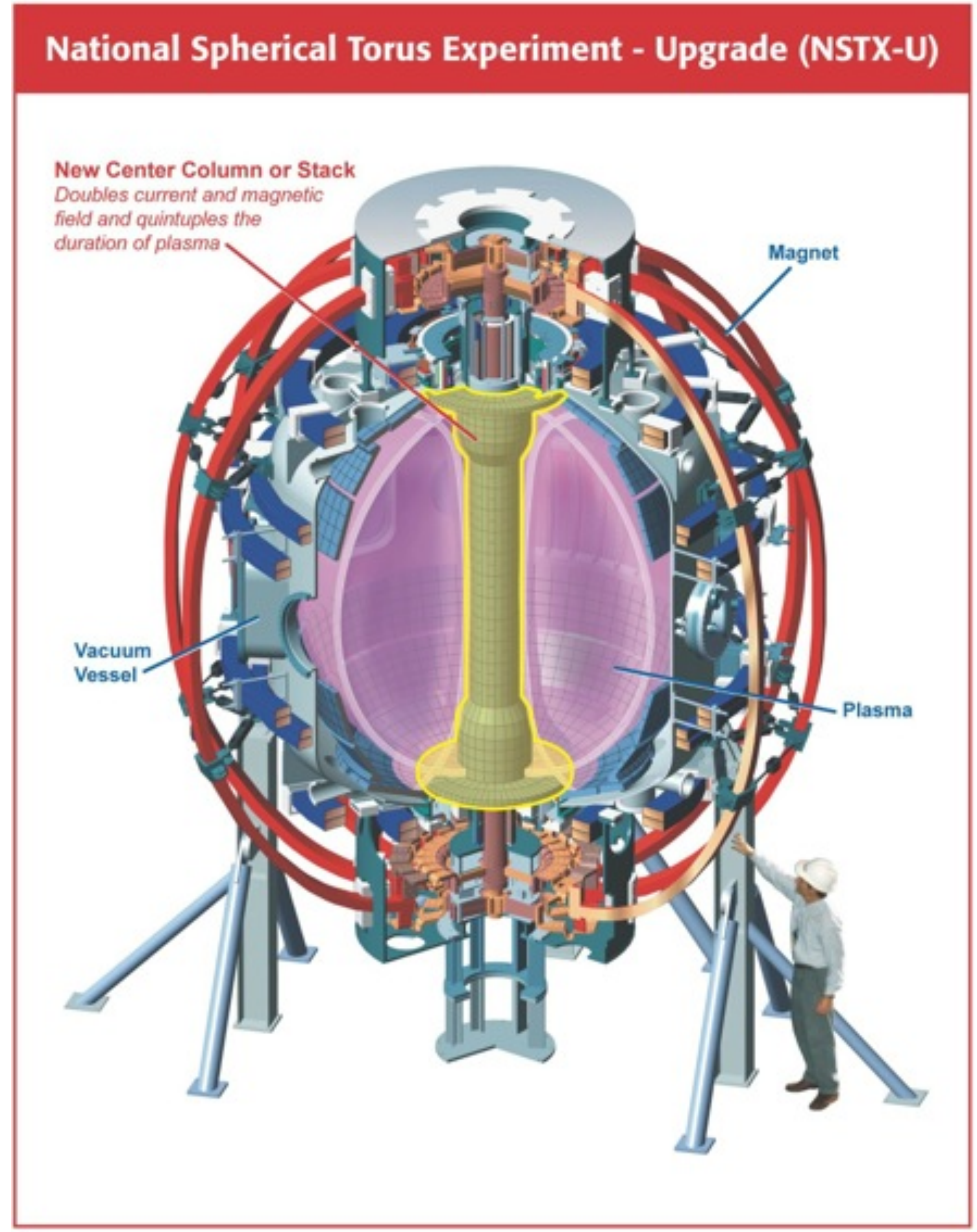
Dynamics of toroidal plasma confinement in tokamaks is extremely complex



- Non-uniformities
- Highly increased transport
- Macroscopic break up

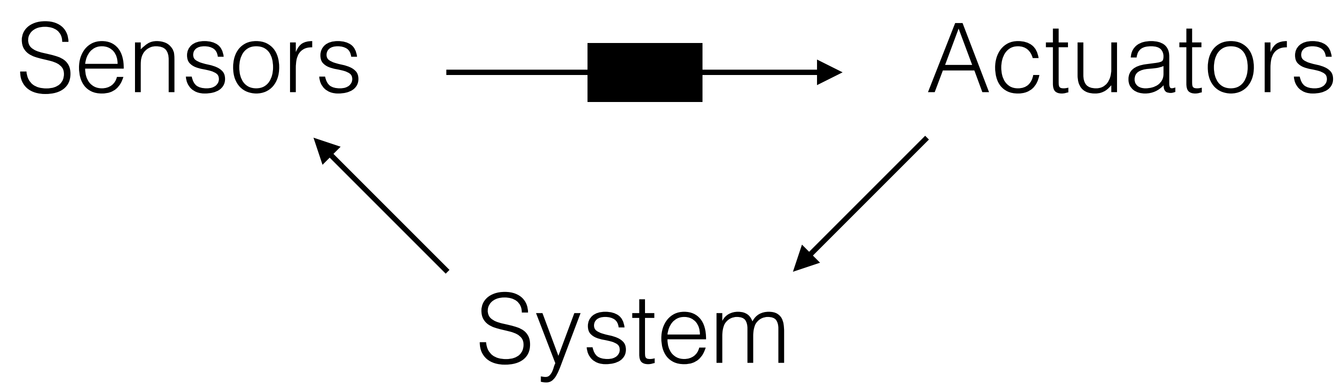
Goal: Preventing *instabilities* and *fluctuations*

The type of questions we want to answer:

- Can we prevent a dynamical system from going unstable?

- Can we drive a dynamical system to a chosen state and stabilize it there?

The answer is *yes*, using feedback control theory



Two examples in plasma

1. Modified Hasegawa-Wakatani equations (MHW)

   Suppress the transition to drift-wave induced turbulence

2. Plasma toroidal rotation

   Control plasma rotation in NSTX device and NSTX-U to maintain plasma stability for long-pulse operation.

Methodology

1- Modeling

- Start by a simple model of the dynamics
- Apply model reduction
- Linearization

2- Building a *linear controller* for the *linear reduced* order model

3- Connect the controller to the original *nonlinear* model

1- MHW problem: the modeling

A - Hasegawa-Wakatani equations: *Model definition*

- Drift waves are a particular type of instability that occur to the plasma inside the tokamak which is spontaneously excited by large ion thermal gradients.
- Zonal flows are then produced from the shearing of the poloidal flow.
- There is a coupling between the drift wave turbulence and the zonal flow:

HW model describes evolution of density fluctuation n and vorticity $\zeta = \nabla^2 \varphi$ (φ: electrostatic potential)

$$\frac{\partial}{\partial t}\zeta + \{\varphi, \zeta\} = \alpha(\varphi - n) - \mu \Delta^2 \zeta$$

$$\frac{\partial}{\partial t} n + \{\varphi, n\} = \alpha(\varphi - n) - \kappa \frac{\partial \varphi}{\partial y} - \mu \Delta^2 n$$

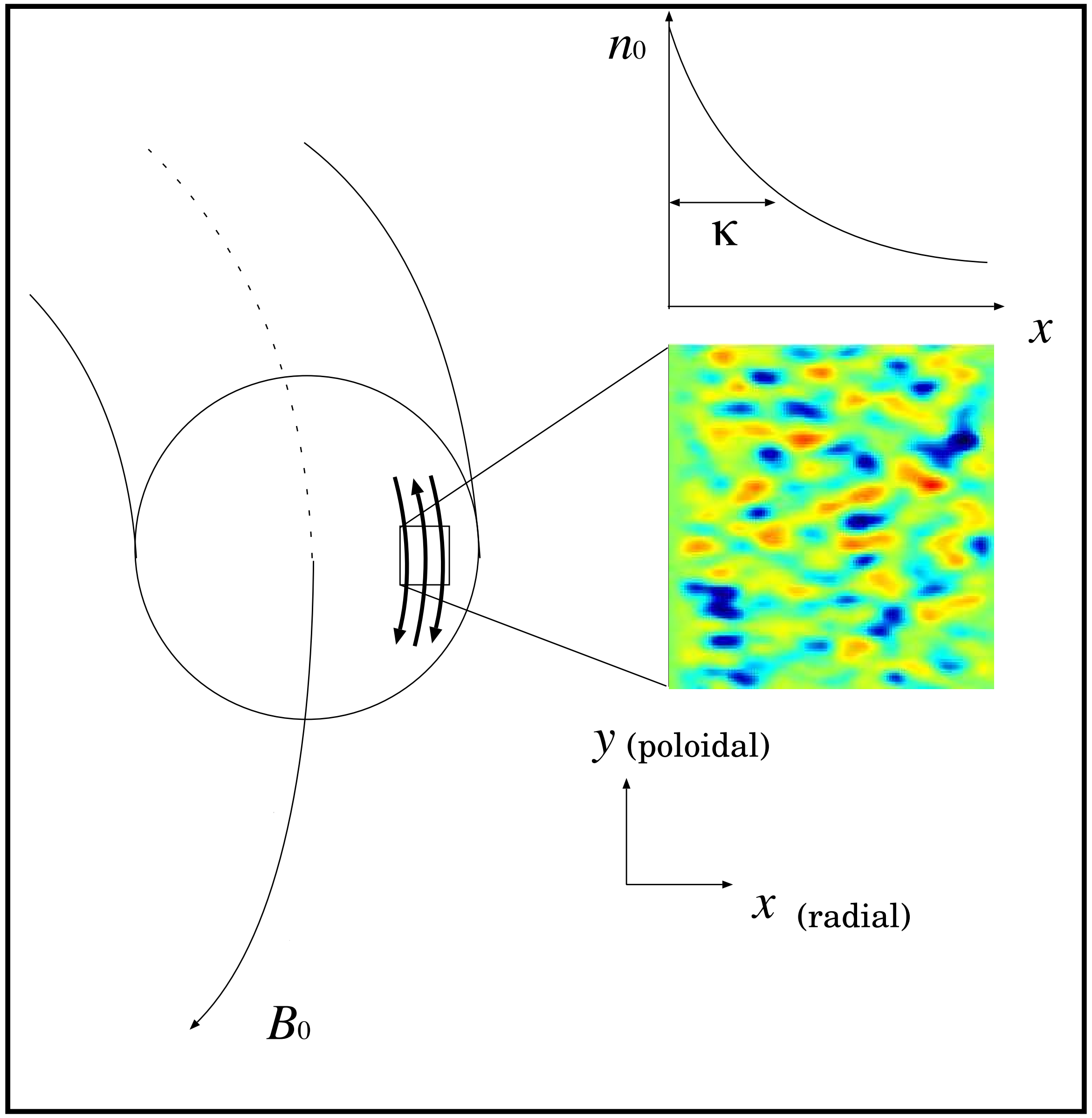
[Hasegawa and Wakatani, 1983]

$$\{a, b\} = (\partial a/\partial x)(\partial b/\partial y) - (\partial a/\partial y)(\partial b/\partial x)$$

$\mu =$ *dissipation coefficient*

$\kappa = -\partial/\partial x \ln n_0$

$\alpha =$ *adiabaticity coefficient*



A - Hasegawa-Wakatani equations: *Model definition*

- Zonal components must be subtracted from resistive coupling term since they $(k_y = k_z = 0)$ do not contribute to this term.

$$\alpha(\varphi - n) \rightarrow \alpha(\tilde{\varphi} - \tilde{n})$$

- Non zonal $\tilde{.}$ and zonal components $< . >$

$$\tilde{\varphi} = \varphi - \langle\varphi\rangle, \quad \tilde{n} = n - \langle n \rangle$$

$$\langle f \rangle = \frac{1}{L_y}\int f dy \quad (f = \varphi, n)$$

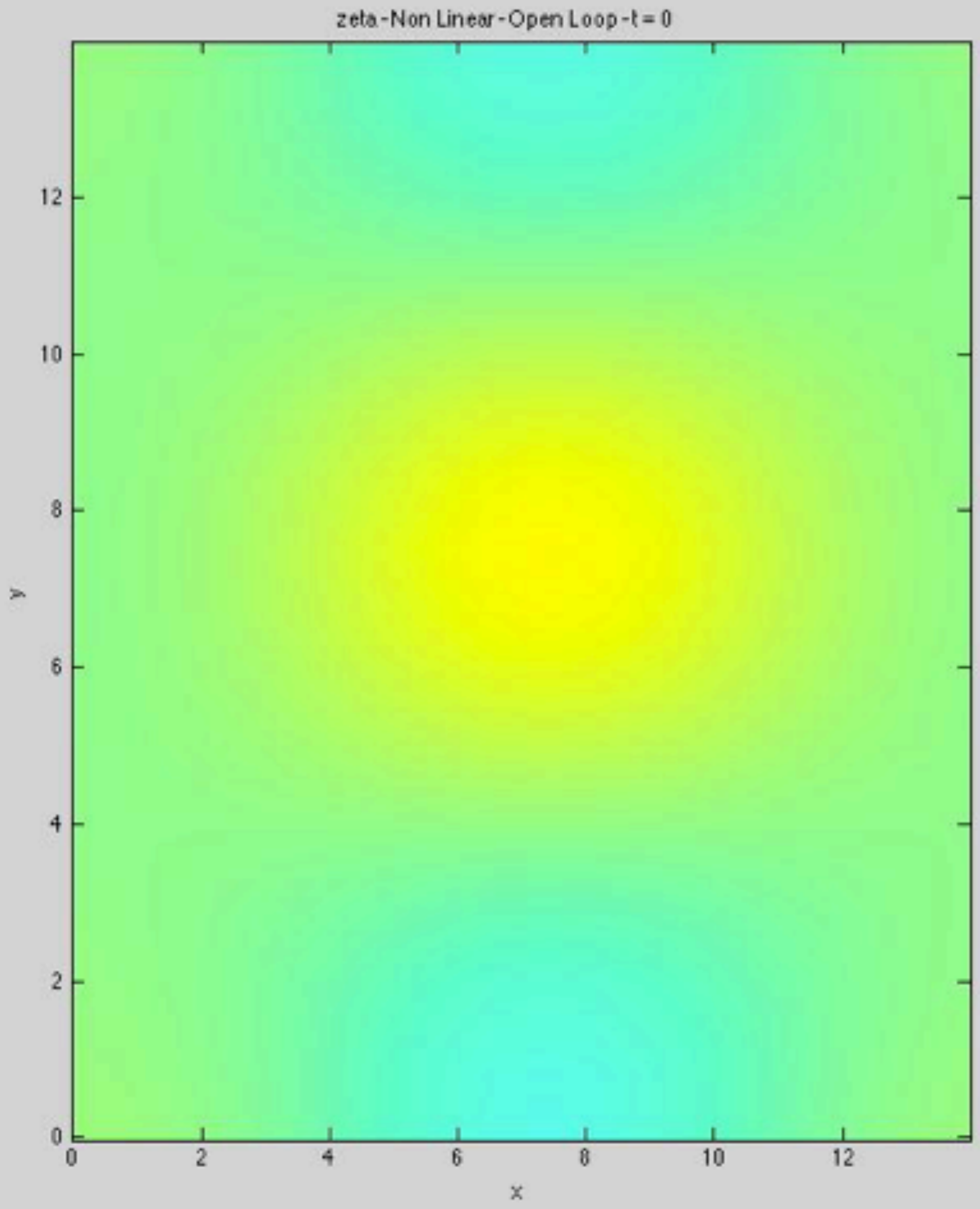
- Modified HW model

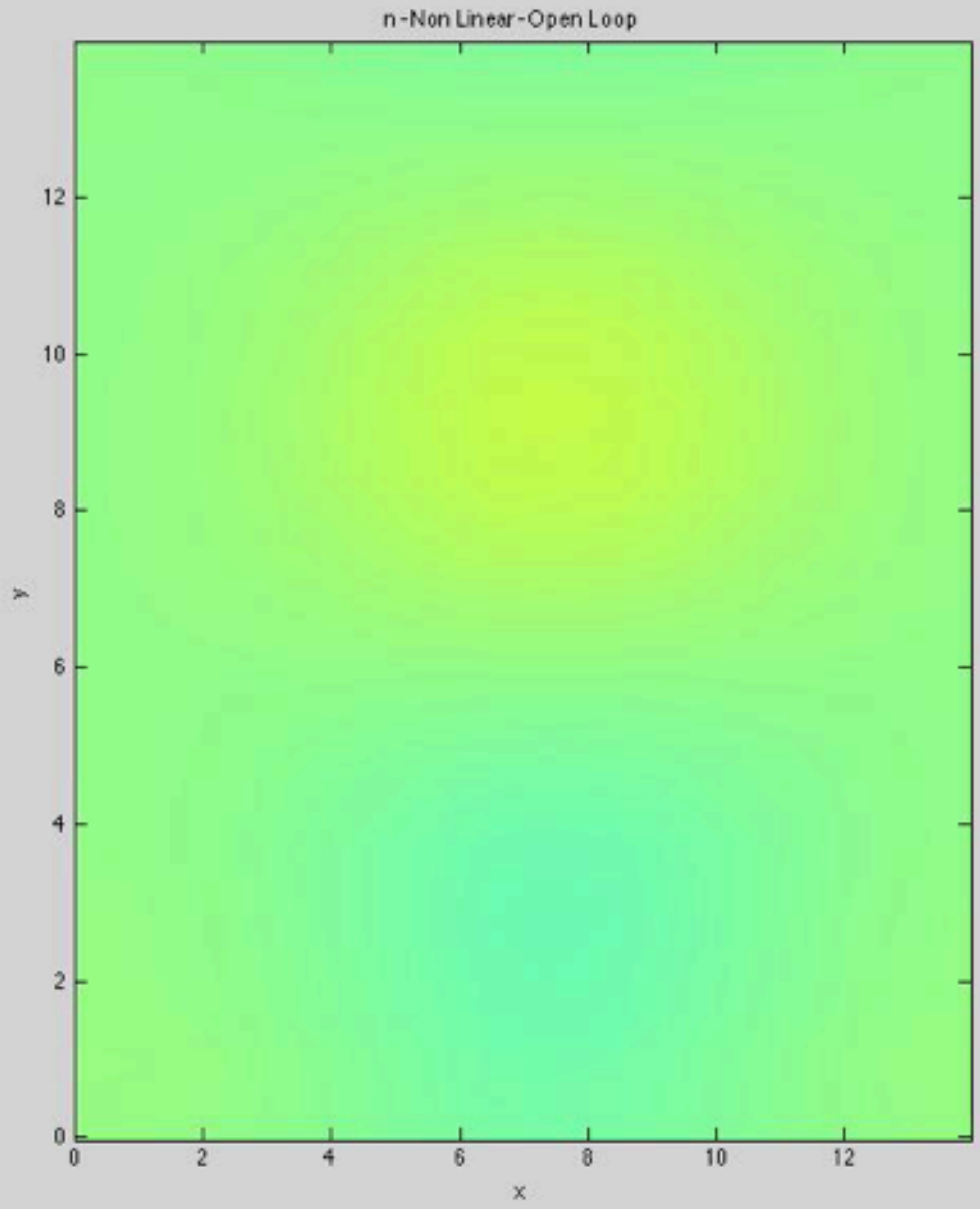
$$\frac{\partial}{\partial t}\zeta + \{\varphi, \zeta\} = \alpha(\tilde{\varphi} - \tilde{n}) - \mu\Delta^2\zeta$$

$$\frac{\partial}{\partial t}n + \{\varphi, n\} = \alpha(\tilde{\varphi} - \tilde{n}) - \kappa\frac{\partial\varphi}{\partial y} - \mu\Delta^2 n$$

A - Hasegawa-Wakatani equations: *Model definition*

- The *numerical simulation* of the MHW equations shows a regime where after an initial transient, *drift waves turbulence* is *suppressed* through *zonal-flow* generation.





A - Hasegawa-Wakatani equations: Linearization

- We add an additional electrostatic potential as our control input into the LMHW model

$$\varphi = \varphi_{int} + \varphi_{ext}$$

$$\varphi_{ext} = \Phi u$$

- Controlled Linearized Hasegawa-Wakatani Model around the equilibrium point (0,0,0):

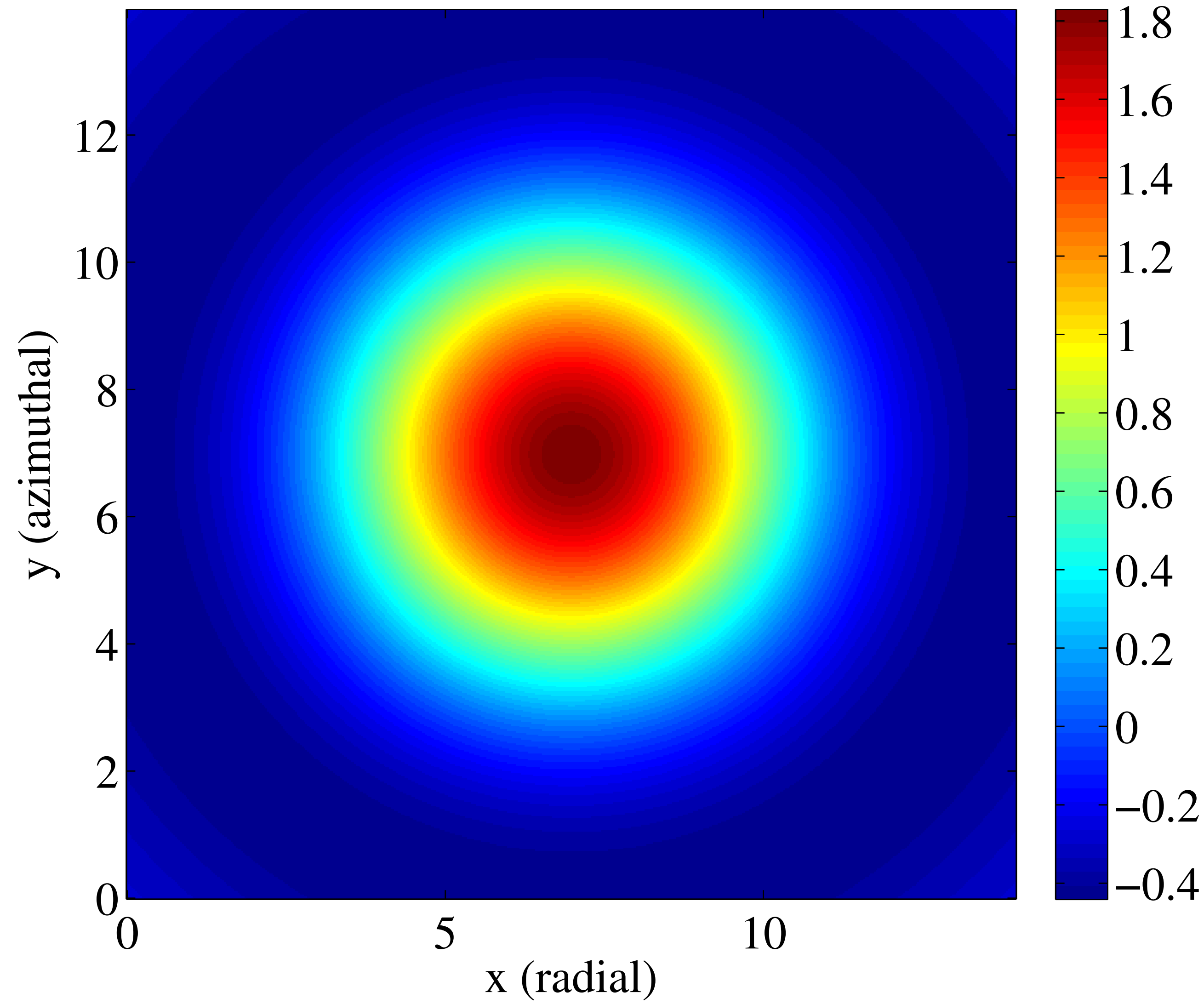
$$\begin{pmatrix} \dot{\zeta} \\ \dot{n} \end{pmatrix} = A \begin{pmatrix} \zeta \\ n \end{pmatrix} + Bu = \begin{pmatrix} \alpha\Delta^{-1} - \mu\Delta^2 & -\alpha \\ \alpha\Delta^{-1} - \kappa\frac{\partial}{\partial y}\Delta^{-1} & -\alpha - \mu\Delta^2 \end{pmatrix} \begin{pmatrix} \zeta \\ n \end{pmatrix} + Bu$$

$$B = \begin{pmatrix} \alpha\tilde{\Phi} \\ \alpha\tilde{\Phi} - \kappa\frac{\partial}{\partial y}\Phi \end{pmatrix}$$

Φ is the distribution of the field φ_{ext}

2D geometry with doubly periodic boundaries.

Distribution of the electrostatic potential φ_{ext}



A - Hasegawa-Wakatani equations: Model reduction

Decoupling the stable and unstable parts of the state

Ψ_u, Ψ_s are the *left* unstable and stable eigenspaces

Φ_u, Φ_s are the *right* unstable and stable eigenspaces

$$x = \begin{pmatrix} \zeta \\ n \end{pmatrix} = \Phi_u a_u + \Phi_s a_s$$

By substituting into $\dot{x} = Ax + Bu$ then, pre-multiplying by Ψ_u^*, Ψ_s^*

$$\frac{d}{dt}\begin{pmatrix} a_u \\ a_s \end{pmatrix} = \begin{pmatrix} \Psi_u^* A \Phi_u & 0 \\ 0 & \Psi_s^* A \Phi_s \end{pmatrix} \begin{pmatrix} a_u \\ a_s \end{pmatrix} + \begin{pmatrix} \Psi_u^* \\ \Psi_s^* \end{pmatrix} Bu$$

$$\frac{d}{dt}\begin{pmatrix} a_u \\ a_s \end{pmatrix} = \begin{pmatrix} A_u & 0 \\ 0 & A_s \end{pmatrix} \begin{pmatrix} a_u \\ a_s \end{pmatrix} + \begin{pmatrix} B_u \\ B_s \end{pmatrix} u$$

A - Hasegawa-Wakatani equations: Model reduction

Balanced truncation applied on the stable part

Cases studied	Original dim. of the state	Reduced dim. of the state
2 RHP	512	6
4 RHP	512	10
8 RHP	512	20

Details can be found in *Goumiri et al., Phys. Plasmas, 2013*

2- MHW problem: the control

Design a *full state feedback* controller

(For the reduced order model)

↓

Design an *observer based* controller

(For the full linear model)

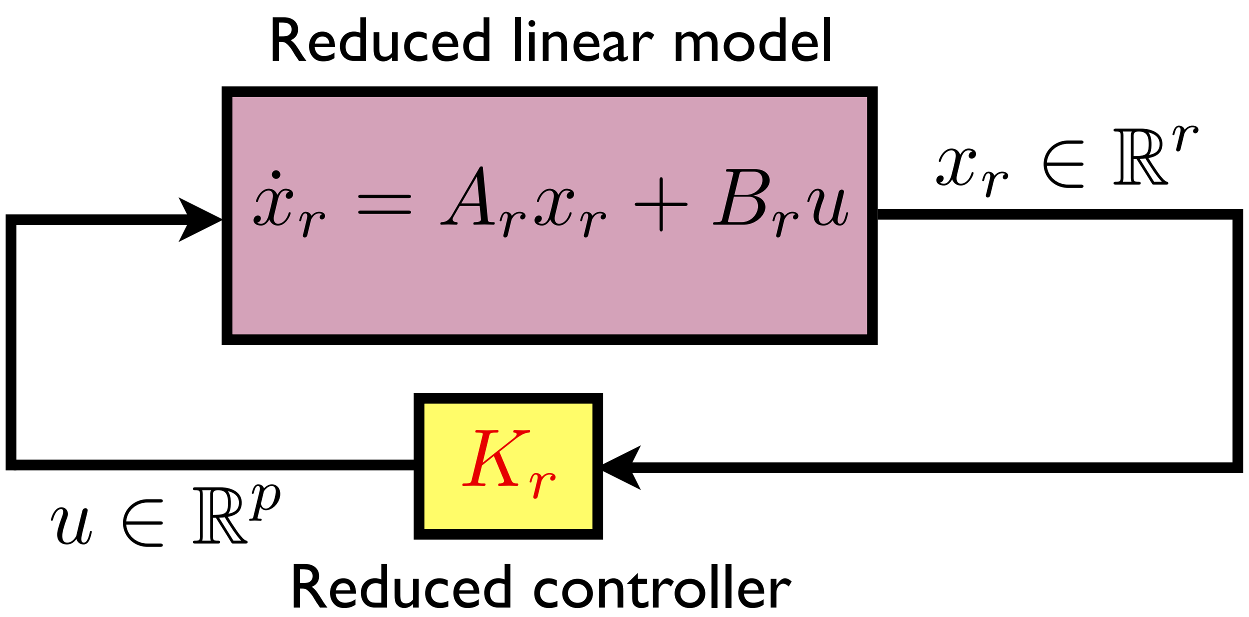
↓

Compensator designed

(For the full nonlinear model)

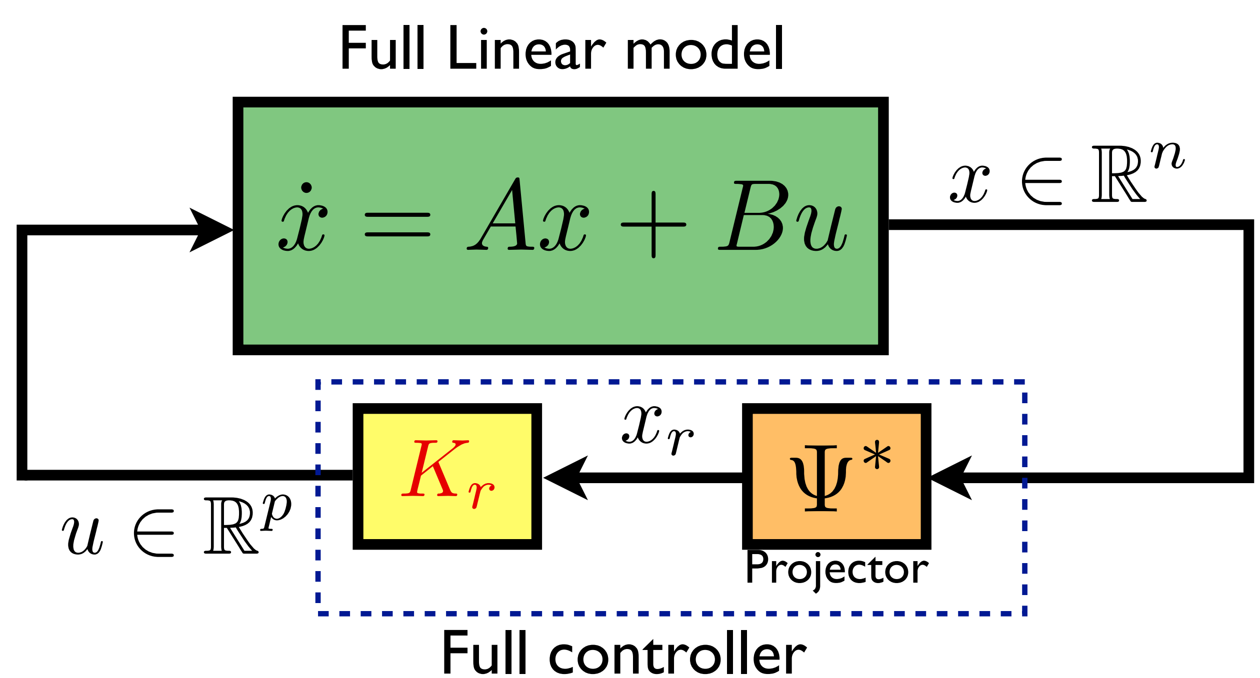
Full state feedback control

$r =$ dim. of reduced state
$n =$ dim. of full state
$p =$ dim. of the input



We start by designing a *controller* for the reduced order model by using Linear Quadratic Regulators (LQR).

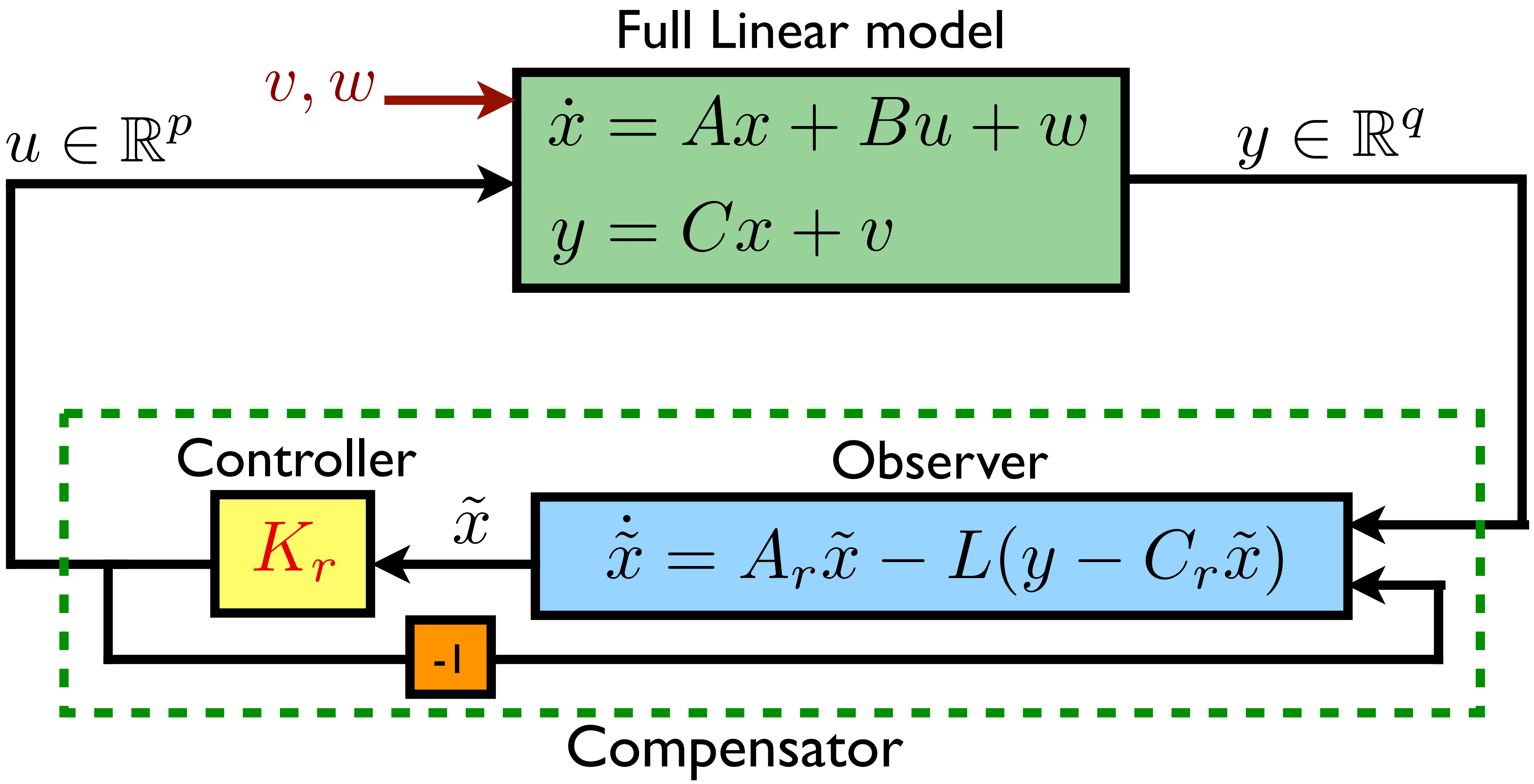
This *controller* is used combined with a projector as the *new controller* of the *full linear model.*



Finally, the *full linear model* is replaced by the *full non linear model*, <u>without</u> changing the design of this new controller.

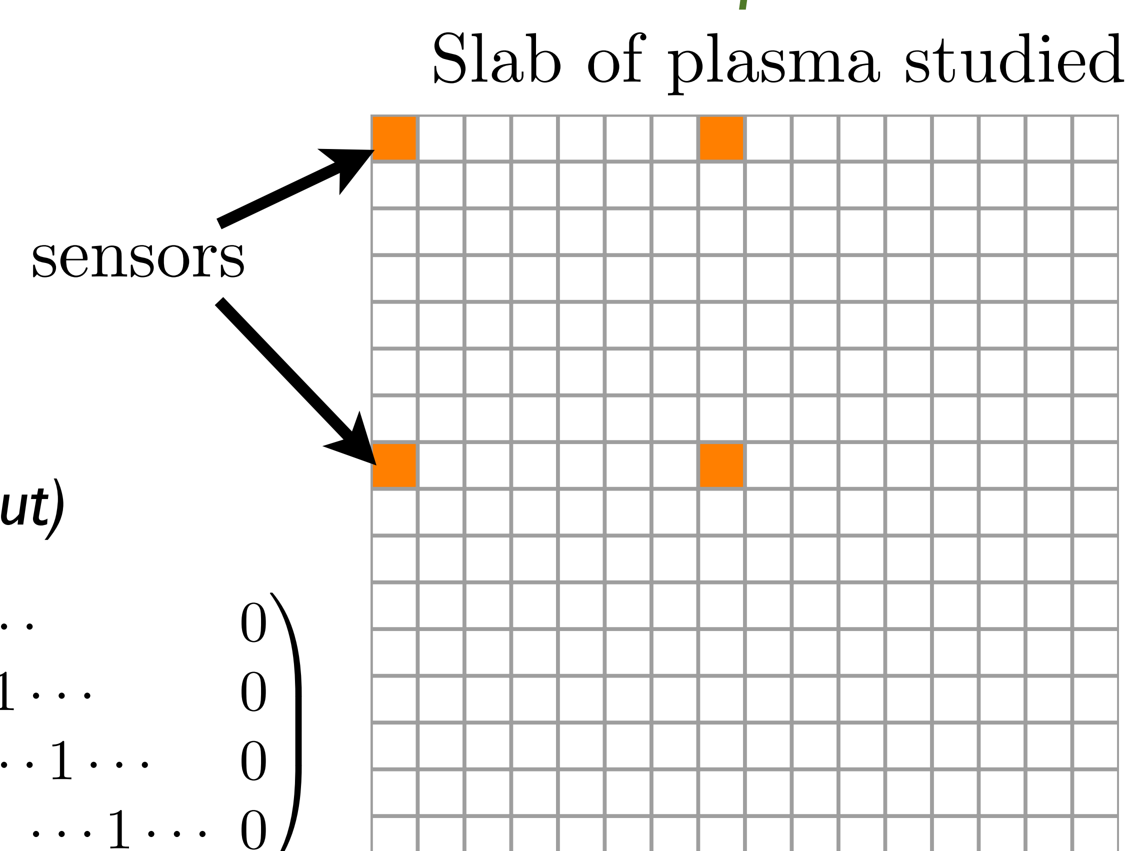
Observer based control

$p =$ dim. of the input
$q =$ dim. of the output
$v =$ sensor noise
$w =$ process noise



We start by designing an *observer* for the linearized model by using *a Kalman filter*, then we combine it with our previously designed controller to obtain a *compensator*.

Finally, the *full linear model* is replaced by the *full non linear model*, without changing the design of this *compensator*. $Ax \to f(x)$

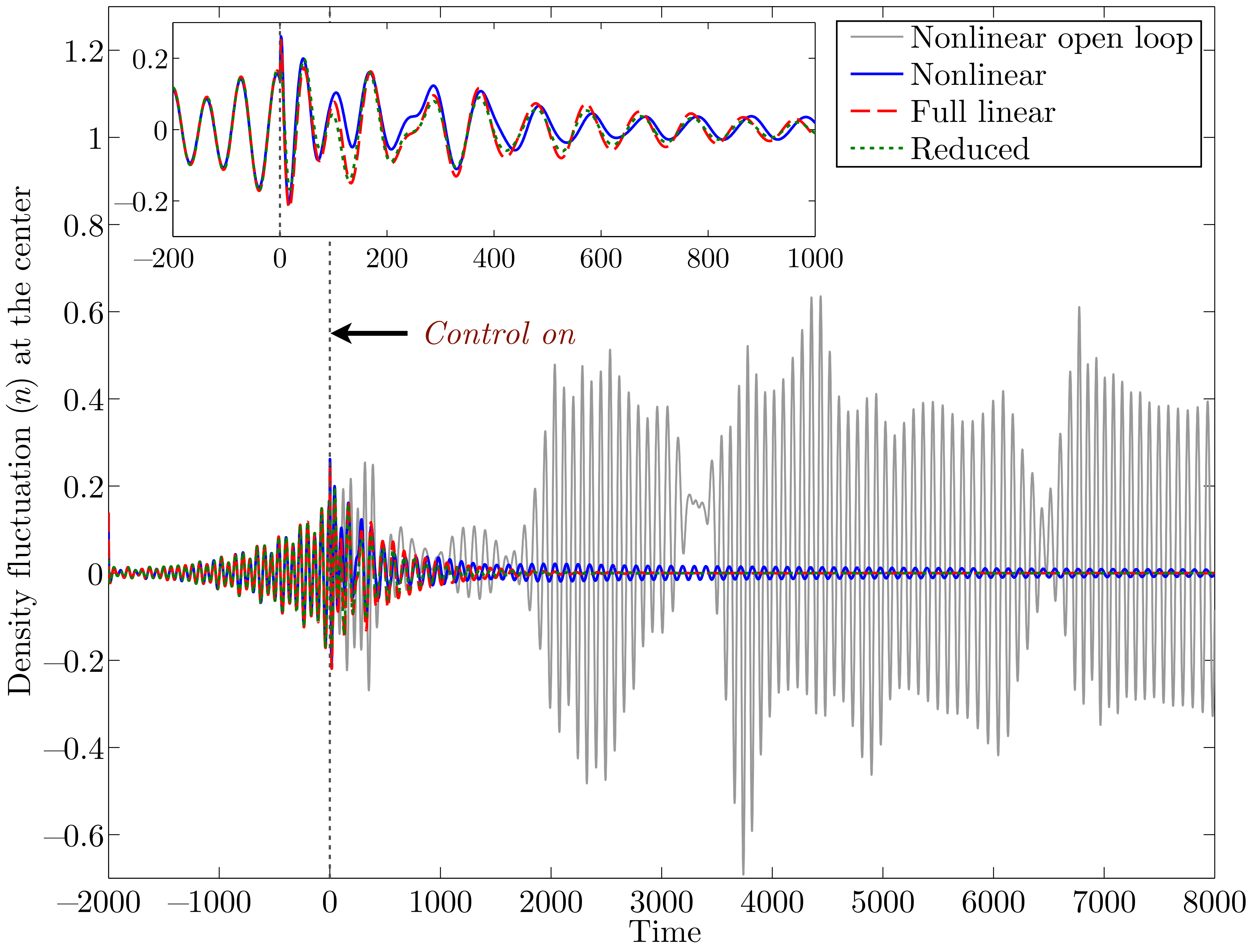


We then *sense* 2 types of *measurements*: *(output)*

- Full density field n: $C = (0 \quad I)$
- 4 pointwise density field n: $C = \begin{pmatrix} 0 & \cdots 1 \cdots & & & & 0 \\ 0 & & \cdots 1 \cdots & & & 0 \\ 0 & & & \cdots 1 \cdots & & 0 \\ 0 & & & & \cdots 1 \cdots & 0 \end{pmatrix}$

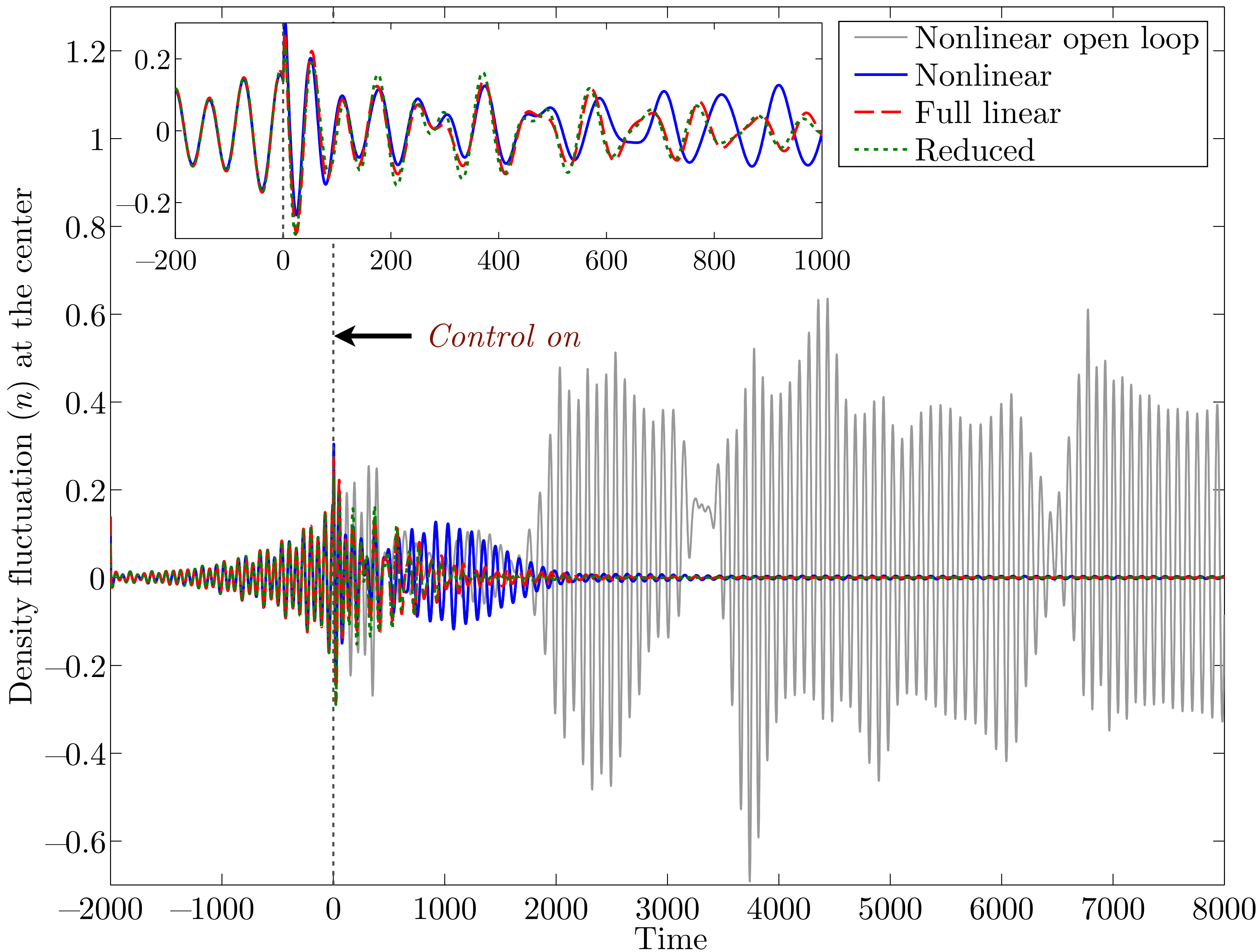
Results for Modified Hasegawa-Wakatani

4 RHP eigenvalues model with full density field sensed



Results for Modified Hasegawa-Wakatani

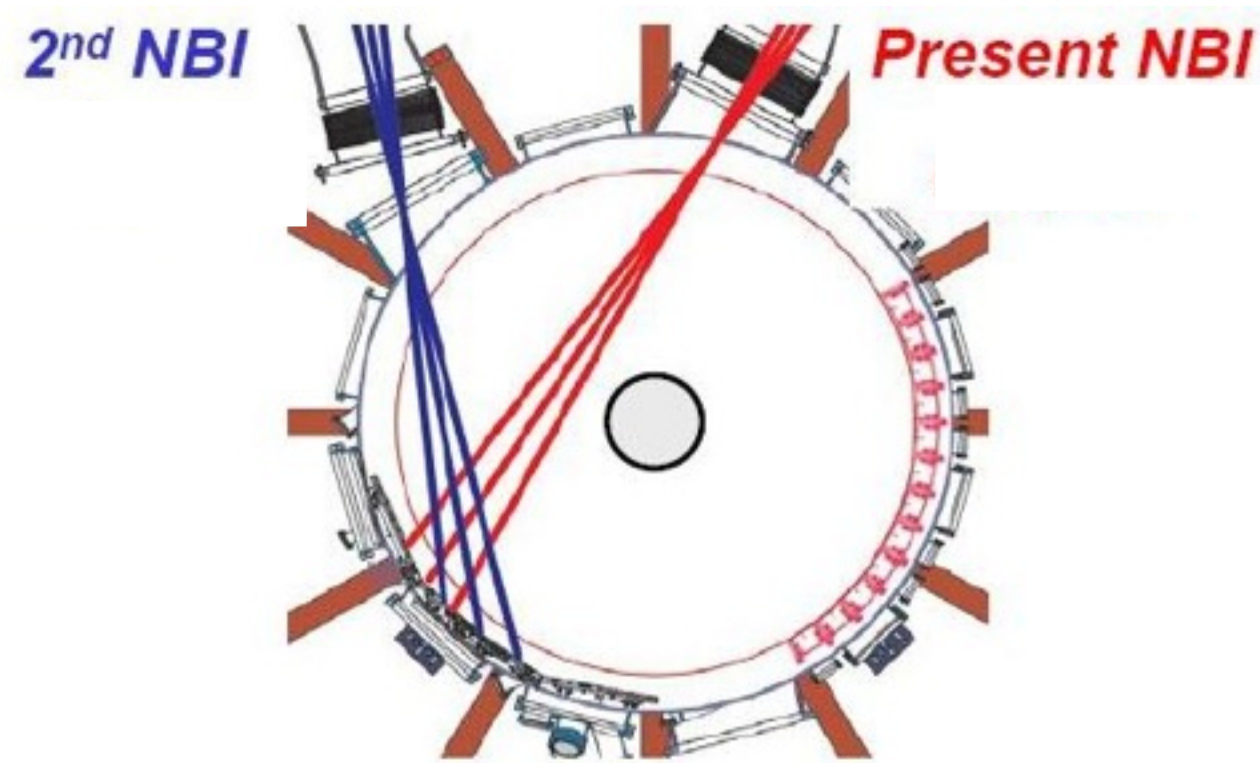
4 RHP eigenvalues model with 4 pointwise density sensors

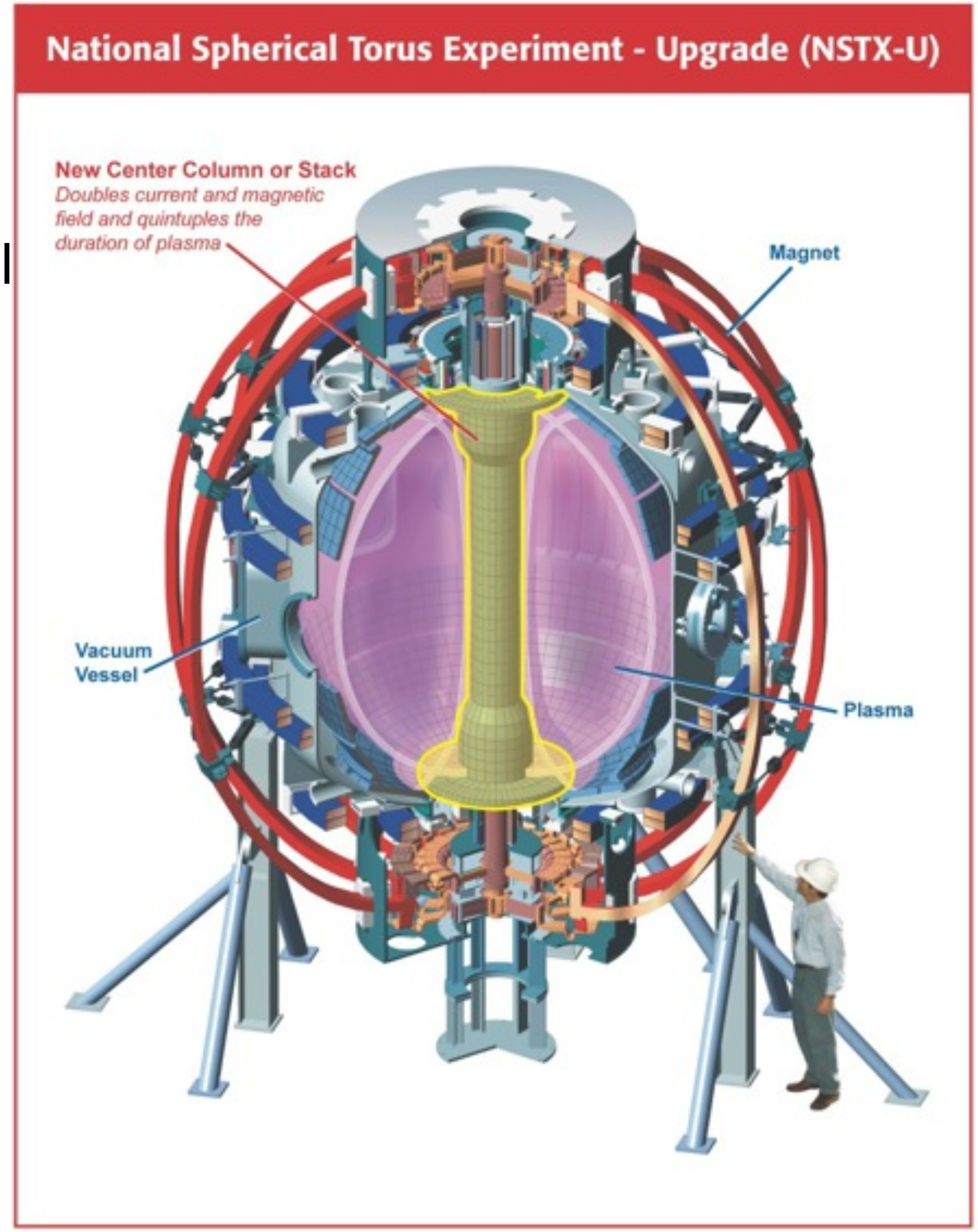


3- Toroidal rotation: the modeling

The primary components of the upgrade:

- The complete **replacement** of the center stack (containing the inner-leg of the toroidal field coils, the ohmic heating solenoid…)
- The **addition** of a second neutral beam injector aimed more tangentially.





The upgrade of NSTX machine will:

- Increase the TF (Toroidal field) capability from 0.55T to 1.0T.
- Increase the maximum plasma current from 1.3 MA to 2 MA.
- Increase auxiliary heating power.
- **Increase neutral beam current drive and the ability to tailor their deposition profiles.**

B- Plasma rotation : *For NSTX and NSTX-U*

Simplified Toroidal Momentum Equation

$$(nm)\left\langle R^2\right\rangle \frac{\partial \omega}{\partial t} = \left(\frac{\partial V}{\partial \rho}\right)^{-1} \frac{\partial}{\partial \rho}\left[\frac{\partial V}{\partial \rho}(nm)\chi_\phi \left\langle R^2(\nabla\rho)^2\right\rangle \frac{\partial \omega}{\partial \rho}\right] + \sum_{i=1}^{4} T_{\mathrm{NBI}i}\left(P_{\mathrm{NBI}}\right) + T_{\mathrm{NTV}}\left(\omega, I^2\right)$$

Energy Equation: β_n control

Energy stored

$$\frac{\partial W}{\partial t} + \frac{W}{\tau_E} = \sum_{i=1}^{4} P_{\mathrm{NBI}i}(t)$$

Empirical Constant coming from H98 code

$$\tau_E = H_{98y,2}\, 0.0562\, I_P^{0.93}\, B_T^{0.15}\, n_e^{0.41}\, P_{Loss(th)}^{-0.69}\, R_0^{1.97}\, \epsilon^{0.58}\, \kappa^{0.78}$$

Boyer, M. D. & al.

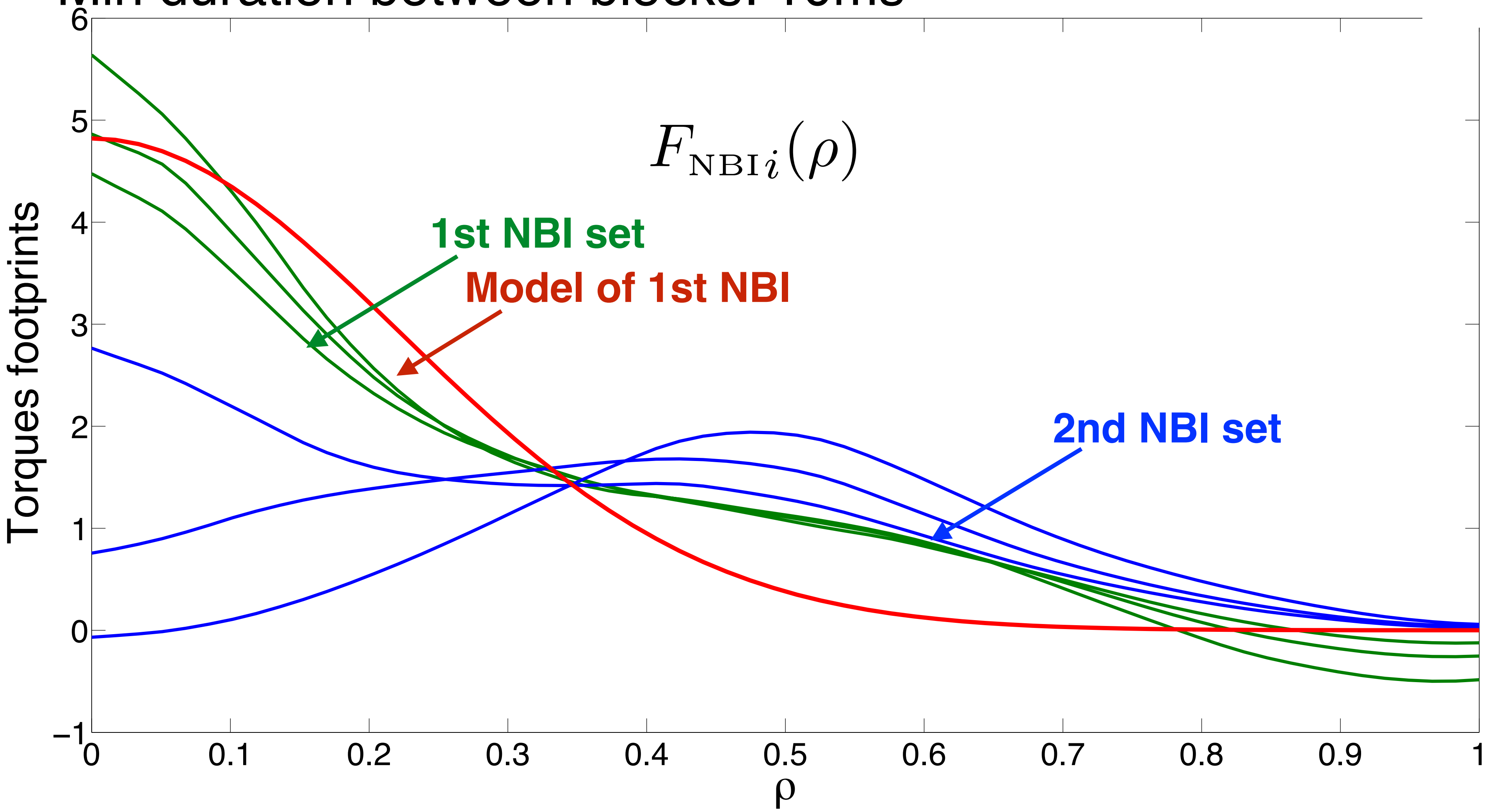
χ_ϕ has to be modeled as well as the background variables

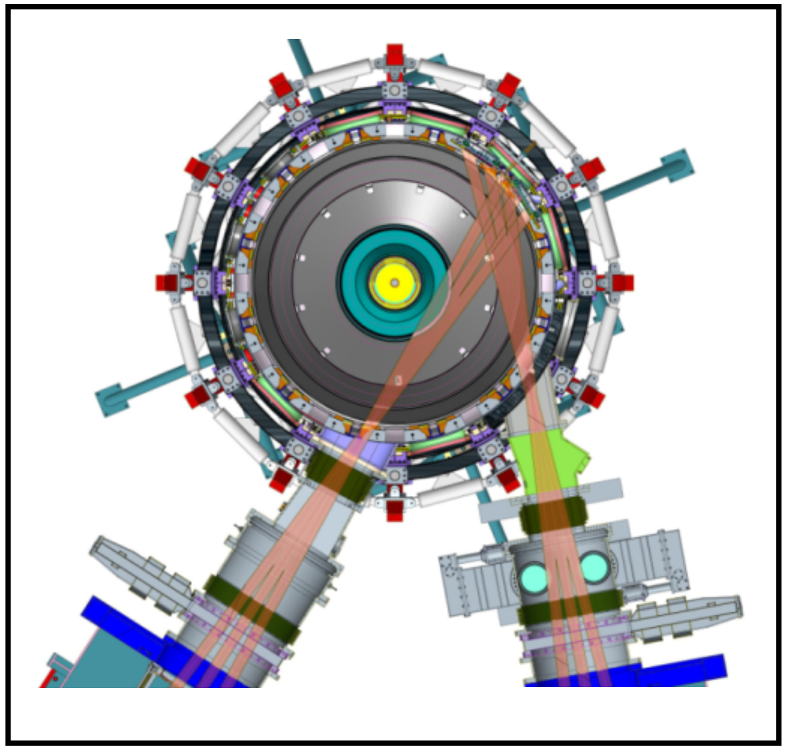
B- Plasma rotation : NBI model *For NSTX & NSTX-U*

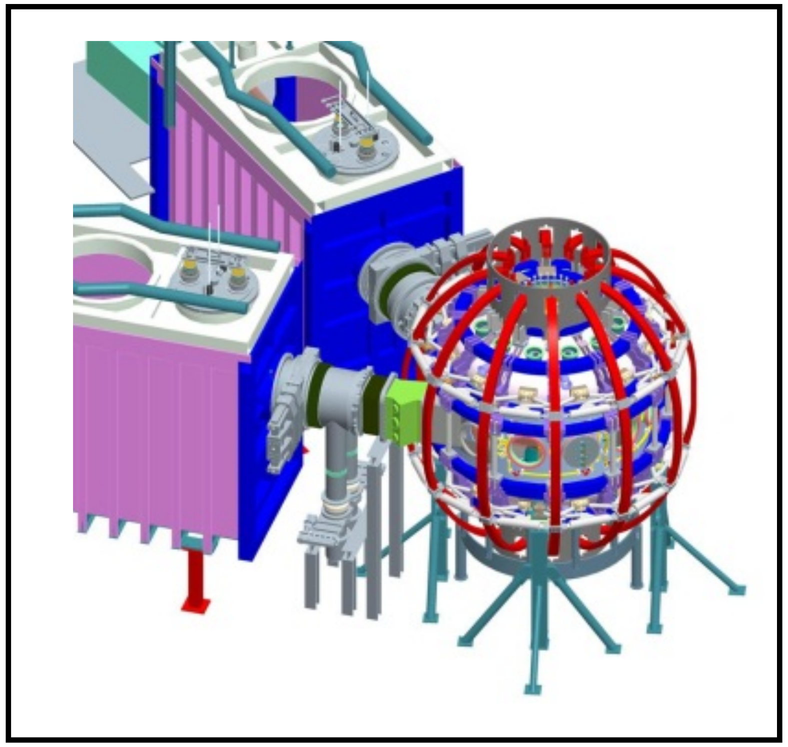
$$T_{\mathrm{NBI}i}(t,\rho) = \overline{T}_{\mathrm{NBI}i}(t) F_{\mathrm{NBI}i}(\rho)$$

$$\frac{\partial \overline{T}_{\mathrm{NBI}i}}{\partial t} + \frac{\overline{T}_{\mathrm{NBI}i}}{\tau_{\mathrm{NBI}i}} = \kappa_{\mathrm{NBI}i} P_{\mathrm{NBI}i}(t)$$

- 6 NBI beams – Power 2MW each: Max **12MW**
- Each beam can be blocked 20 times max.
- Block min duration: 10ms
- Min duration between blocks: 10ms



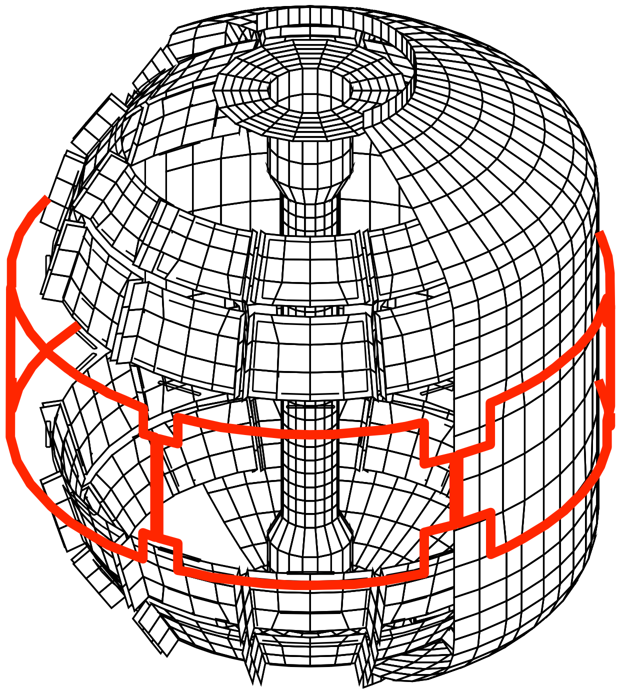


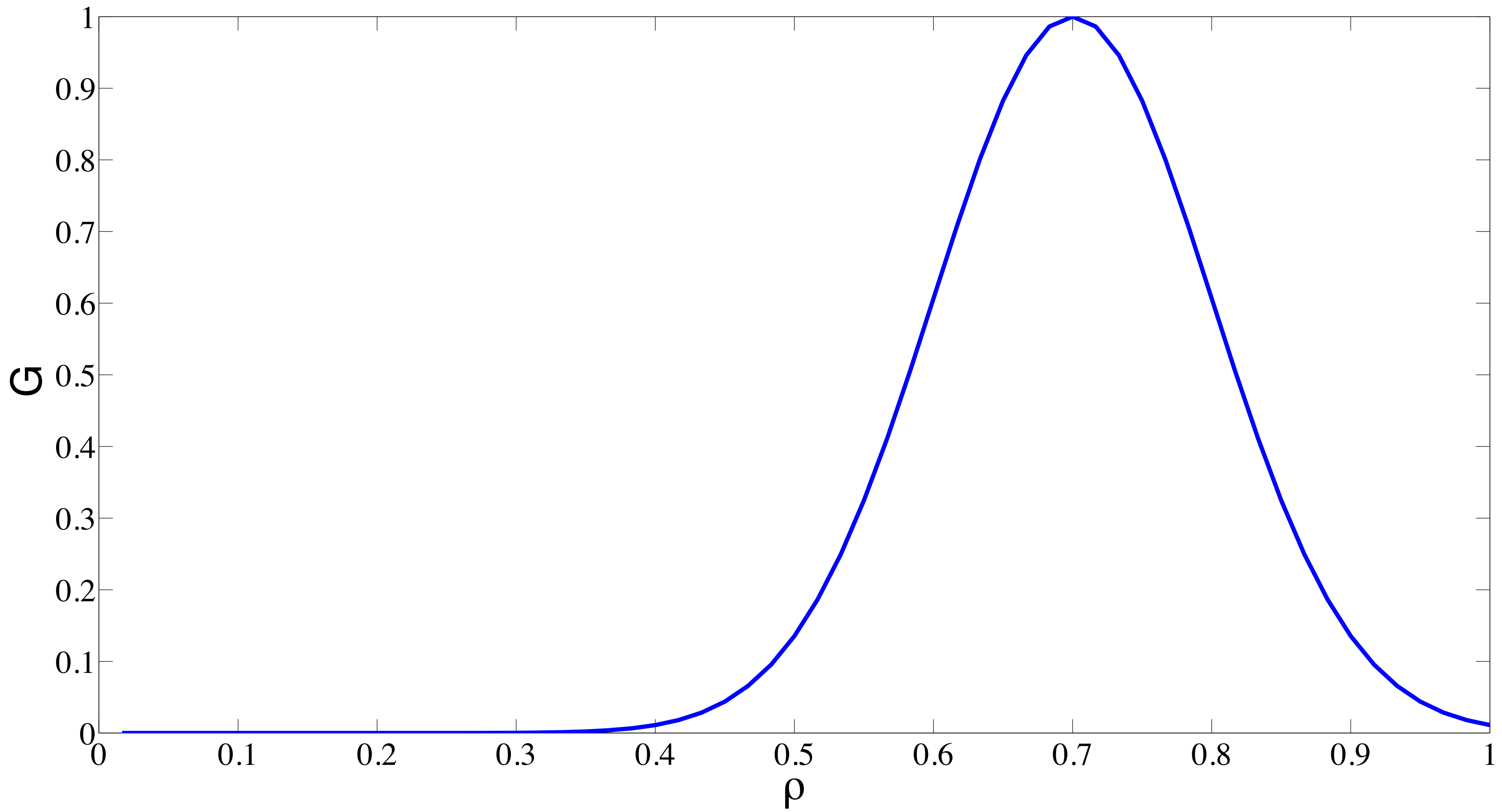


B- Plasma rotation : NTV model *For NSTX & NSTX-U*

- Same model used for NSTX & NSTX-U: Max. Current = 3kA

$$\boxed{T_{\mathrm{NTV}}(t,\rho) = K\,G(\rho)\,\langle R^2\rangle\,I^2(t)\,\omega(t,\rho)}$$





B- Plasma rotation : Model reduction

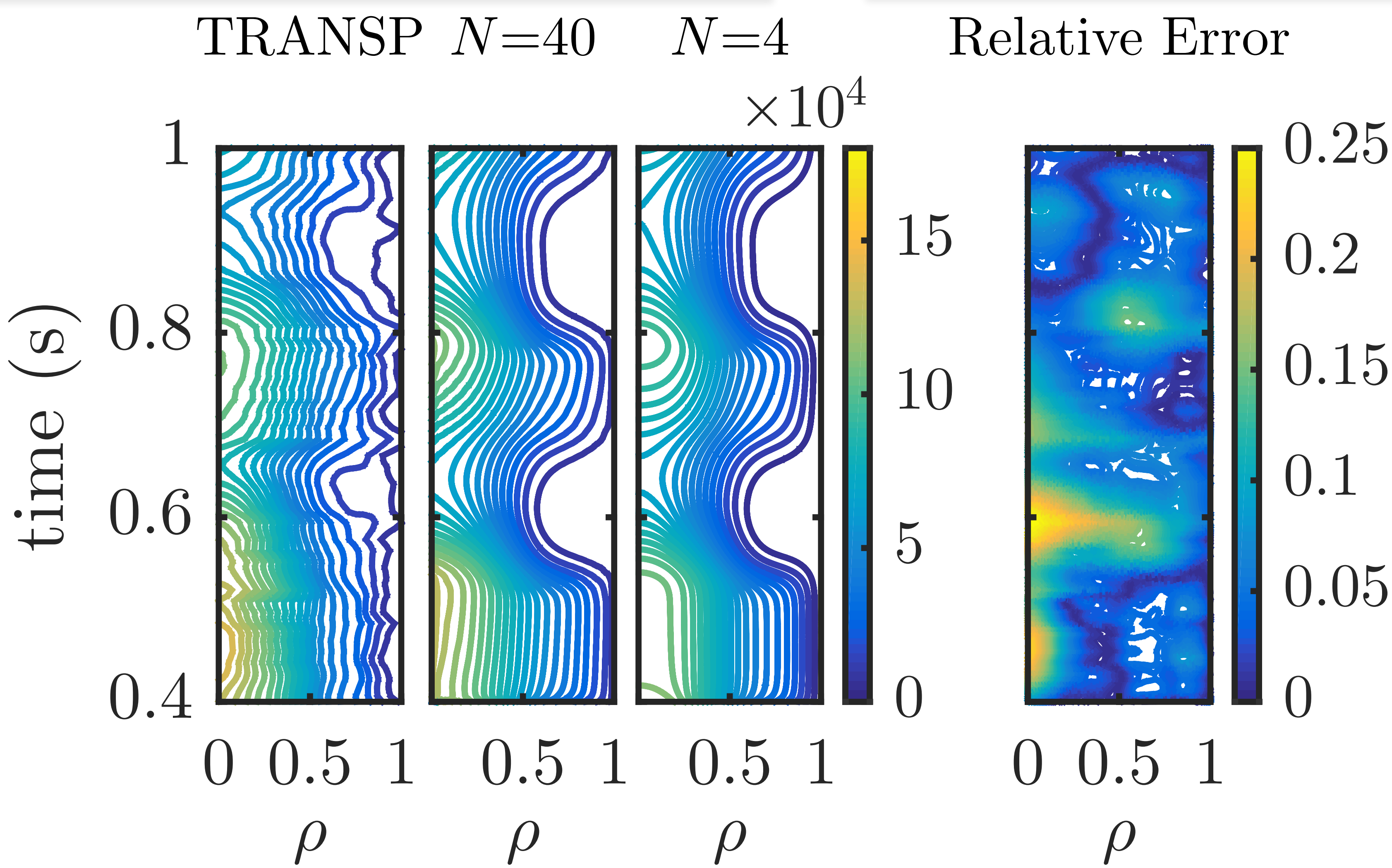
$$\omega(\rho, t) = \sum_{n=1}^{N} a_n(t)\varphi_n(\rho)$$

$$\varphi_n(\rho) = J_0(k_n\rho), \qquad n = 1, \dots, N$$

J_0 denotes the Bessel function of the first kind and k_n denotes the n-th root of J_0

$$\langle \varphi_n, \varphi_m \rangle = 0, \qquad \text{for } m \neq n$$

$$\langle f, g \rangle = \int_0^1 \rho\, f(\rho)\, g(\rho)\, d\rho$$



$$\omega(t,\rho) = \omega_0(\rho) + \omega_1(t,\rho)$$
$$P_{\mathrm{NBI}i}(t) = P_{0i} + P_{1i}(t)$$
$$I(t) = I_0 + I_1(t)$$
$$W = W_0 + W_1$$

Building the state space realization

$$\frac{\partial}{\partial t}\begin{bmatrix}\omega_1\\ \overline{T}_1\\ \overline{T}_2\\ \overline{T}_3\\ \overline{T}_4\\ W\end{bmatrix} = \overbrace{\begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} & 0\\ 0 & a_{22} & 0 & 0 & 0 & 0\\ 0 & 0 & a_{33} & 0 & 0 & 0\\ 0 & 0 & 0 & a_{44} & 0 & 0\\ 0 & 0 & 0 & 0 & a_{55} & 0\\ 0 & 0 & 0 & 0 & 0 & a_{66}\end{pmatrix}}^{\mathbf{A}}\begin{bmatrix}\omega_1\\ \overline{T}_1\\ \overline{T}_2\\ \overline{T}_3\\ \overline{T}_4\\ W\end{bmatrix}$$

$$+\underbrace{\begin{pmatrix} b_{11} & 0 & 0 & 0 & 0\\ 0 & b_{22} & 0 & 0 & 0\\ 0 & 0 & b_{33} & 0 & 0\\ 0 & 0 & 0 & b_{44} & 0\\ 0 & 0 & 0 & 0 & b_{55}\\ 0 & 1 & 1 & 1 & 1\end{pmatrix}}_{\mathbf{B}}\begin{bmatrix}I_1^2\\ P_1\\ P_2\\ P_3\\ P_4\end{bmatrix}$$

$$\dot{x}_1 = Ax_1 + Bu_1$$
$$y_1 = Cx_1$$

$$a_{11} = \frac{1}{nm\left\langle R^2\right\rangle}\left[\left(\frac{\partial V}{\partial \rho}\right)^{-1}\frac{\partial}{\partial \rho}\left[\frac{\partial V}{\partial \rho}(nm)\chi_\phi\left\langle R^2(\nabla\rho)^2\right\rangle\frac{\partial}{\partial \rho}\right] - KG(\rho)\langle R^2\rangle I_0^2\right]$$

$$a_{1,i+1} = \frac{F_{NBI_i}(\rho)}{nm\left\langle R^2\right\rangle}$$

$$a_{i+1,i+1} = -\frac{1}{\tau_{NBI_i}}$$

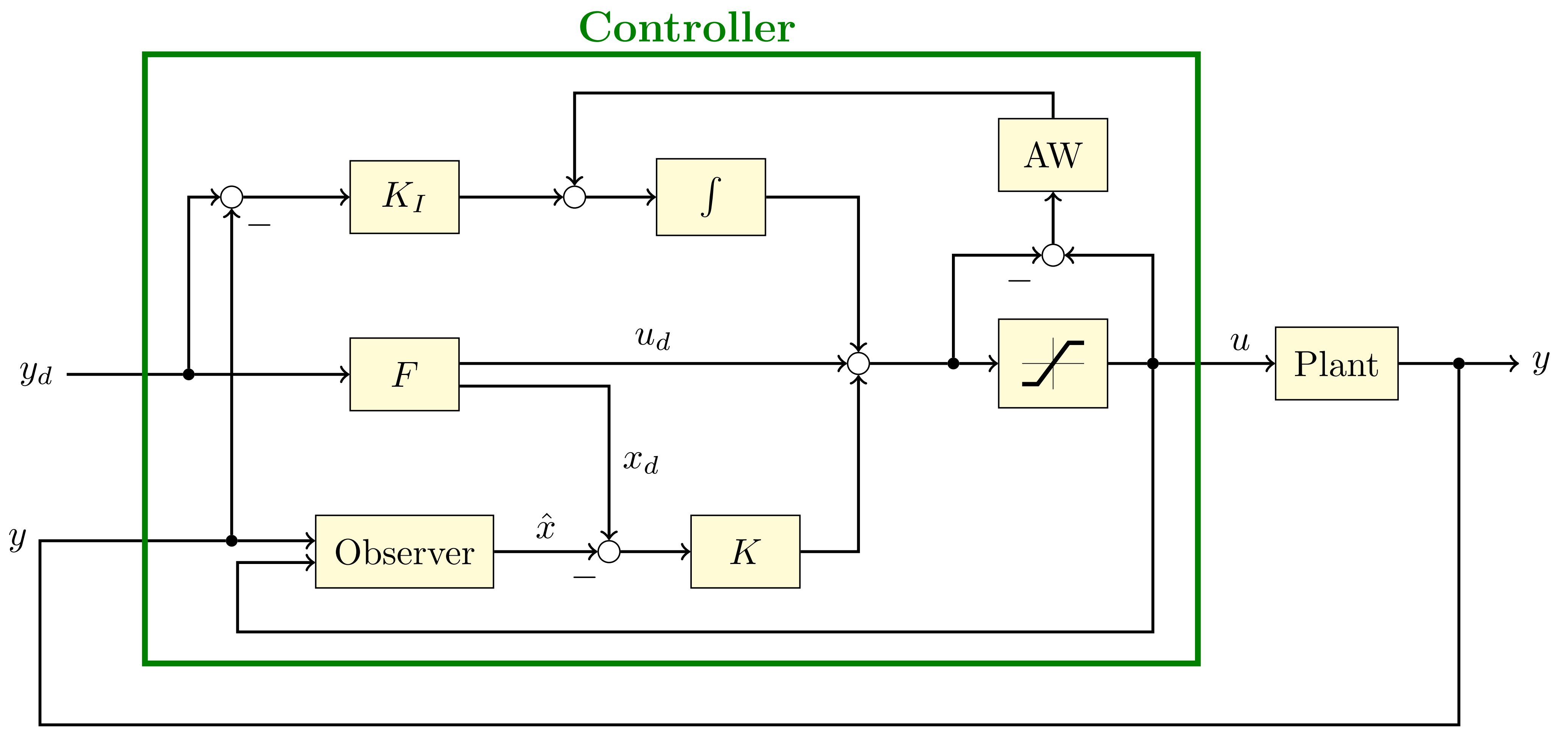
$$a_{6,6} = -\frac{1}{\tau_E}$$

$$b_{11} = -\frac{1}{nm\left\langle R^2\right\rangle}KG(\rho)\langle R^2\rangle\omega_0$$

$$b_{i+1,i+1} = \kappa_{NBI_i}$$

4- Toroidal rotation: the control

Controller design (LQG) for NSTX & NSTX-U:



•Feedforward (F)
Set-point of desired profile

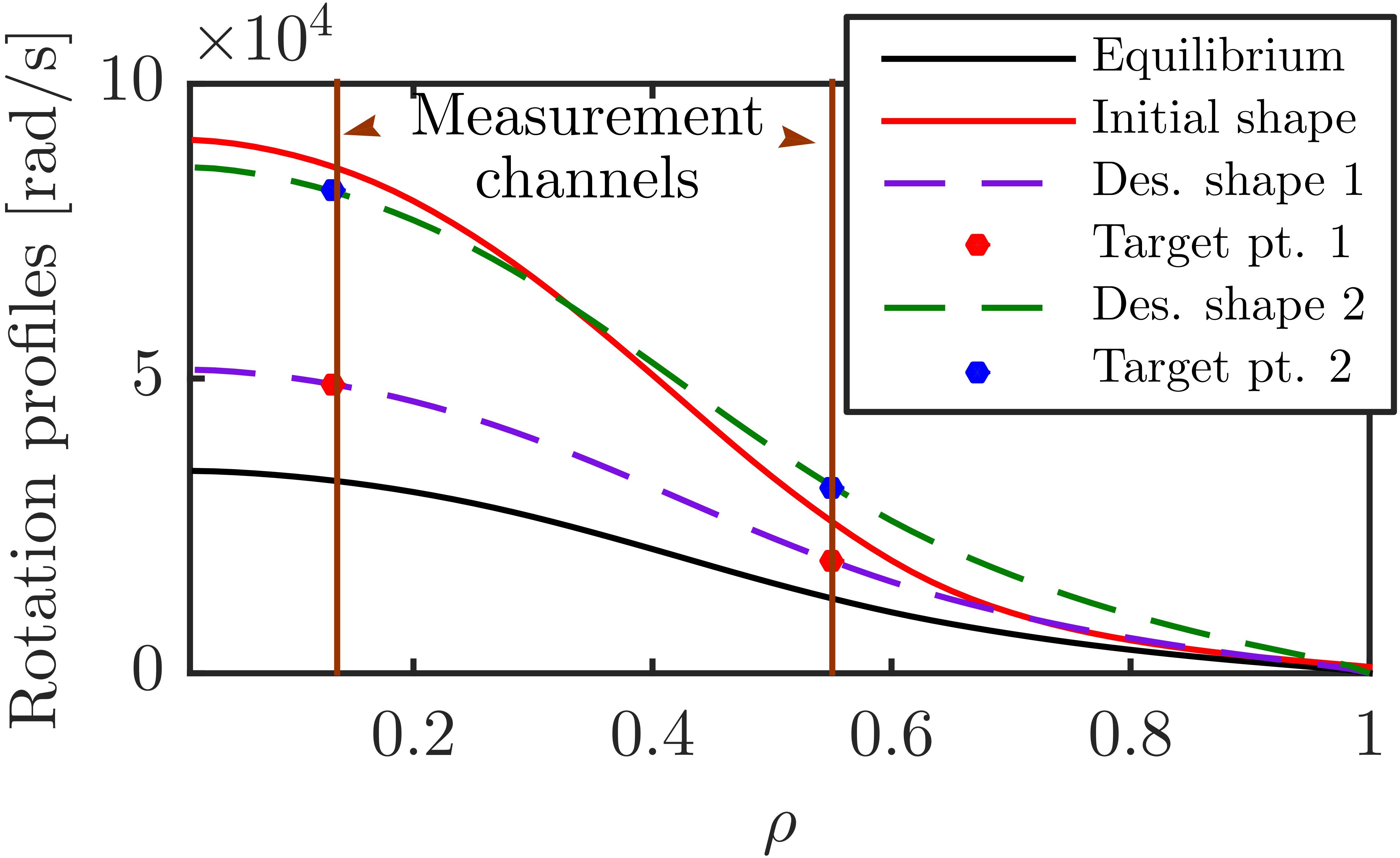
•Linear Quadratic Regulator (K)
Minimize cost function: $\mathcal{J} = \int_{t_0}^{\infty} \left(x^T Q x + u^T R u\right) dt$

•Linear Quadratic Integrator (K_I)
Integrate error to remove steady-state error

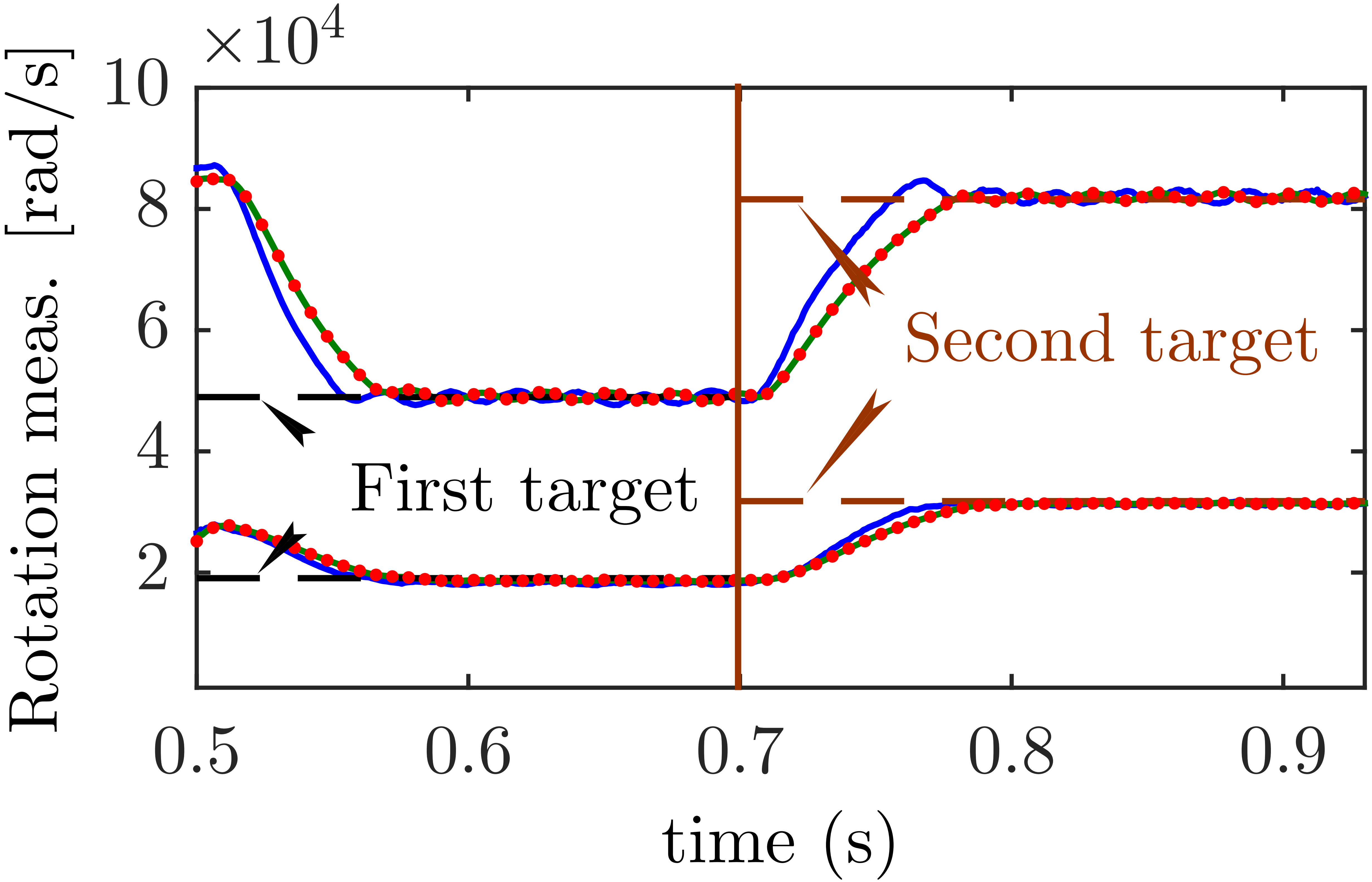
•Anti-windup (AW)
Prevent actuator windup due to saturation

•Observer
Predict full state based on point-wise measurements

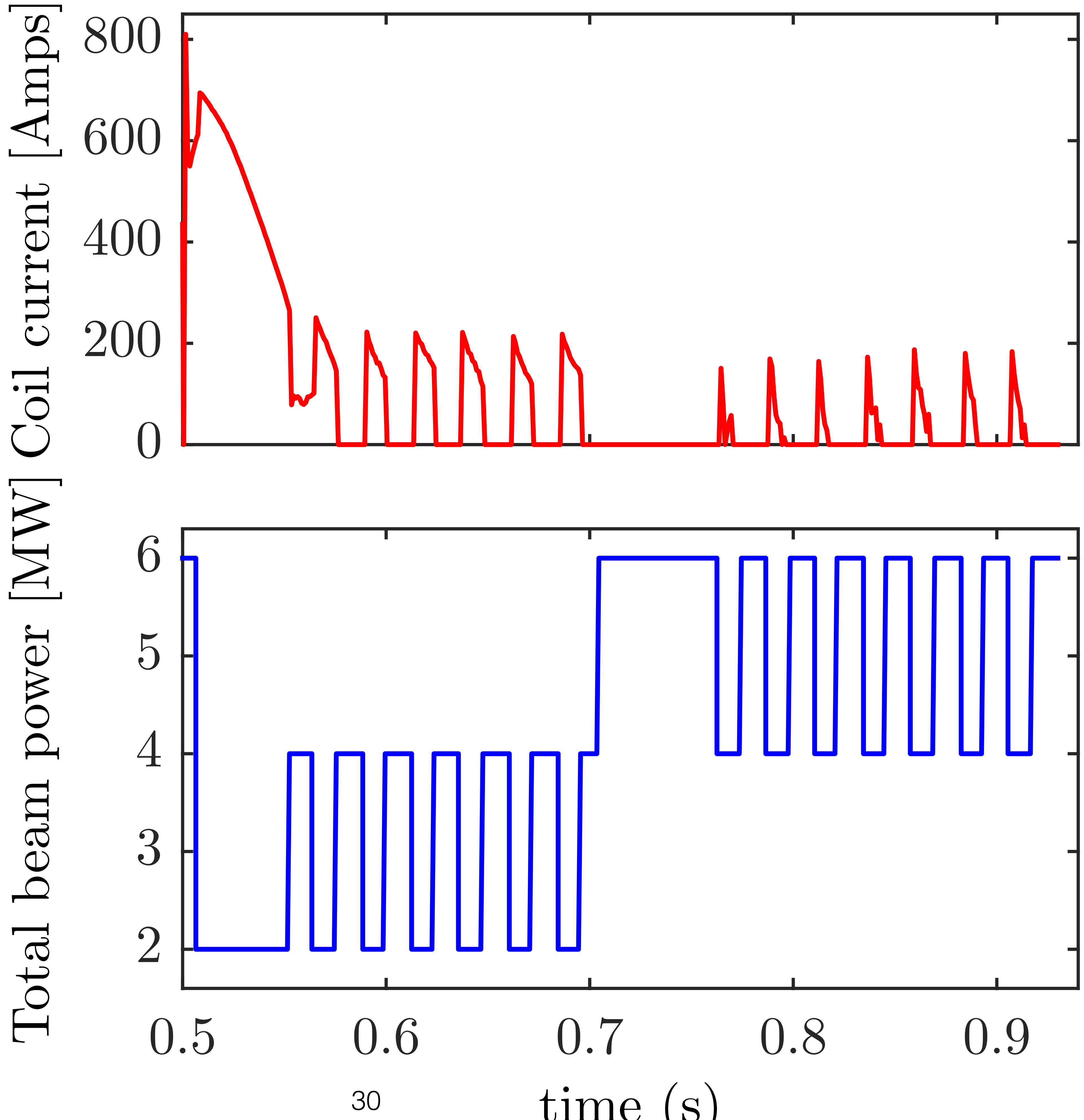
Results for NSTX



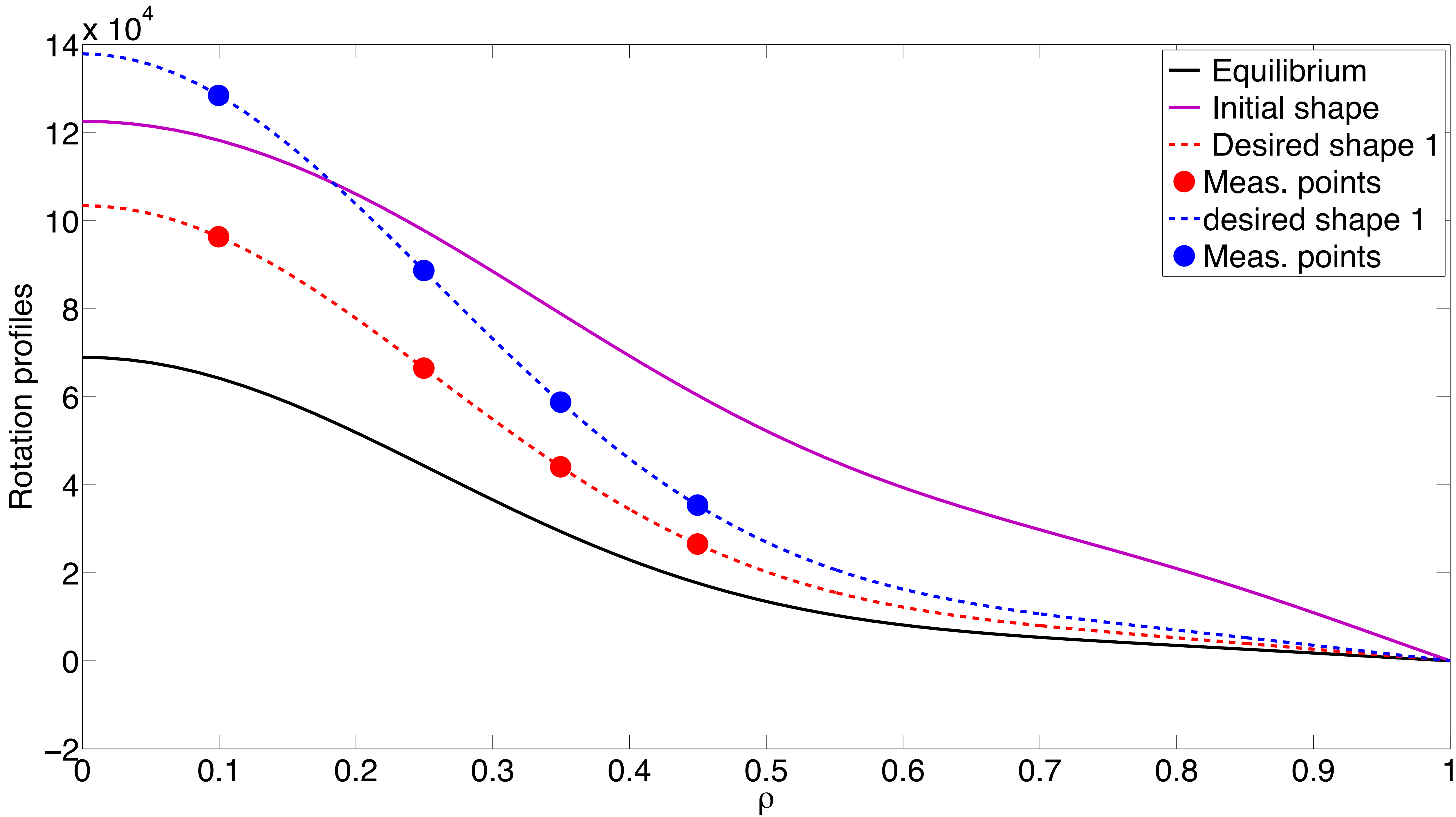
Results for NSTX



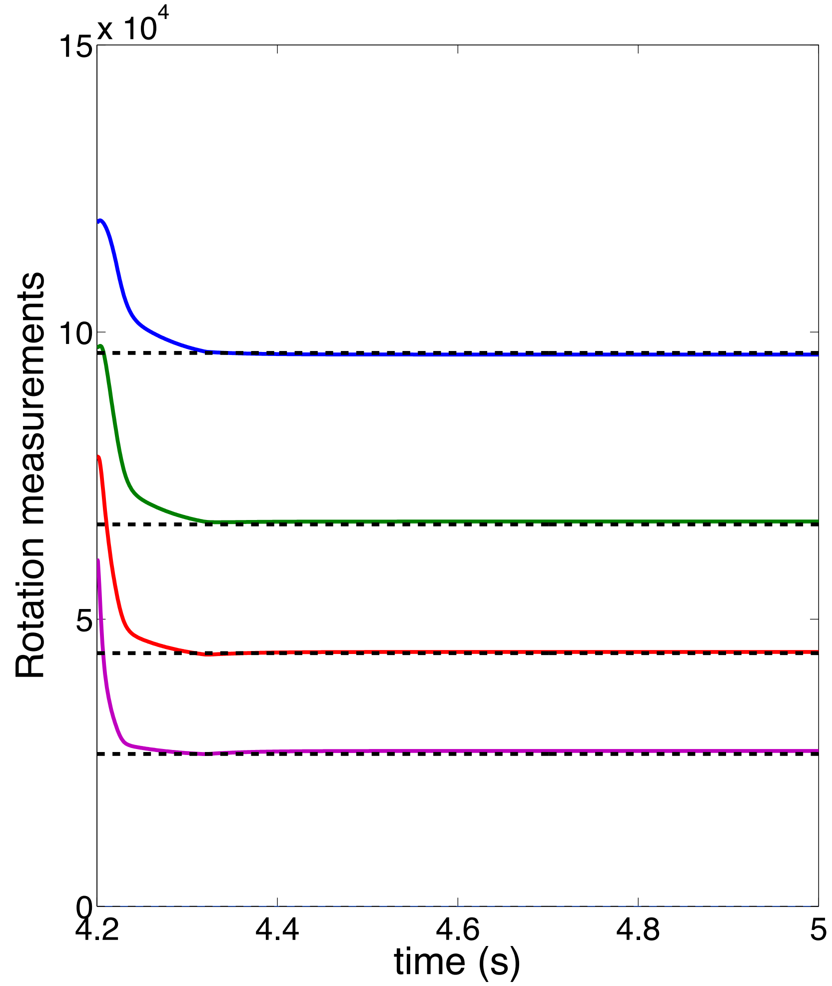
Inputs needed for NSTX

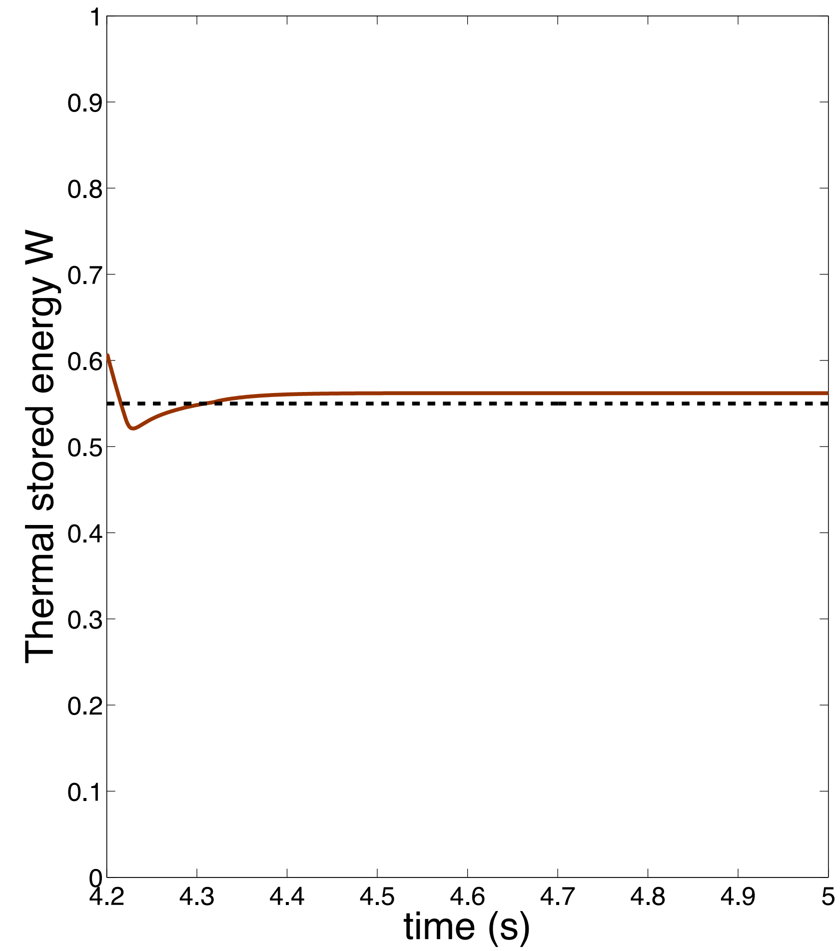


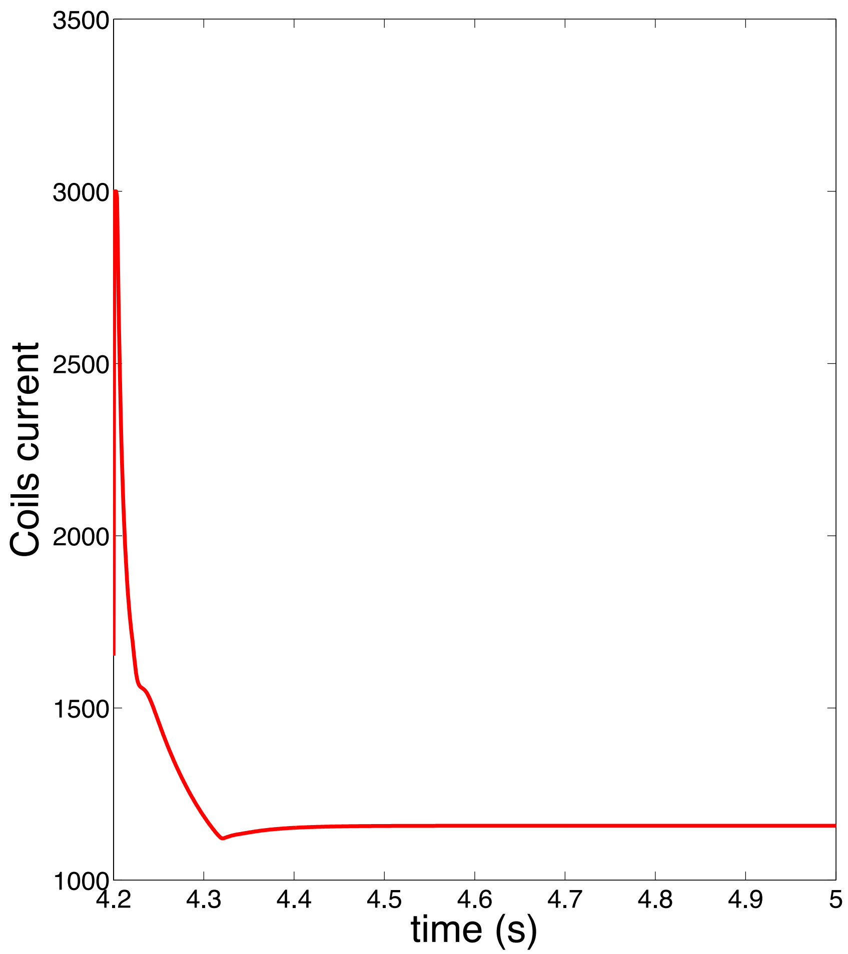
Initiation states for NSTX-U:

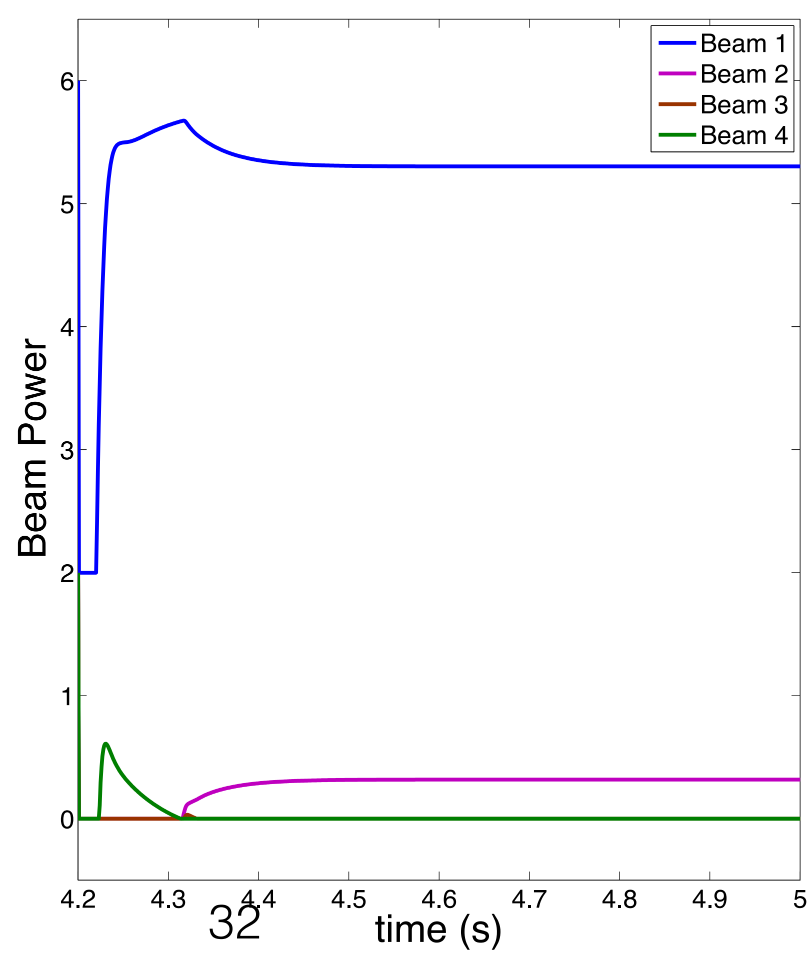


Nonlinear model inputs and outputs

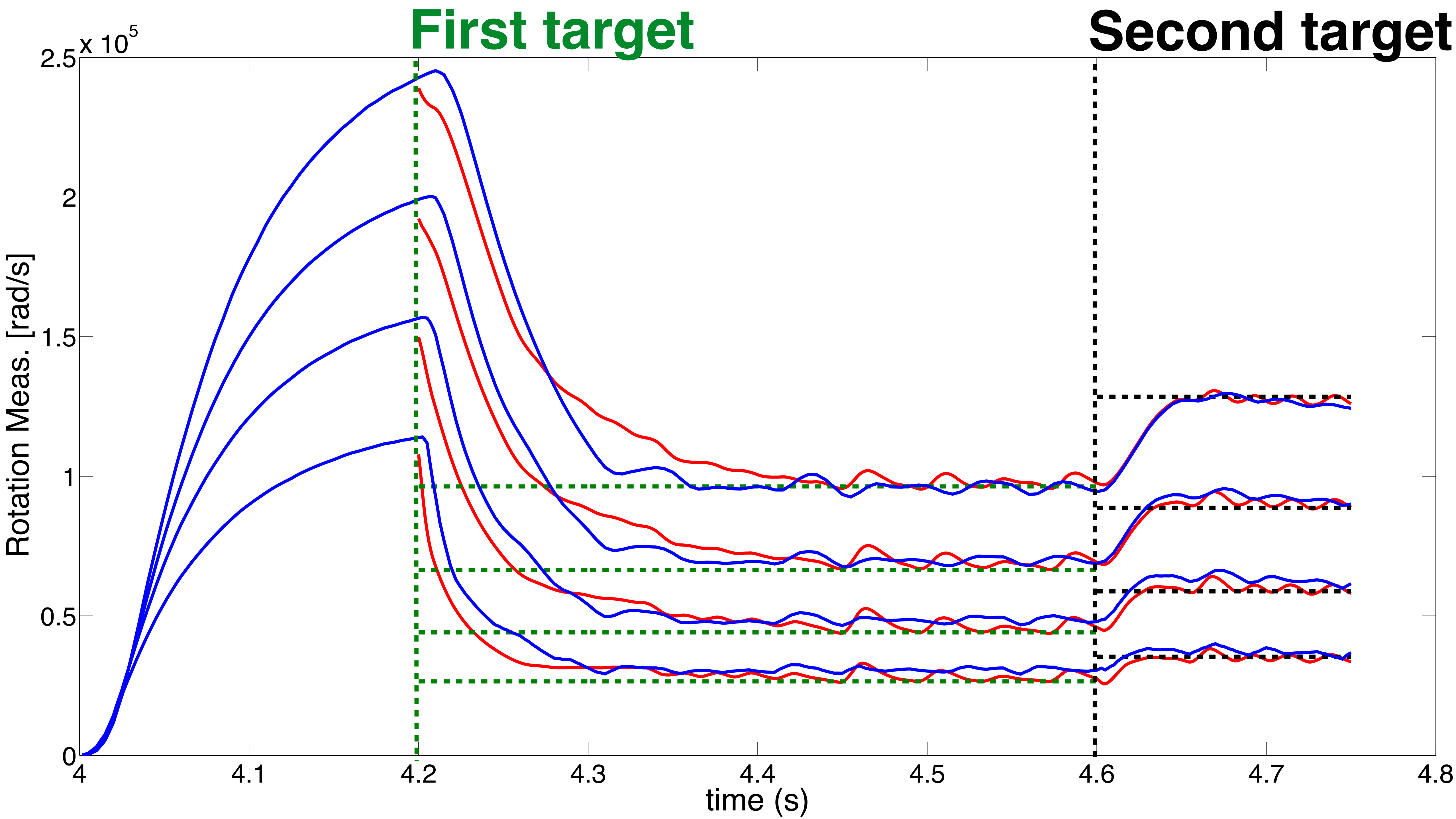




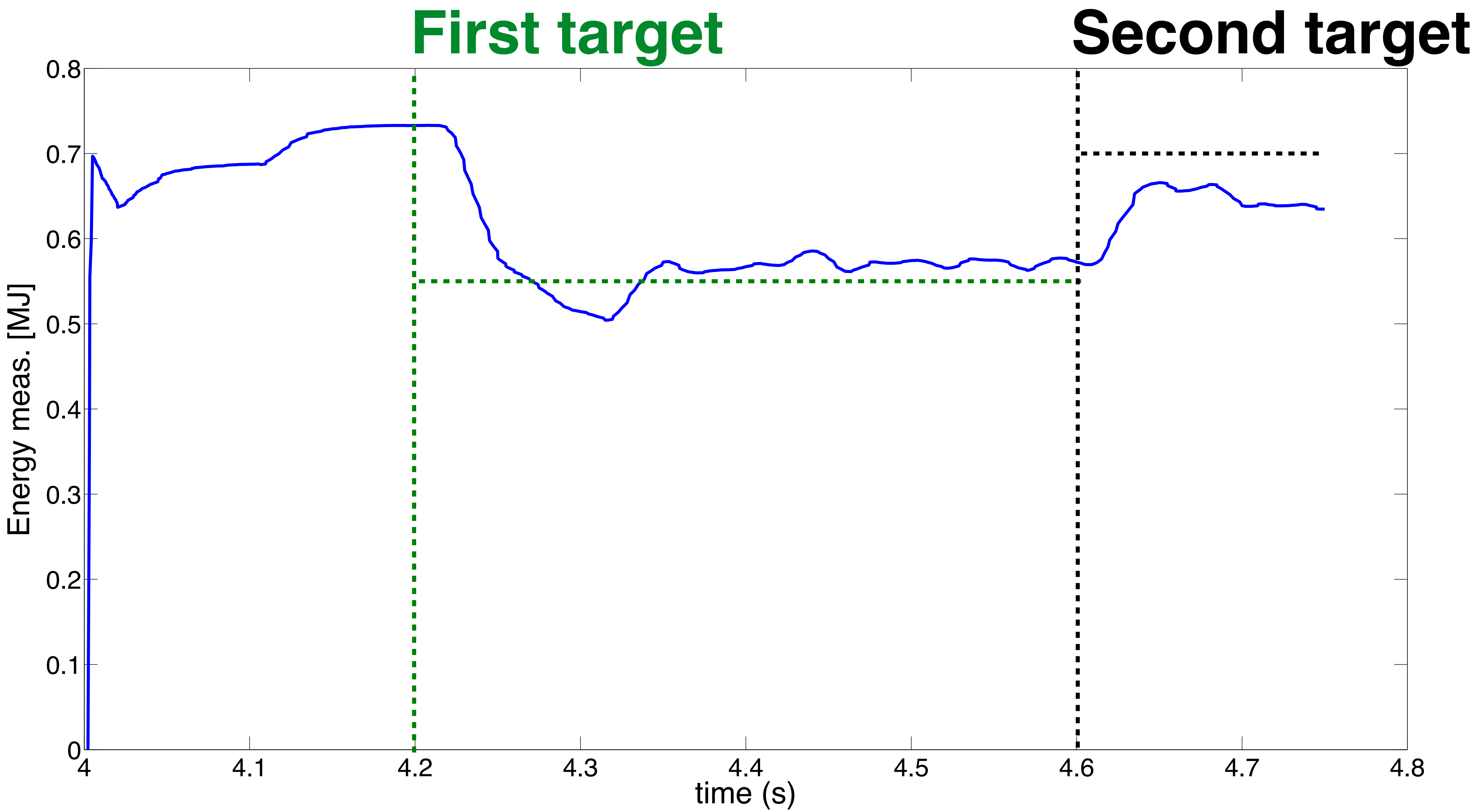




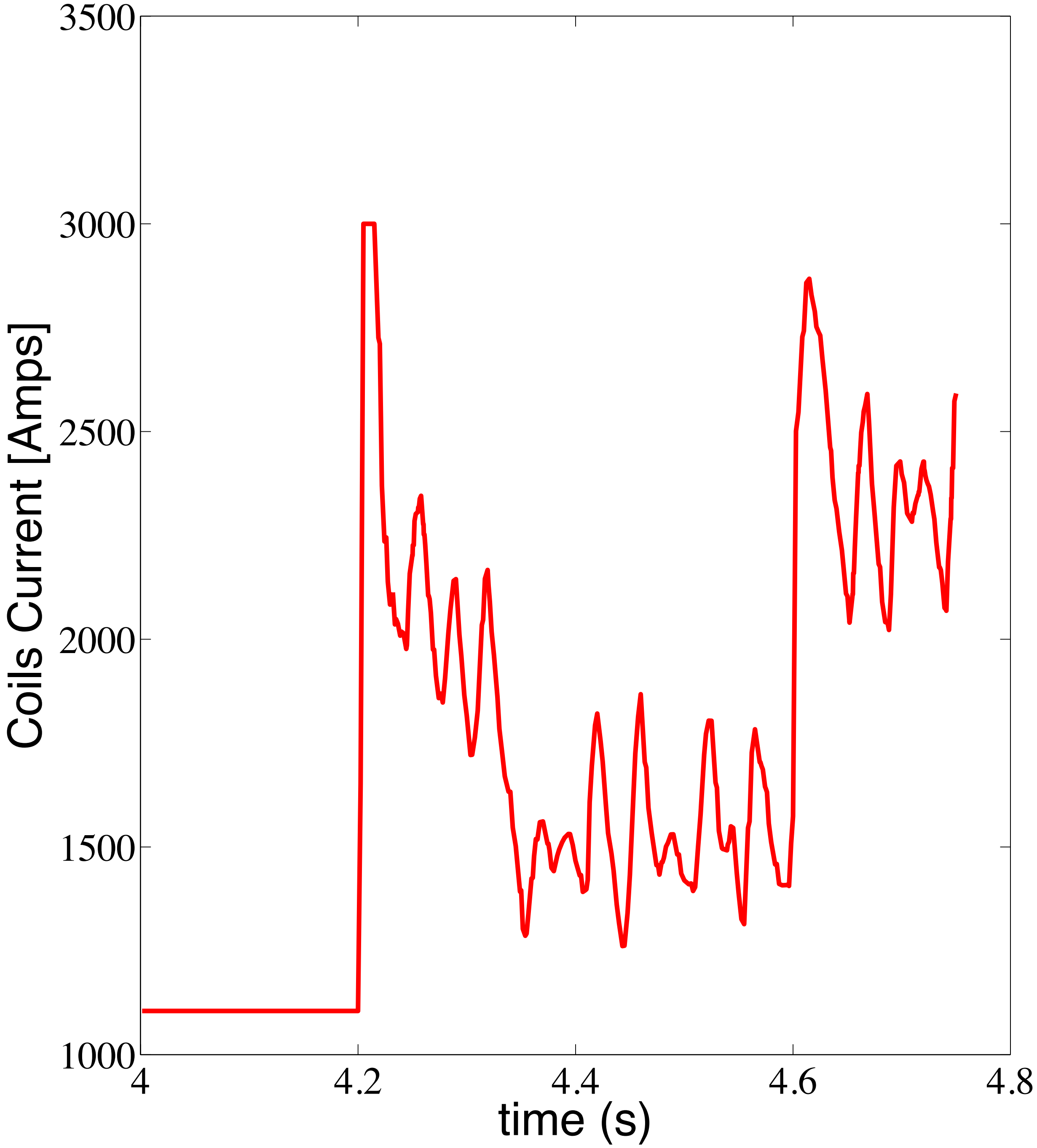
Results: TRANSP model - Measured toroidal rotation

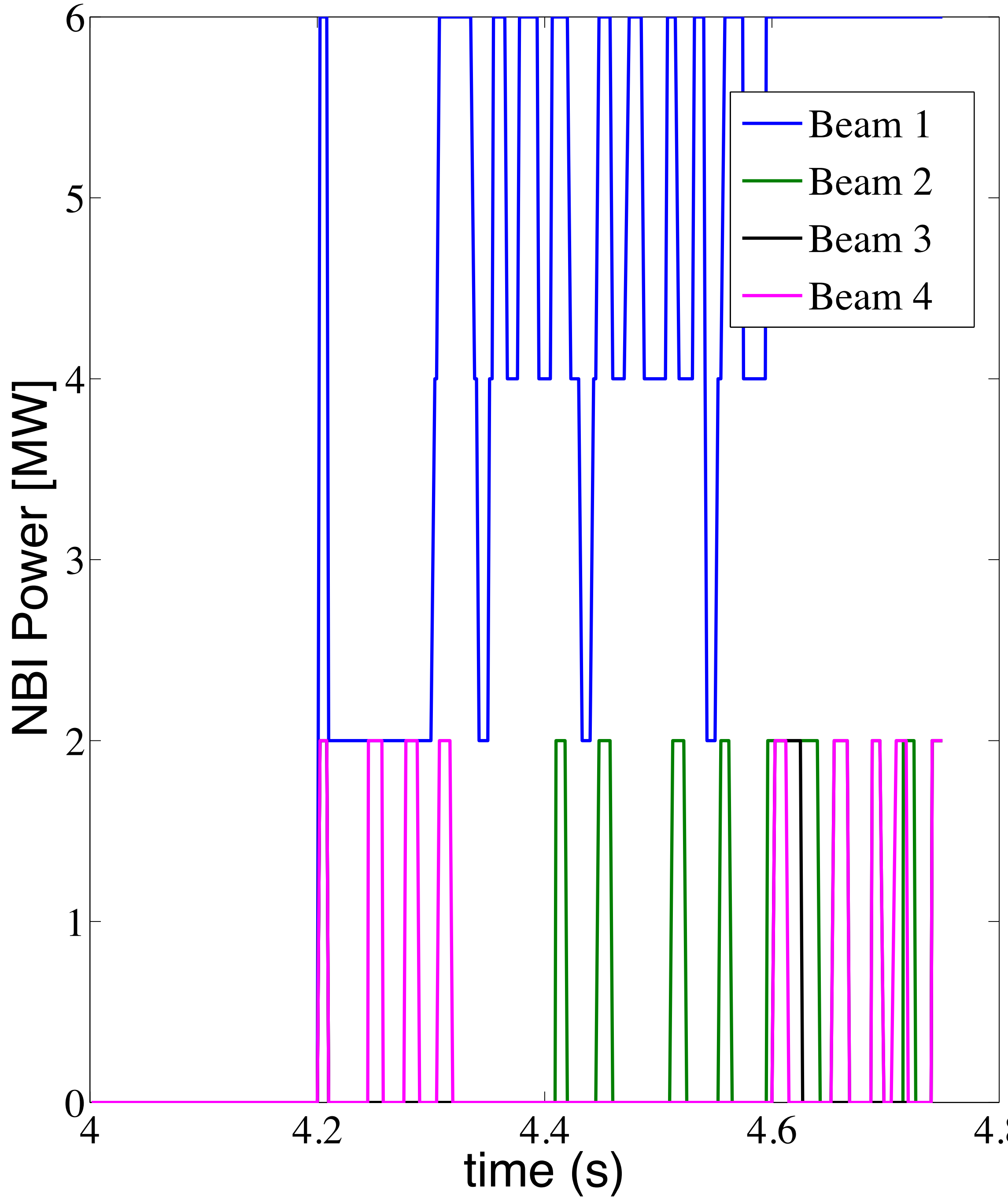


Results: TRANSP model - Measured Stored thermal energy W



Results: TRANSP model - Coil current and Beam power needed





Summary

- NSTX and NSTX Upgrade devices
- Linear control tools
- Based on reduced order model
- Only few measurement points
- Strict constraints on the actuators
- Second line of three neutral beams, spatially more extended
- No data available, 100% model based approach
- Satisfactory Control
- Implemented in TRANSP

Work to do

- Study robustness in stability and performance
- Implementing in real machine through PCS