Exploring spectral energy transfer in nonlinear gyrokinetic simulations to understand zonal flow drive

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Thanks to Jason Parisi, Jack Berkery and Toby Adkins

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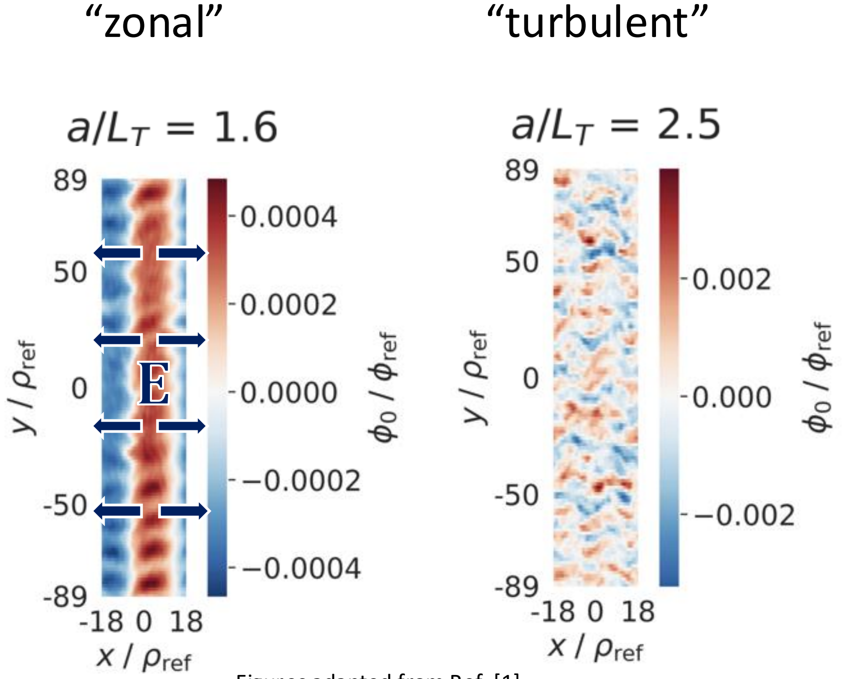
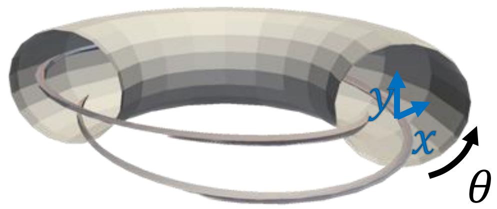
The role of zonal flows in regulating turbulence

- Zonal flows are radially-sheared $\mathbf{E} \times \mathbf{B}$ poloidal flows that cause turbulent transport to saturate at lower levels
- Linearly stable but driven nonlinearly through the Reynolds-Maxwell stress $R_{xy} = \langle v_x v_y \rangle - \langle b_x b_y \rangle / \mu_0 \bar{\rho}$
- Regulate turbulent transport in two ways:
  - Radial flow shear tilts and stretches eddies
  - Energy transferred into zonal flows also reduces energy of the turbulence



Figures adapted from Ref. [1]

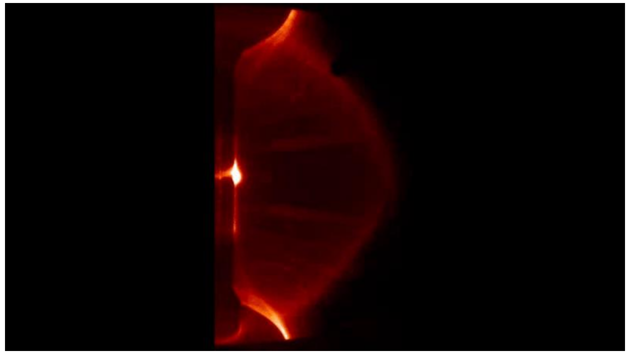


[1] S. Biggs-Fox (2022). *Spatial structure of micro-instabilities in tokamak plasmas: zonal flows and global effects in local gyrokinetic simulations*. PhD thesis, University of York.

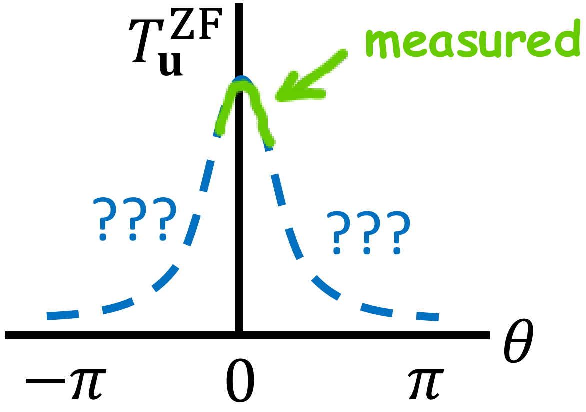
Zonal flows as a trigger mechanism for the L-H transition

- Cziegler et al. [2, 3] estimated the nonlinear transfer of energy from turbulence to zonal flows to be a factor $\sim 3.5\times$ larger than the reduction in turbulent energy.

- "this is as good an agreement as can be experimentally expected" without detailed knowledge of the poloidal structure of the zonal transfer [3]



Credit: Steven Lisgo on YouTube



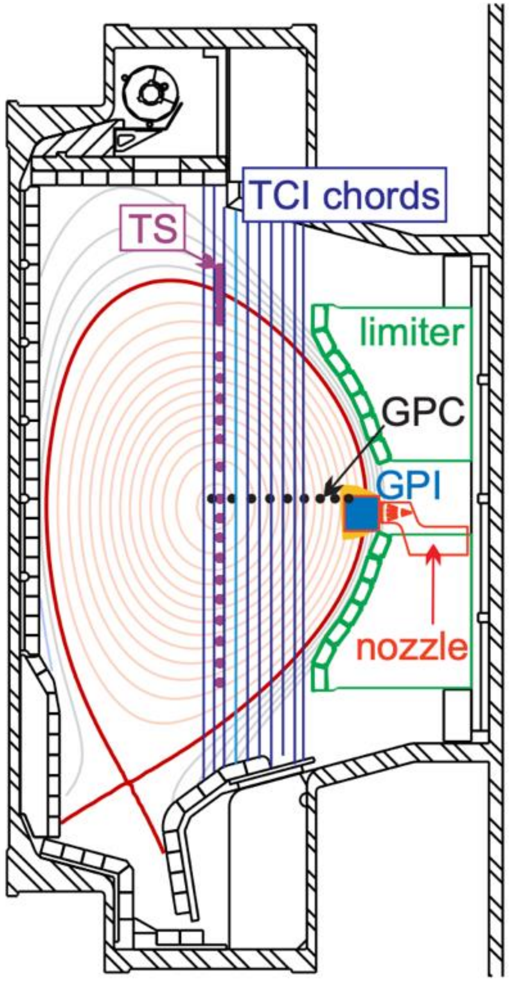


Figure from Ref. [3]

[2] I. Cziegler et al. (2014). *Zonal flow production in the L–H transition in Alcator C-Mod*. Plasma Physics and Controlled Fusion, 56(7), 075013.

[3] I. Cziegler et al. (2015). *Nonlinear transfer in heated L-modes approaching the L–H transition threshold in Alcator C-Mod*. Nuclear Fusion, 55(8), 083007.

Questions we'd like to answer

- What is the poloidal structure of ZF drive?
- Why does it exhibit the observed structure?
- How does it depend on shaping? (e.g. elongation κ and triangularity δ)
- Given the observed structure, can we explain the discrepancy between the energy transferred into ZFs and reduction in turbulent activity observed by Cziegler et al.?
- How does the ZF drive and turbulence saturation level change with increasing β?
- What are the most favourable operating conditions (κ, κ', δ, δ', q, $\hat{s}$, L_T, L_n, ...) that lead to an increased β_{NZT}

Transfer functions

Momentum equation:

$$n_s m_s \left[\frac{\partial \mathbf{u_s}}{\partial t} + (\mathbf{u_s} \cdot \nabla)\mathbf{u_s}\right] + \nabla p_s + \nabla \cdot \Pi_s = Z_s e n_s (\mathbf{E} + \mathbf{u_s} \times \mathbf{B}) + \sum_{s' \neq s} \mathbf{F}_{ss'}$$

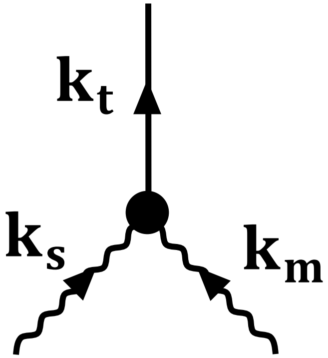
Nonlinear kinetic energy transfer function [1]

$$T_{\mathbf{u}} \equiv \frac{\partial |\mathbf{u_k}|^2}{\partial t} = \frac{2[(\mathbf{k_t} \times \mathbf{k_s}) \cdot \hat{\mathbf{z}})](\mathbf{k_t} \times \hat{\mathbf{z}}) \cdot (\hat{\mathbf{z}} \times \mathbf{k_s})}{B^3} \mathrm{Re}[\phi^*(\mathbf{k_t})\, \phi(\mathbf{k_m})\, \phi(\mathbf{k_s})]$$

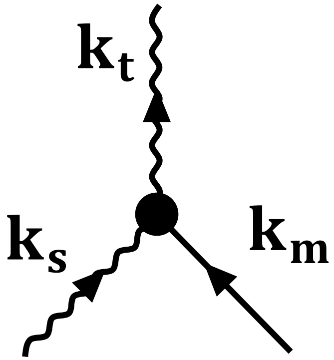
- $T_{\mathbf{u}}$ describes the energy transfer caused by the quadratic nonlinearity $(\mathbf{u} \cdot \nabla)\mathbf{u}$ via the three-wave interaction

$$\mathbf{k_t} = \mathbf{k_s} + \mathbf{k_m}$$

target, source, mediator



zonal drive



zonal shearing

- Above expression comes from taking the $\mathbf{E} \times \mathbf{B}$ velocity to be $\mathbf{u_k} = (\hat{\mathbf{z}} \cdot \nabla_\perp \phi_\mathbf{k})/B_0$, which does not include $\delta A_\parallel$ or $\delta B_\parallel$.

$$T_{\mathbf{u}} = T_{\mathbf{u}}(t, \theta, k_{xs}, k_{ys}, k_{xt}, k_{yt})$$

[1] S. Biggs-Fox (2022). *Spatial structure of micro-instabilities in tokamak plasmas: zonal flows and global effects in local gyrokinetic simulations*. PhD thesis, University of York.
[4] M. Xu et al. (2009). *Study of nonlinear spectral energy transfer in frequency domain*. Physics of Plasmas

Code implementation in a cartoon

Extend $\phi(k_x, k_y, z)$
to include $k_y < 0$ through
$\phi(-k_x, -k_y, z) = \phi^*(k_x, k_y, z)$

→

Loop through all (k_{xt}, k_{xs}, k_{ys})
and determine mediators
$\mathbf{k_m} = \mathbf{k_t} - \mathbf{k_s}$

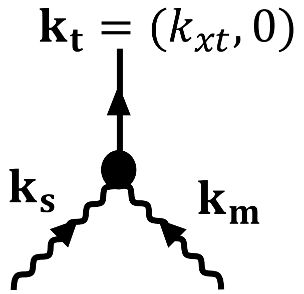
→

If valid: compute transfer

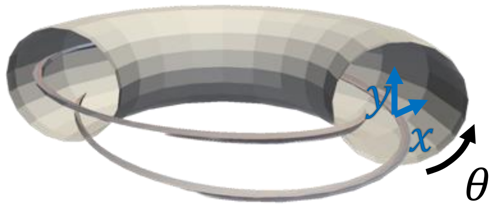
Else: zero transfer

$$T_\mathbf{u} \equiv \frac{\partial |\mathbf{u_k}|^2}{\partial t} = \underbrace{\frac{2[(\mathbf{k_t} \times \mathbf{k_s}) \cdot \hat{\mathbf{z}})](\mathbf{k_t} \times \hat{\mathbf{z}}) \cdot (\hat{\mathbf{z}} \times \mathbf{k_s})}{B^3}}_{k_{xt}^2 k_{xs} k_{ys} / B_0^3 \text{ as } k_{yt} = 0} \mathrm{Re}[\phi^*(\mathbf{k_t})\, \phi(\mathbf{k_m})\, \phi(\mathbf{k_s})]$$

$$T_\mathbf{u} = T_\mathbf{u}(t, \theta, k_{xs}, k_{ys}, k_{xt})$$



zonal drive



Analysis of NETFs

- $T_{\mathbf{u}} = T_{\mathbf{u}}\left(t, \theta, k_{xs}, k_{ys}, k_{xt}, k_{yt}\right)$

- Facilitates the following types of analysis:

$$\left\langle \sum_{k_{xs}, k_{ys}, k_{xt}} T_{\mathbf{u}}(k_{yt} = 0) \right\rangle_t$$

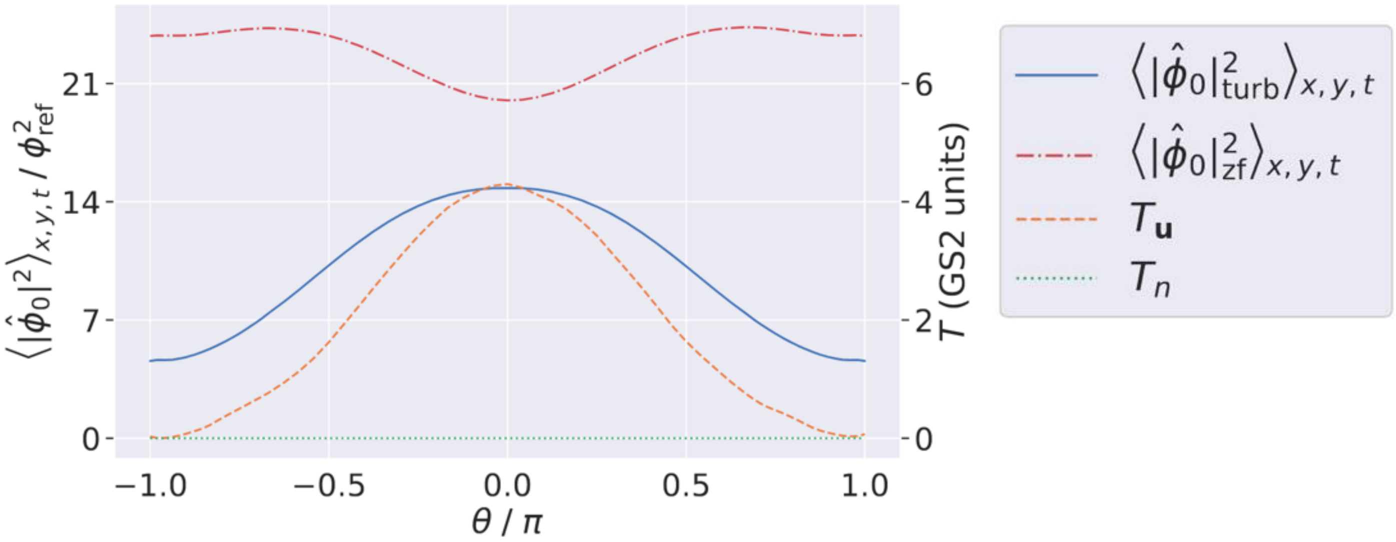
poloidal structure of ZF drive

$$\sum_{k_{xs}, k_{ys}, k_{xt}} T_{\mathbf{u}}(\theta = 0, k_{yt} = 0)$$

time trace ZF drive at a particular poloidal angle

$$\left\langle T_{\mathbf{u}}\left(\theta = 0, k_{ys} = k_{yt} = 0.35\right)\right\rangle_t$$

k_x spectrum of transfer



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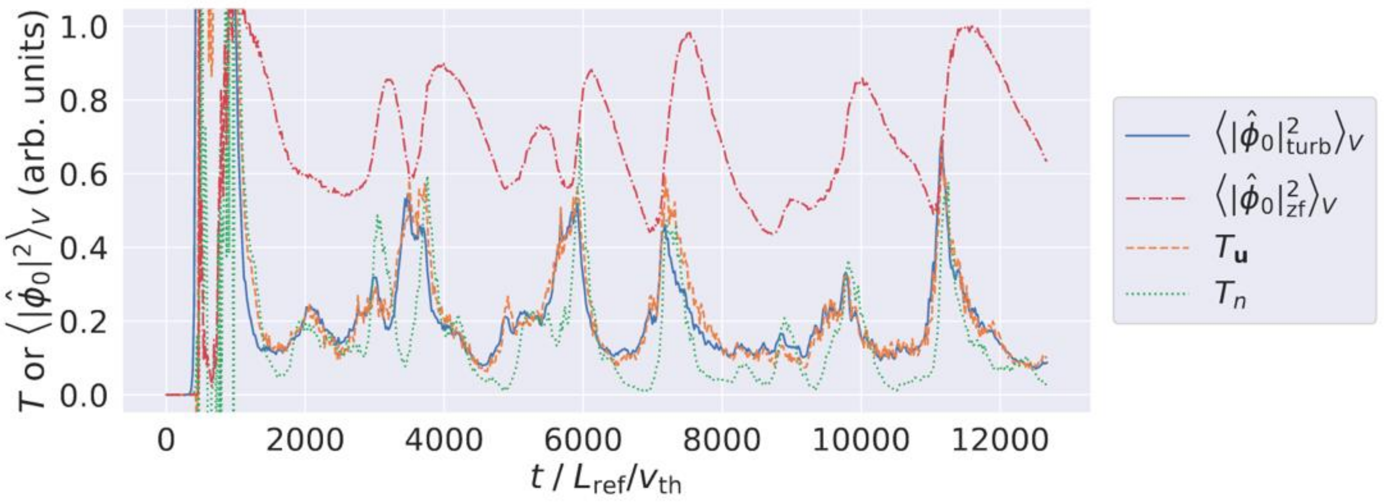

$$\langle T_{\mathbf{u}}(\theta = 0, k_{ys} = k_{yt} = 0.35)\rangle_t$$

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$$\left\langle \sum_{k_{xs}, k_{ys}, k_{xt}} T_{\mathbf{u}}(k_{yt} = 0) \right\rangle_t$$ poloidal structure of ZF drive

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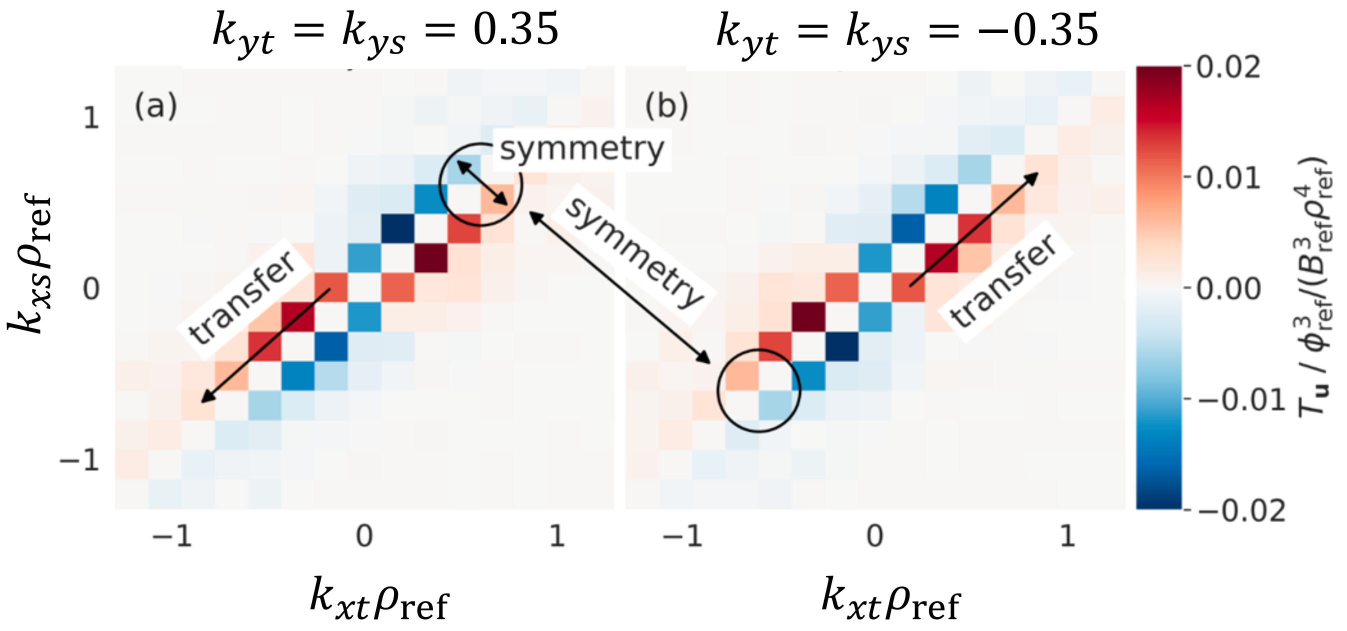

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GX vs GS2 comparison

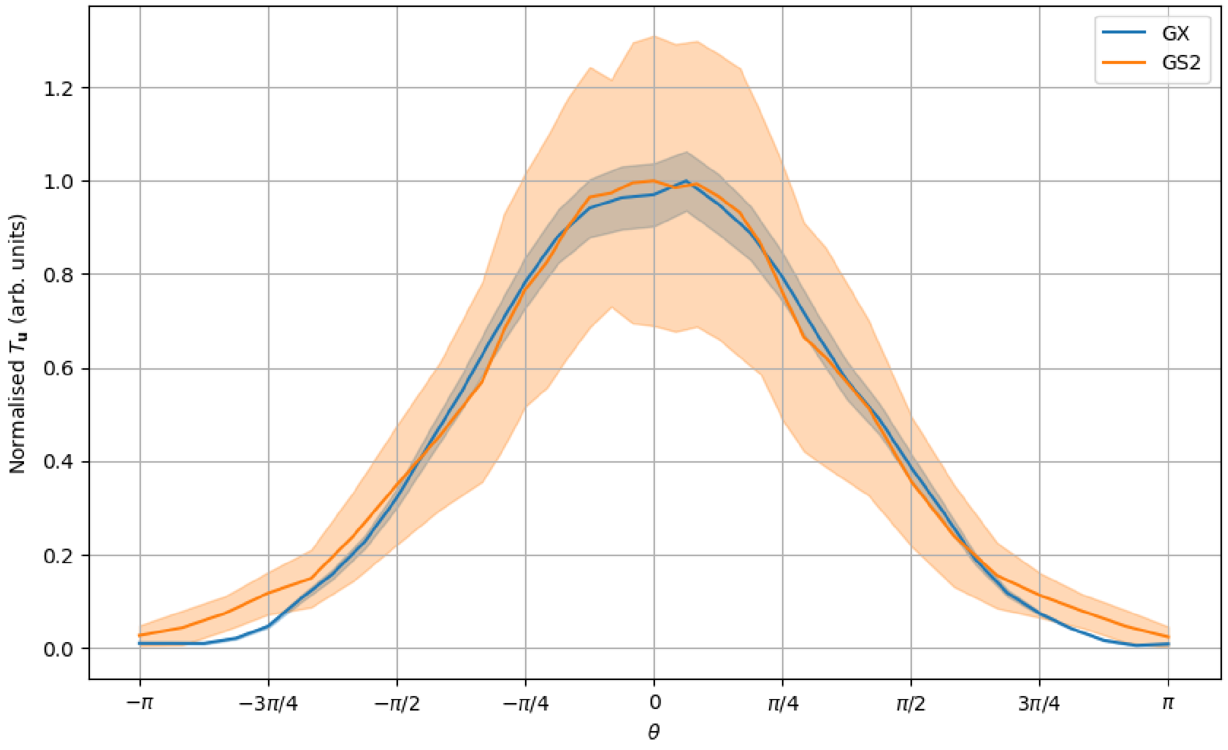
- Nonlinear kinetic energy transfer diagnostics added by Tobias Schuett to the local δf gyrokinetic code GS2 [5]. Paper in review!

- I've added the same diagnostic to GX [6], a GPU-native local δf gyrokinetic code.

- Good agreement between codes for CBC parameters (electrostatic, adiabatic electrons)

```
[species]
  z = [ 1.0,]
  mass = [ 1.0,]
  dens = [ 1.0,]
  temp = [ 1.0,]
  tprim = [ 6.0,]
  fprim = [ 0.81,]
  vnewk = [ 0.0,]
  type = [ "ion",]
```

```
[Geometry]
  geo_option = "miller"
  rhoc = 0.8
  Rmaj = 2.72
  R_geo = 2.72
  qinp = 1.4
  shat = 0.78
  shift = 0.0
  akappa = 1.0
  akappri = 0.0
  tri = 0.0
  tripri = 0.0
  betaprim = 0.0
```



[5] GS2 v.8.2.1. See src/diagnostics/diagnostics_kinetic_energy_transfer.fpp
[6] GX branch ZF-energy-transfer-diagnostic. Working as of commit 096fc9f.

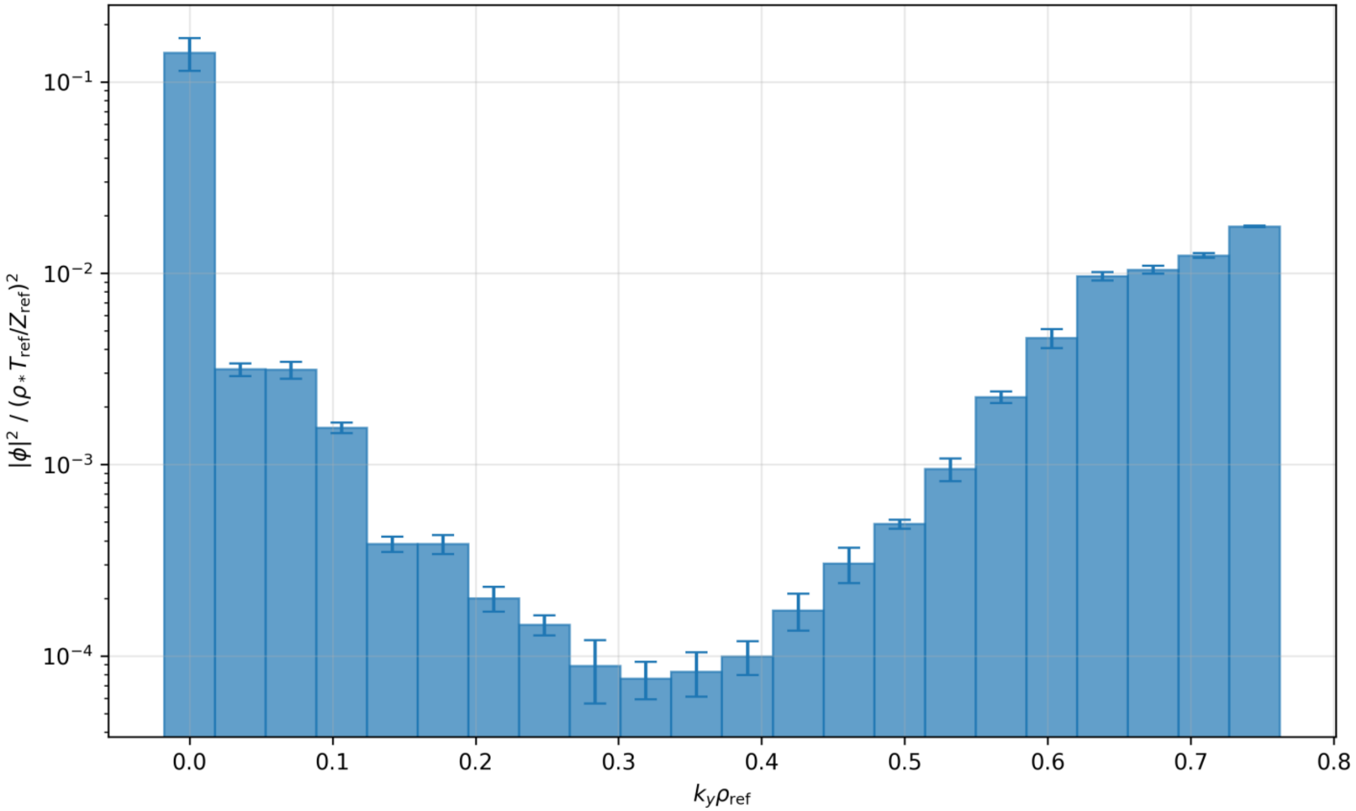
NSTX shots 132543 and 132588

- Jason provided me with equilibria and profiles for shots 132543 and 132588.

- **<u>User</u> issue with GX*:** $|\phi|^2$ is peaking at small scales, suggesting the box size is too small and/or there's not enough dissipation.

```
[Dimensions]
  ntheta = 16
  nperiod = 1
  nx = 128
  ny = 64
  nhermite = 12
  nlaguerre = 8
  nspecies = 1
```

```
y0 = 28.200001
x0 = 27.917713
jtwist = 7
shat = 1.137375
```



- Once resolved, I'll apply the diagnostic to these shots

* *my fault, not GX!*

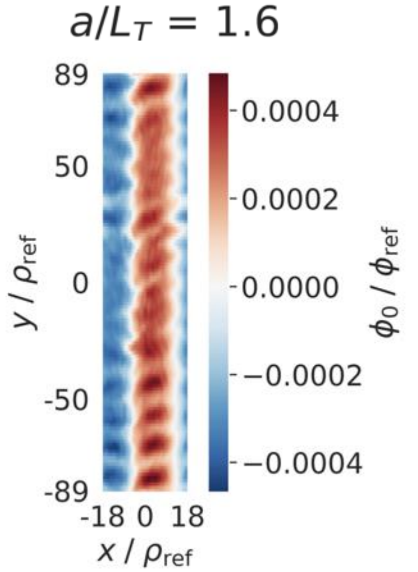
Effect of magnetic fluctuations on ZFs?

$$\beta = \frac{n_0 T_0}{B_0^2/2\mu_0}$$

- Reactor-relevant plasmas will operate at a higher β than present devices ⇒ magnetic fluctuations more important
- Finite-β considerations for ZF drive:
  - ZFs driven through the Reynolds stress, but Maxwell stress also contributes. Relative sign matters!
  - Radial magnetic field fluctuations short the zonal potential

$$R_{xy} = \langle v_x v_y \rangle - \frac{\langle b_x b_y \rangle}{\mu_0 \bar{\rho}}$$

- At some critical β, the non-zonal transition (NZT) occurs, and turbulent transport saturates at much higher levels
- Rath & Peeters [5] observed slowly-growing "mesoscale" ZFs ($k_x \rho_i \approx 1/70$) in local gradient-driven gyrokinetic simulations that provide access to improved-β regimes. This paints a more optimistic picture for the NZT, but needs to be investigated further.



[5] F. Rath and A. G. Peeters (2022). *Transport hysteresis in electromagnetic microturbulence caused by mesoscale zonal flow pattern-induced mitigation of high β turbulence runaways*. Physics of Plasmas, 29(4).

Effect of magnetic fluctuations on ZFs?

1) Write the numerator and denominator of $\mathbf{u}_{\mathbf{k}_\perp} = \left(\delta\mathbf{E}_{\mathbf{k}_\perp} \times \mathbf{B}_{\mathbf{k}_\perp}\right)/\left|\mathbf{B}_{\mathbf{k}_\perp}\right|^2$ as

$$\delta\mathbf{E}_{\mathbf{k}_\perp} \times \mathbf{B}_{\mathbf{k}_\perp} = i\delta\phi_{\mathbf{k}_\perp}(B_0 + \delta B_{\|,\mathbf{k}_\perp})(\hat{\mathbf{b}} \times \mathbf{k}_\perp) - k_\perp^2 \delta\phi_{\mathbf{k}_\perp}\delta A_{\|,\mathbf{k}_\perp}\hat{\mathbf{b}} + i\left(\frac{B_0 + \delta\mathbf{B}_{\|,\mathbf{k}_\perp}}{k_\perp^2}\partial_t\delta B_{\|,\mathbf{k}_\perp} - \delta A_{\|,\mathbf{k}_\perp}\partial_t\delta A_{\|,\mathbf{k}_\perp}\right)\mathbf{k}_\perp$$

$$|\mathbf{B}_{\mathbf{k}_\perp}|^2 = (B_0 + \delta B_{\|,\mathbf{k}_\perp})^2 - k_\perp^2\delta A_{\|,\mathbf{k}_\perp}^2$$

2) Substitute $\mathbf{u}_{\mathbf{k}_\perp}$ into general formular for NETF: $T_\mathbf{u} = -2\mathrm{Re}[\,\boldsymbol{u}_\mathbf{k}^* \cdot (\mathbf{u}_{\mathbf{k}-\mathbf{k}'} \cdot \nabla_\perp)\mathbf{u}_{\mathbf{k}'}]$

3) ???

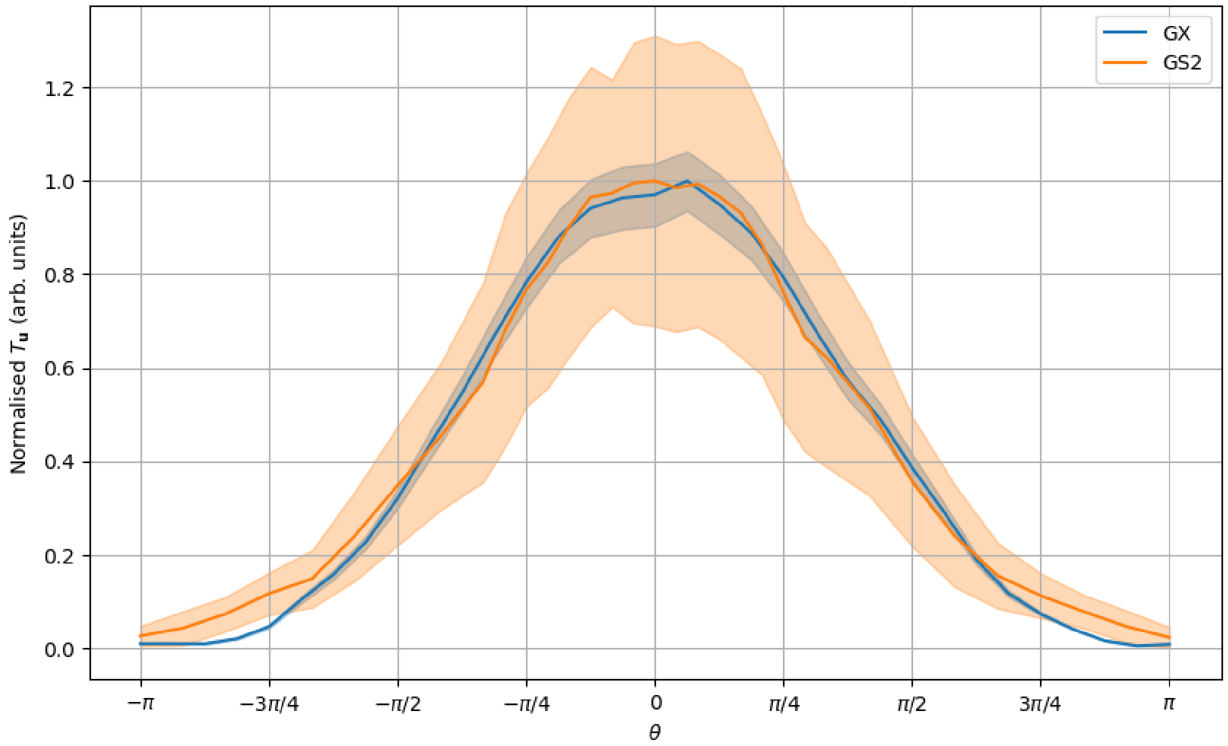
4) Obtain electromagnetic version of

$$T_\mathbf{u} \equiv \frac{\partial|\mathbf{u}_\mathbf{k}|^2}{\partial t} = \frac{2[(\mathbf{k_t} \times \mathbf{k_s}) \cdot \hat{\mathbf{z}})](\mathbf{k_t} \times \hat{\mathbf{z}}) \cdot (\hat{\mathbf{z}} \times \mathbf{k_s})}{B^3}\mathrm{Re}[\phi^*(\mathbf{k_t})\,\phi(\mathbf{k_m})\,\phi(\mathbf{k_s})]$$

Summary & to-dos

Summary:

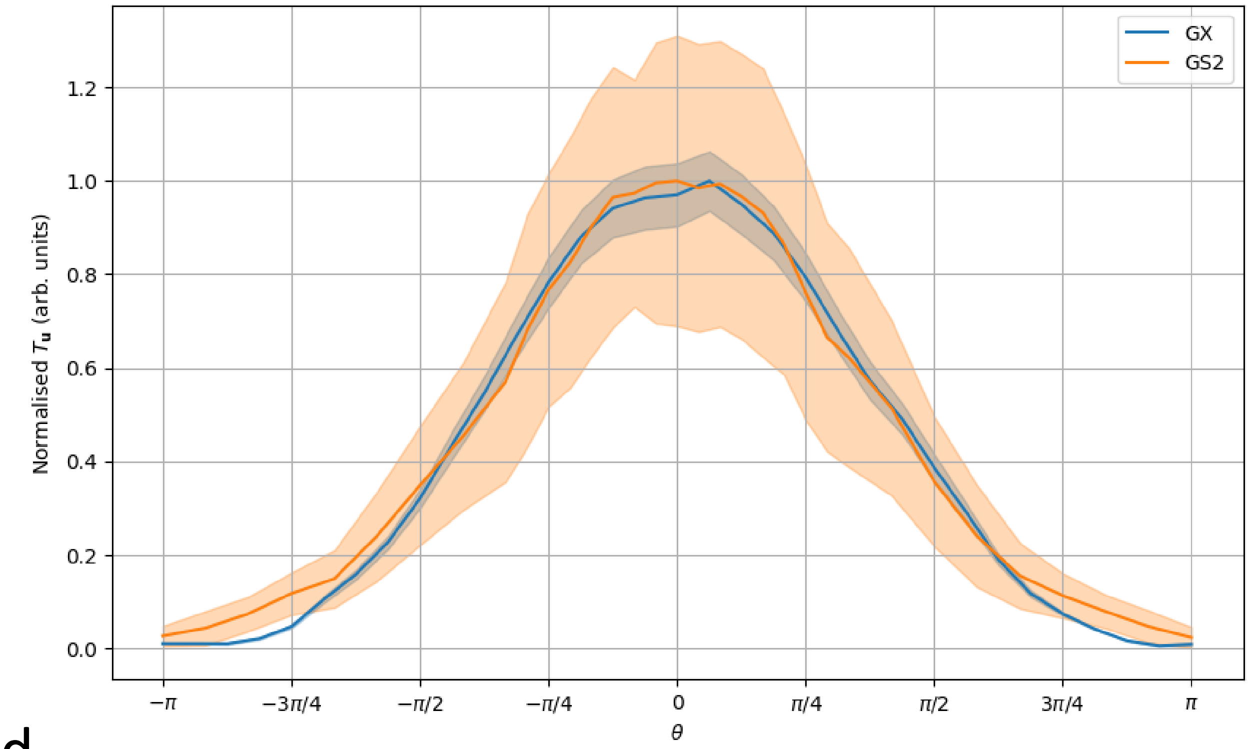
- A new diagnostic has been added to the gyrokinetic code GX that calculates the spectrally resolved nonlinear energy transferred to zonal flows.
- The diagnostic shows good agreement with an existing diagnostic in GS2
- No insights to share re: NSTX shots 132543 and 132588… yet!

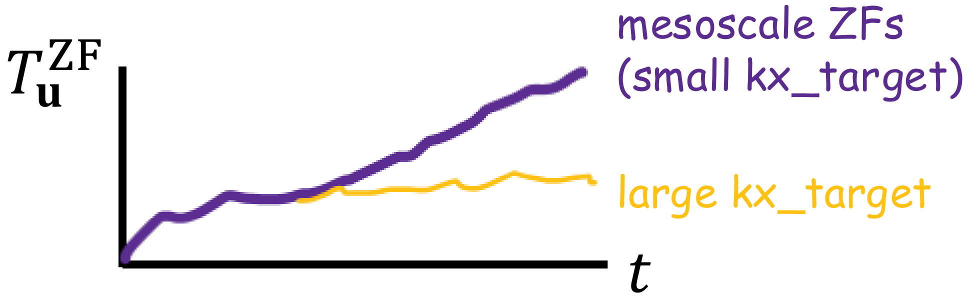


Summary & to-dos

To-do:

- Resolve numerical issues with NSTX simulations and compare GX and GS2 diagnostic outputs for shots 132543 and 132588
- Derive electromagnetic $T_{\mathbf{u}}^{\mathrm{ZF}}$ and add it to GS2 and GX diagnostics
- Investigate Rath & Peeters mesoscale ZFs at $\beta \sim 1\%$ by running electromagnetic CBC and NSTX cases for a long time ($\sim 10^3\ a/v_{\mathrm{th},i}$)
- Investigate effects of shaping on ZF drive in the presence of electromagnetic turbulence





Thank you!

Any questions?

Backup slides

Brief (!) derivation of electrostatic NETF (1/2)

Ion momentum:

$$n_i m_i \left[\frac{\partial \mathbf{u_i}}{\partial t} + (\mathbf{u_i} \cdot \nabla)\mathbf{u_i}\right] + \nabla p_i + \nabla \cdot \Pi_i = Z_i e n_i (\mathbf{E} + \mathbf{u_i} \times \mathbf{B}) + \sum_{s \neq i} \mathbf{F}_{si}$$

Retain only quadratic nonlinearity

$$\frac{\partial \mathbf{u}}{\partial t} = -(\mathbf{u} \cdot \nabla_\perp)\mathbf{u}$$

Fourier transform

$$\mathcal{F}\left[\frac{\partial \mathbf{u}}{\partial t}\right] = -\mathcal{F}[(\mathbf{u} \cdot \nabla_\perp)\mathbf{u}]$$

Convolution theorem

$$\frac{\partial \mathbf{u_k}}{\partial t} = -\int_{-\infty}^{\infty} \mathrm{d}\mathbf{k}' \, (\mathbf{u_{k-k'}} \cdot \nabla_\perp)\mathbf{u_{k'}}$$

Dot both sides with $\mathbf{u_k^*}$:

$$\boldsymbol{u}_\mathbf{k}^* \cdot \frac{\partial \mathbf{u_k}}{\partial t} = -\int_{-\infty}^{\infty} \mathrm{d}\mathbf{k}' \, \boldsymbol{u}_\mathbf{k}^* \cdot (\mathbf{u_{k-k'}} \cdot \nabla_\perp)\mathbf{u_{k'}}$$

Add this expression to its complex conjugate: $z + z^* = (a + ib) + (a - ib) = 2\mathrm{Re}[z]$

$$2\mathrm{Re}\left[\boldsymbol{u}_\mathbf{k}^* \cdot \frac{\partial \mathbf{u_k}}{\partial t}\right] = -2\mathrm{Re}\left[\int_{-\infty}^{\infty} \mathrm{d}\mathbf{k}' \, \boldsymbol{u}_\mathbf{k}^* \cdot (\mathbf{u_{k-k'}} \cdot \nabla_\perp)\mathbf{u_{k'}}\right]$$

Product rule: since $|z|^2 = zz^*$

$$\frac{\partial |z|^2}{\partial t} = z^* \frac{\partial z}{\partial t} + z \frac{\partial z^*}{\partial t}$$

$$\frac{\partial |\mathbf{u_k}|^2}{\partial t} = -2\mathrm{Re}\left[\int_{-\infty}^{\infty} \mathrm{d}\mathbf{k}' \, \boldsymbol{u}_\mathbf{k}^* \cdot (\mathbf{u_{k-k'}} \cdot \nabla_\perp)\mathbf{u_{k'}}\right]$$

Drop integration over $\boldsymbol{k}'$ and identify $T_u \equiv \partial |\mathbf{u_k}|^2 / \partial t$

$$T_\mathbf{u} = -2\mathrm{Re}[\, \boldsymbol{u}_\mathbf{k}^* \cdot (\mathbf{u_{k-k'}} \cdot \nabla_\perp)\mathbf{u_{k'}}]$$

[4] M. Xu et al. (2009). *Study of nonlinear spectral energy transfer in frequency domain*. Physics of Plasmas

Brief (!) derivation of electrostatic NETF (2/2)

$$T_{\mathbf{u}} = -2\mathrm{Re}[\, \boldsymbol{u}_{\mathbf{k}}^{*} \cdot (\mathbf{u}_{\mathbf{k}-\mathbf{k}'} \cdot \nabla_{\perp})\mathbf{u}_{\mathbf{k}'}]$$

$$\mathbf{u}_{\mathbf{k}}^{*} = \frac{\hat{\mathbf{z}} \cdot \nabla_{\perp}\phi_{\mathbf{k}}^{*}}{B_0},$$

$$\mathbf{u}_{\mathbf{k}-\mathbf{k}'} = \frac{\hat{\mathbf{z}} \cdot \nabla_{\perp}\phi_{\mathbf{k}-\mathbf{k}'}}{B_0}$$

$$\mathbf{u}_{\mathbf{k}'} = \frac{\hat{\mathbf{z}} \cdot \nabla_{\perp}\phi_{\mathbf{k}'}}{B_0}$$

$$T_{\mathbf{u}} = \frac{2(\mathbf{k} \times \mathbf{k}' \cdot \hat{\mathbf{z}})(\mathbf{k} \times \hat{\mathbf{z}}) \cdot (\hat{\mathbf{z}} \times \mathbf{k}')}{B_0^3}\mathrm{Re}[\phi_{\mathbf{k}}^{*}\, \phi_{\mathbf{k}-\mathbf{k}'}\, \phi_{\mathbf{k}'}]$$

Relabel

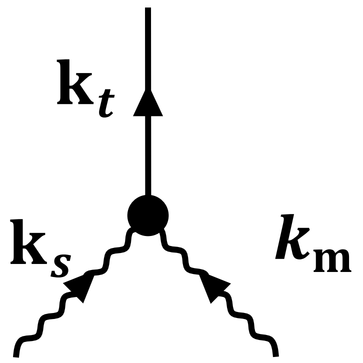
$$T_{\mathbf{u}} = \frac{2(\mathbf{k_t} \times \mathbf{k_s} \cdot \hat{\mathbf{z}})(\mathbf{k_t} \times \hat{\mathbf{z}}) \cdot (\hat{\mathbf{z}} \times \mathbf{k_s})}{B_0^3}\mathrm{Re}[\phi^{*}(\mathbf{k_t})\phi(\mathbf{k_m})\, \phi(\mathbf{k_s})]$$

- $T_{\mathbf{u}}$ describes the energy transfer caused by the quadratic nonlinearity $(\mathbf{u} \cdot \nabla_{\perp})\mathbf{u}$ via the three-wave interaction

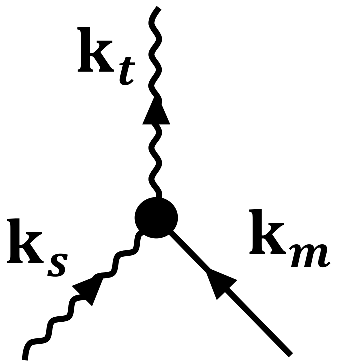
$$\mathbf{k}_t = \mathbf{k}_s + \mathbf{k}_m$$

target, source, mediator

- 'Electrostatic' in the sense that it only considers $\delta\phi$ (i.e. $\delta A_{\parallel} = \delta B_{\parallel} = 0$)



zonal drive



zonal shearing

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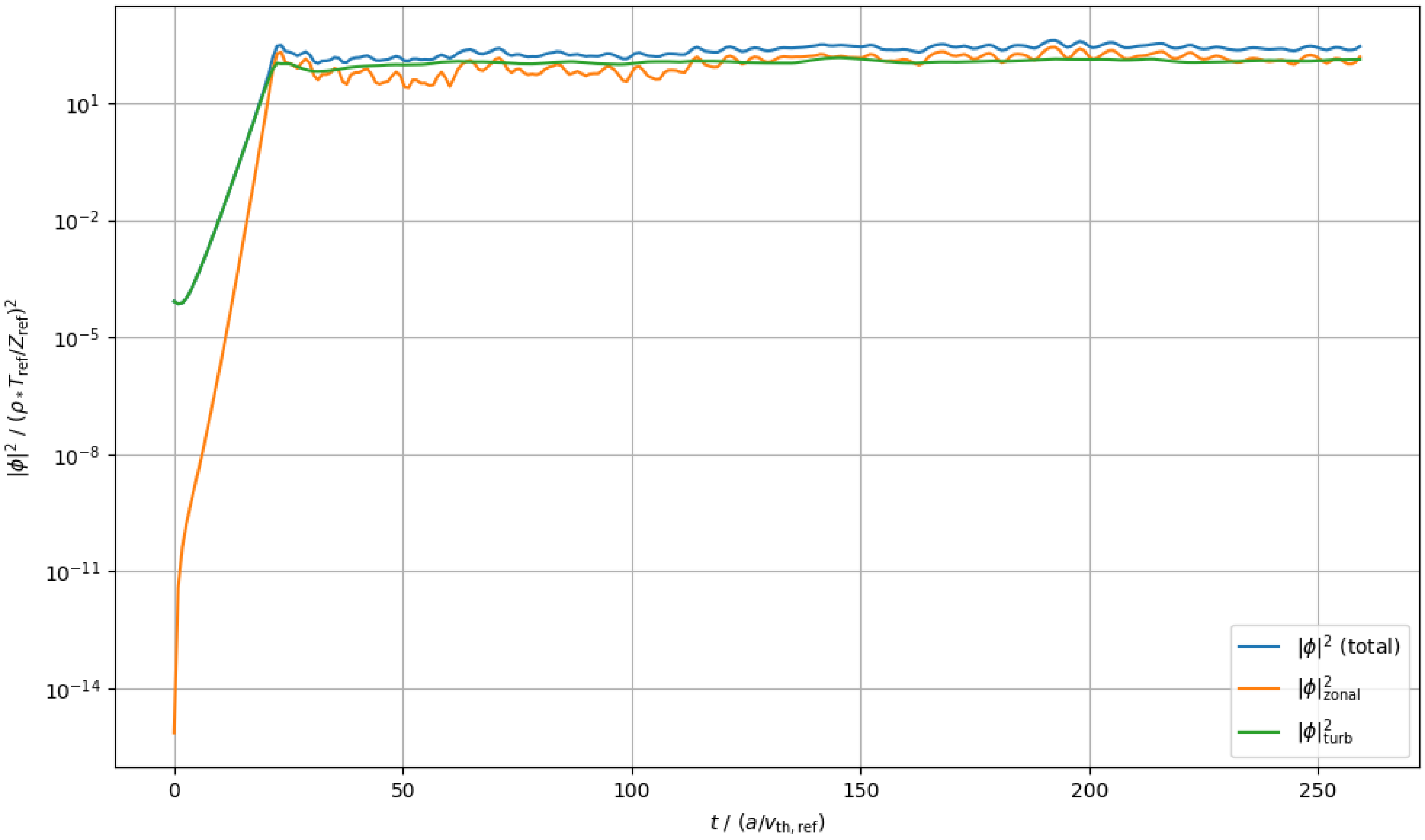
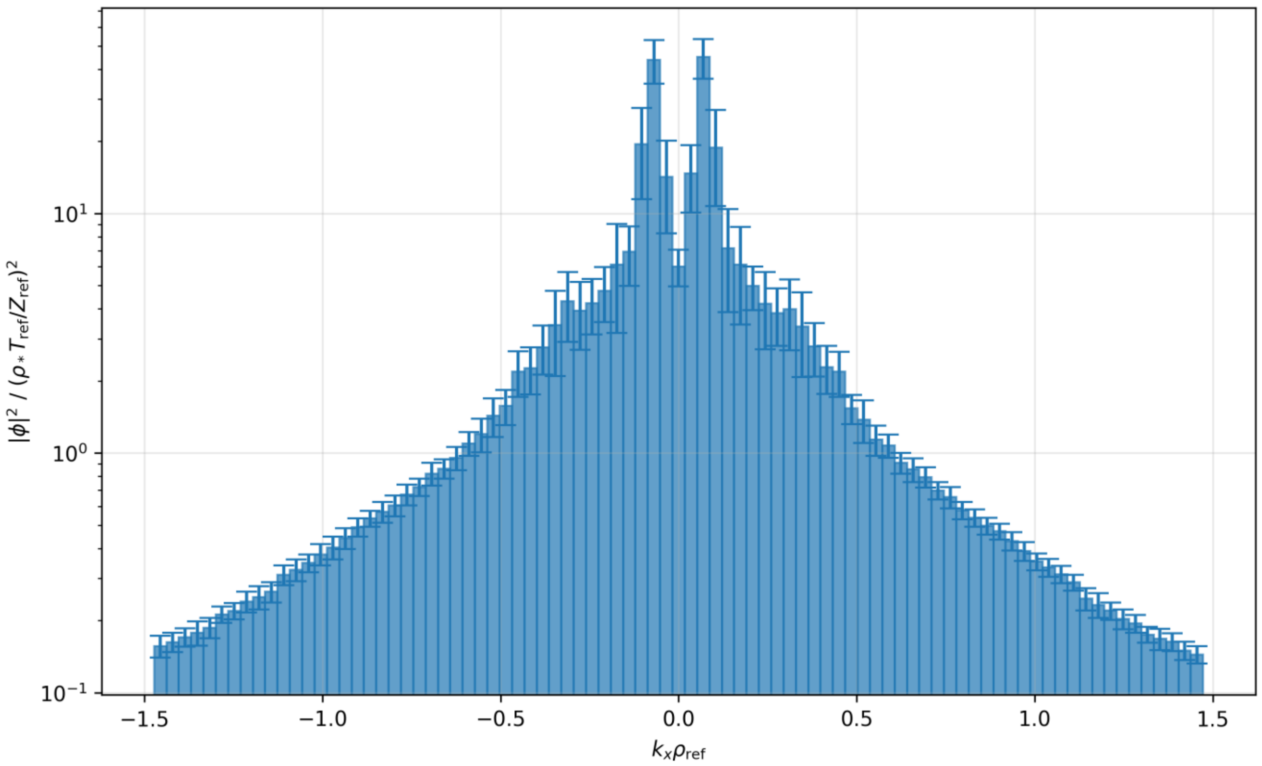
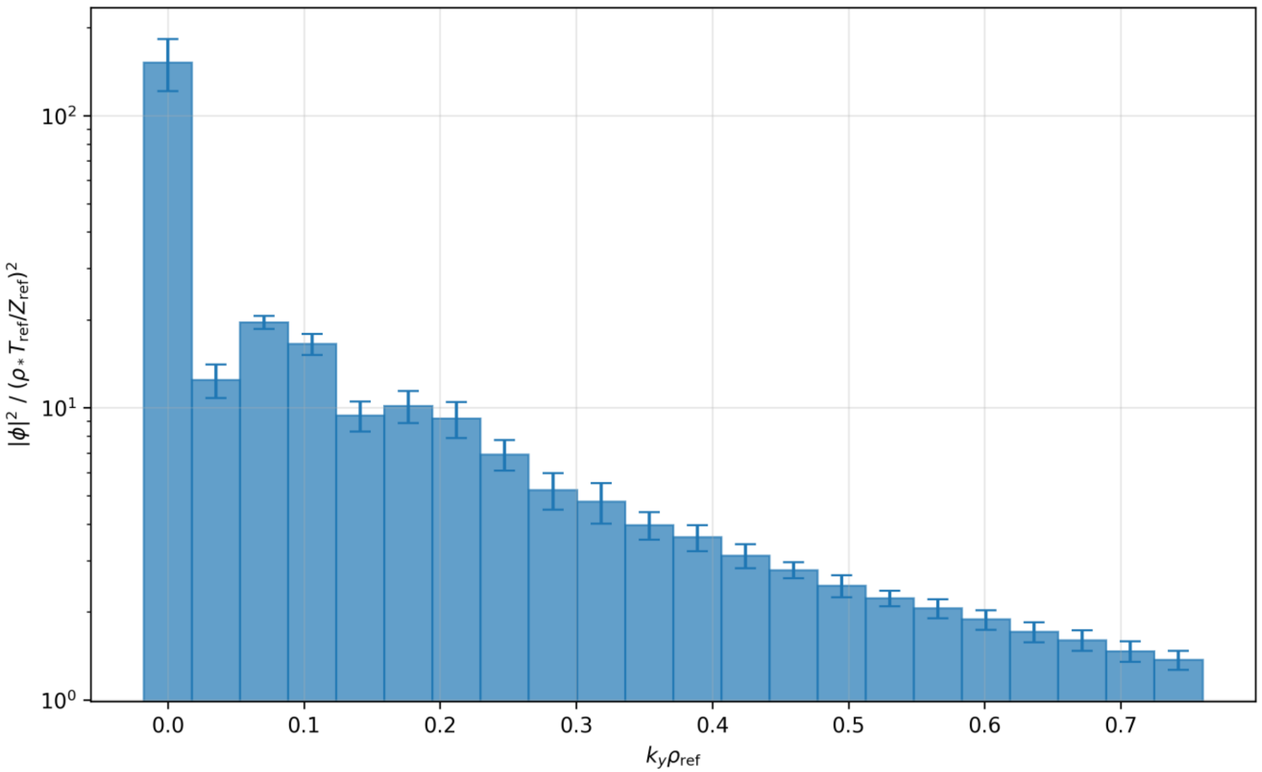
Plots for GX data used in GS2 comparison (1/4)

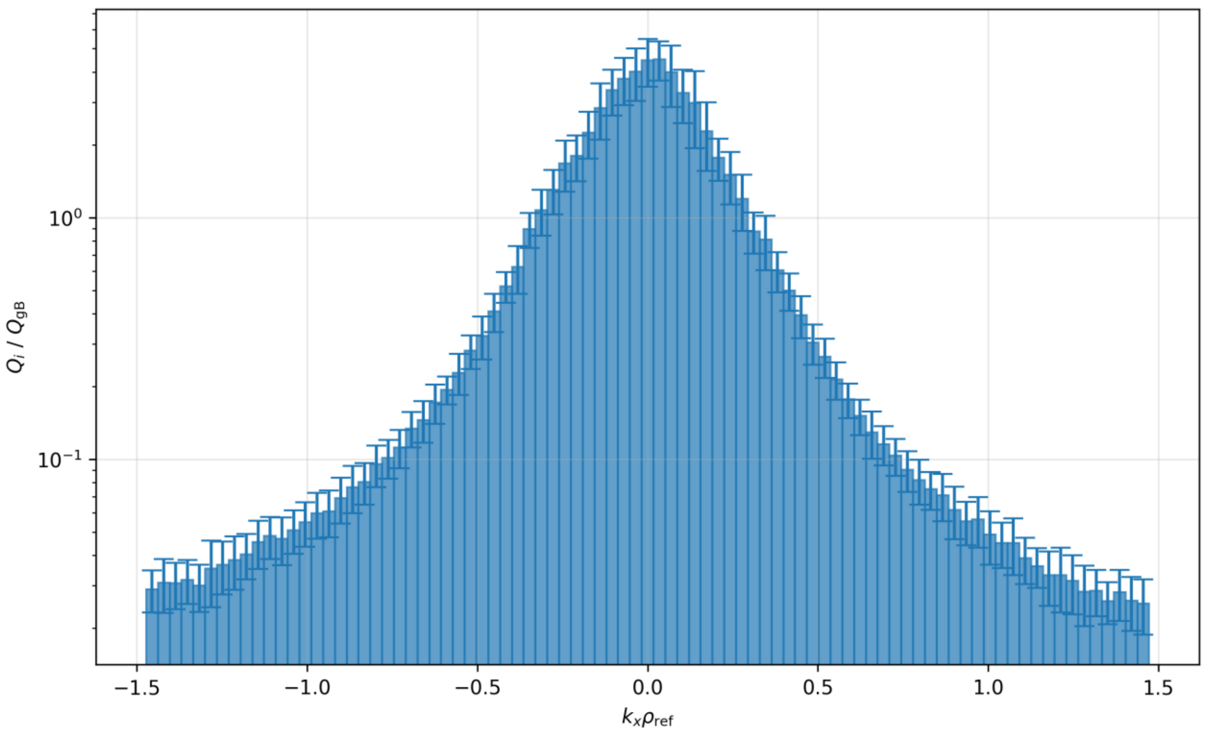


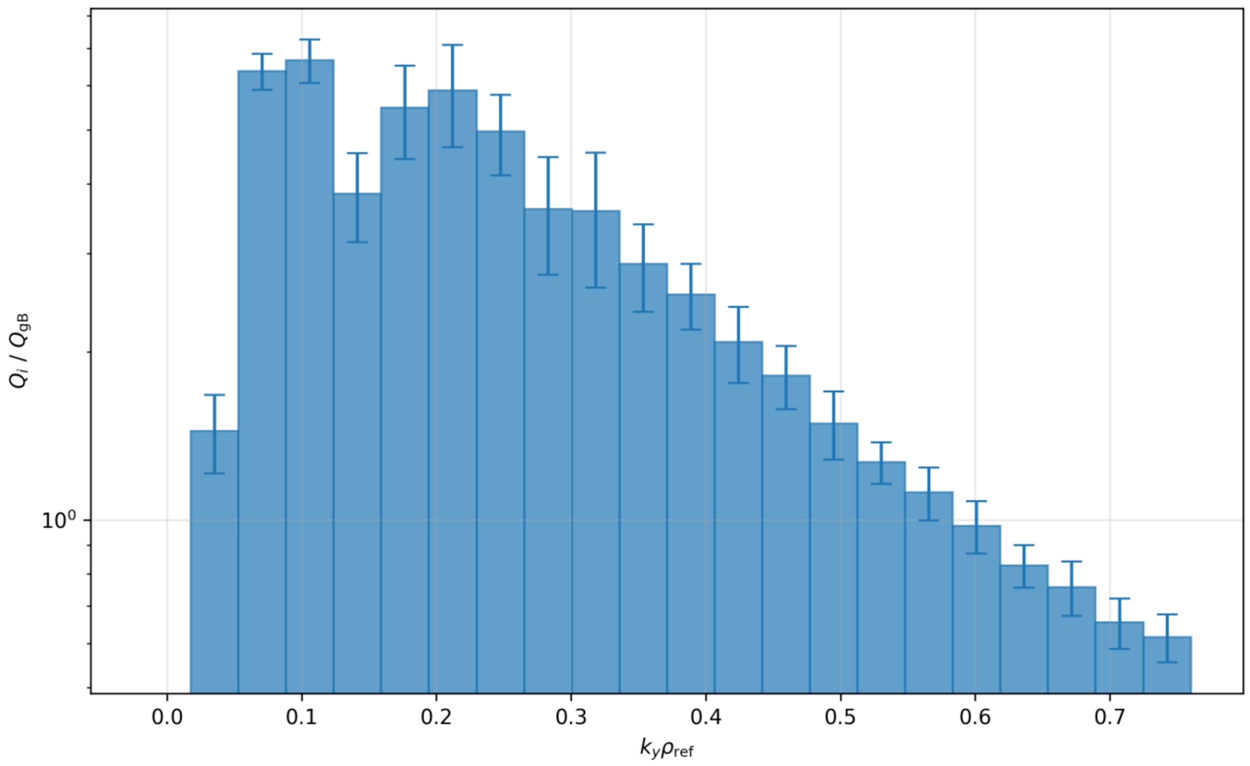
Plots for GX data used in GS2 comparison (2/4)





Plots for GX data used in GS2 comparison (3/4)





Plots for GX data used in GS2 comparison (4/4)