Model-based Optimal Feedforward Current Profile Control in NSTX-U

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Philosophy for Control of Plasma Profile Dynamics

- For ITER to achieve scientific objectives, it **needs active, model-based, closed-loop control of magnetic and kinetic profile evolutions**.
  - Maintain plasma in high-performance, MHD-stable, (steady) state.
- **Model-based control** is motivated by multivariable, coupled, nonlinear, distributed dynamics of plasma.
- Closed-loop **(feedforward + feedback)** model-based algorithm is proposed to control plasma profile dynamics.
  - **Feedforward:** Computed off-line and designed to achieve desired plasma state. It can handle arbitrary model complexity (accuracy-speed tradeoff).
  - **Feedback:** Computed on-line and designed to add robustness to control scheme (reject variability in wall conditions, impurities, drifts due external disturbances and IC perturbations, and account for uncertainties in model used for control synthesis). It is critical to ensure repeatability of scenarios.
- Controllers derived by embedding dynamic model into design process:
  - **Know in which direction to actuate to generate desired response.**
  - **Can be designed to share available actuation capabilities.**
  - **Eliminate trial-and-error tuning of control scheme, which will be impractical for use on ITER where discharges are at a premium.**

Uses of First-principles-driven Control-oriented Models

Control Design = Control Synthesis + Control Simulation

- Control Simulation: Run CL simulation “fast" $\rightarrow$ effective iterative design.
- Control Synthesis: Complexity of models needs to be further reduced.
  - feedforward controller, feedback controller, state observer.

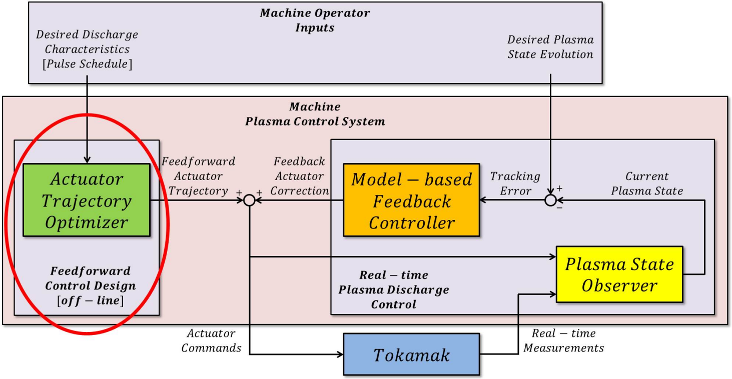

Fig. 1: Overal plasma profile control architecture.

Feedforward Control: Actuator Trajectory Optimization

- **Objective:** Reach **target plasma scenario** at some time t_f by designing **actuator waveforms** subject to **plasma dynamics** and **constraints**.
  - Defines nonlinear, constrained optimization problem.
- **Target scenario:** Defined in terms of q profile (T_e, n_e profiles (or β_N/W)). Proximity of achieved state to target formulated into to-be-minimized cost functional J. A measure of steadiness can be incorporated in J.
- **Plasma dynamics:** Governed by first-principles-driven (FPD), physics-based, control-oriented, transport model.
- **Constraints:** Actuators (magnitude and rate) and state (e.g., q_{min}).
- **Solution method:** Sequential quadratic programming (SQP)
  - Embed control-oriented transport model + constraints in nonlinear optimization algorithm.
  - Solve quadratic programming problem at each iteration of algorithm to update solution of original nonlinear programming problem.

SQP Algorithm to Solve Optimization Problem

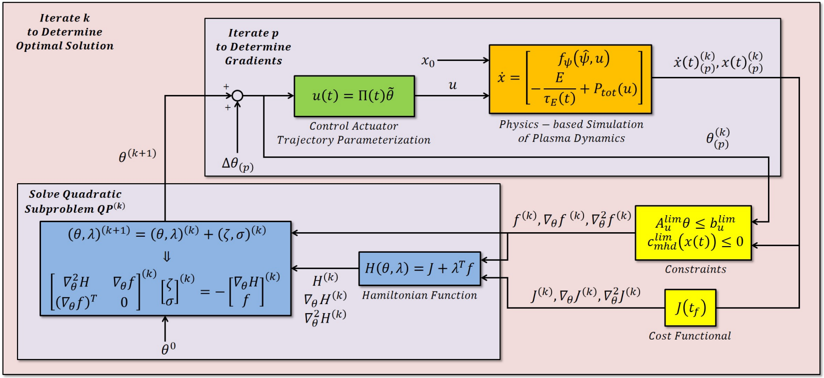

Fig. 2: Schematic of sequential quadratic programming (SQP) algorithm to solve actuator trajectory optimization problem [1, 2].

[1] C. Xu, Y. Ou, J. Dalessio, E. Schuster, T.C. Luce, J.R. Ferron, M.L. Walker and D.A. Humphreys, IEEE Trans. Plasma Sci., vol. 38, no 2, pp. 163-173, 2010.

[2] J.E. Barton, W. Shi, E. Schuster, T.C. Luce, J.R. Ferron, M.L. Walker, et. al., in 19th IFAC World Congress, pp. 671-676, August 24-29, 2014.

First-principles-driven Control-oriented Transport Model

- Poloidal magnetic flux dynamics given by Magnetic Diffusion Equation:

$$\frac{\partial \psi}{\partial t} = \frac{\eta(T_e)}{\mu_0 \rho_b^2 \hat{F}^2} \frac{1}{\hat{\rho}} \frac{\partial}{\partial \hat{\rho}} \left(\hat{\rho} \hat{F} \hat{G} \hat{H} \frac{\partial \psi}{\partial \hat{\rho}} \right) + R_0 \hat{H} \eta(T_e) \left[j_{aux} + j_{bs}(n_e, T_e) \right], \qquad (1)$$

  with boundary conditions $\frac{\partial \psi}{\partial \hat{\rho}}(0,t) = 0$, $\frac{\partial \psi}{\partial \hat{\rho}}(1,t) = -\frac{\mu_0}{2\pi} \frac{R_0}{\hat{G}(1)\hat{H}(1)} I_p$.

- Electron temperature dynamics give by Heat Transport Equation:

$$\frac{3}{2} \frac{\partial}{\partial t} [n_e T_e] = \frac{1}{\rho_b^2 \hat{H}} \frac{1}{\hat{\rho}} \frac{\partial}{\partial \hat{\rho}} \left[\hat{\rho} \frac{\hat{G} \hat{H}^2}{\hat{F}} \left(\chi_e(\psi, n_e, T_e) n_e \frac{\partial T_e}{\partial \hat{\rho}} \right) \right] + Q_e^{\bar{aux}} + Q_e^{aux} \qquad (2)$$

  with boundary conditions $\frac{\partial T_e}{\partial \hat{\rho}}(0,t) = 0$, $T_e(1,t) = T_{e,bdry}$. A singular perturbation approximation is possible by exploiting time scale difference between energy confinement and resistive diffusion ($\tau_E << \tau_R$).

- **Actuators for q profile control:** Auxiliary heating ($Q_e \propto P_{aux}$) and current-drive ($j_{aux} \propto P_{aux}$), total plasma current (I_p), electron density $n_e(\hat{\rho}, t) = n_e^{prof}(\hat{\rho}) \bar{n}_e(t)$, where P_{aux} denotes the power injected through auxiliary sources and $Q_e^{\bar{aux}} = Q_e^{ohm} + Q_e^{rad} + Q_e^{fus}$.

Goals of the Experiment and its Significance

- Goal is to implement **optimal feedforward control laws** developed for the regulation of the q-profile evolution with the **ultimate goal of achieving specific plasma scenarios.** Different targets will be used.
  - Accomplished by embedding model in nonlinear optimization algorithm.
- **Developed control-oriented transport model [3]** needed for actuator trajectory optimization will be refined by using NSTX-U data from previous experiment (Current Profile Controllability Scoping Study (Boyer)).
  - Core of control-oriented transport models are **fundamental physical laws.**
    - Magnetic diffusion equation, heat transport equation.
  - Goal is to convert physics models into form **suitable for control design.**
    - Extract relevant physics in equation parameters to develop control-level model.
- **Control feasibility evaluation** through use of **feedforward** trajectories + **control-oriented transport model validation** $\rightarrow$ key prerequisites for the next step: **feedback** controller for q-profile regulation around target.
- Assessment of potential of model-based optimal feedforward profile control as a **systematic approach for scenario planning** in NSTX-U.

[3] Z. Ilhan, J. Barton, W. Shi, E. Schuster, D. Gates, S. Gerhardt, E. Kolemen and J. Menard, “Physics-based Control-oriented Modeling of the Current Profile Evolution in NSTX-Upgrade," 55th Division of Plasma Physics (DPP) Annual Meeting of the American Physical Society (APS), Denver, Colorado, USA, Nov 11-15, 2013.