

Physics of fast flux closure in coaxial helicity injection experiments in NSTX

Fatima Ebrahimi¹

Thanks to: R. Raman⁴, B. Hooper²,
C. Sovinec³, A. Bhattacharjee¹, and R. Kulsrud¹

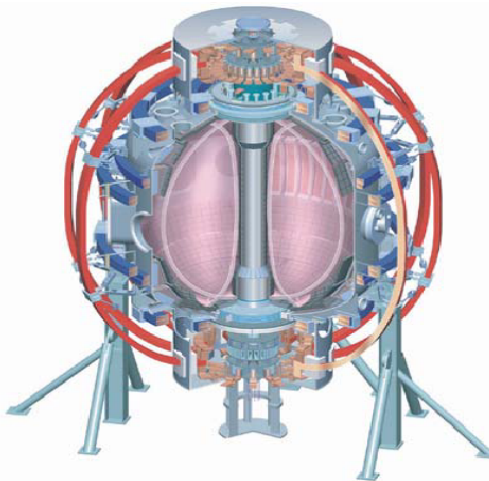
¹Princeton University, ²Lawrence Livermore National Laboratory, ³University of Wisconsin-Madison, ⁴University of Washington

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Motivation

- Conventional tokamak design rely on a central solenoid to generate an inductive initial current.
- Solenoid free non-inductive techniques to form a startup plasma enable lower aspect ratio configurations and long pulse or steady-state current-drive for future facilities.
- Helicity injection is one way to sustain the plasma non-inductively.



- Coaxial Helicity Injection (CHI) is a promising candidate for plasma start-up current formation in NSTX.

Low aspect ratio tokamaks are formed and sustained through electrostatic helicity injection

- Magnetic helicity, $K = \int \mathbf{A} \cdot \mathbf{B} d\mathbf{v} - \phi_z \phi_p$
- The rate of change of magnetic helicity is

$$\frac{\partial K}{\partial t} = \underbrace{2\phi_z v_z}_{\text{ac/inductive injection}} - \underbrace{2 \int \Phi \mathbf{B} \cdot d\mathbf{s}}_{\text{dc injection}} - \underbrace{2 \int \eta \mathbf{J} \cdot \mathbf{B} d\mathbf{v}}_{\text{dissipation}}$$



ac/inductive injection



dc injection



dissipation

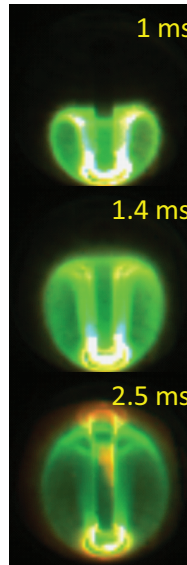
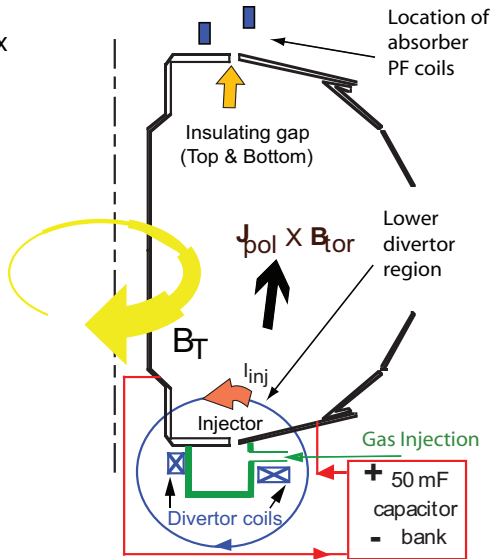
- Φ is the applied electrostatic potential on the plasma surface, second term represents the intersection of a field line with a surface held at a constant electric potential, when $B_N(a) \neq 0$
- A bias voltage V_{inj} , should be applied to the magnetic flux ψ_{inj} that penetrates two electrodes, $\dot{K} = 2\psi_{\text{inj}} V_{\text{inj}}$

Jensen and Chu 1984

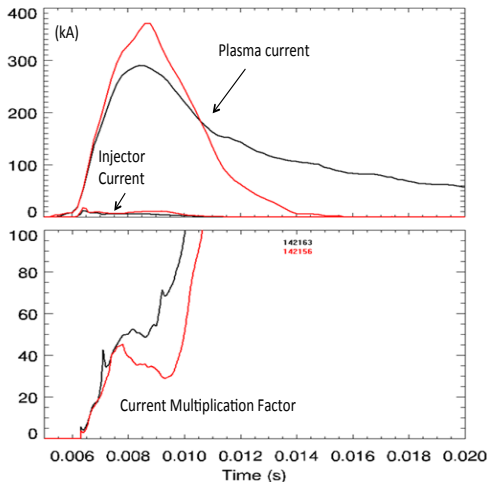


In transient CHI, axisymmetric reconnection generates a high quality closed flux start-up equilibrium in NSTX

- Injected toroidal flux links the poloidal flux
flux $\dot{K} = 2\psi_{inj} V_{inj}$
- The helicity and plasma expands into the vessel:
- if I_{inj} exceeds a threshold value.
- when the magnetic compression across the current layer exceeds the field-line tension of the injector flux;
 $\Delta B_{tor}^2/2 > B \cdot \nabla B$



NSTX has demonstrated CHI-generated plasma current up to 300kA



Injector current



- $I_p = I_{inj}(\Psi_T / \Psi_{inj})$



Toroidal flux
[T. R. Jarboe (1989)]

- Current multiplication increases with toroidal field. Scaling confirmed with TSC simulations [Raman, Jardin, et al. 2011]

Raman et al. PRL 2006

Helicity injection simulations are performed using the extended-MHD NIMROD code

Resistive MHD simulations have been successful in the study of helicity injection in fusion plasmas. Simulations to provide:

- understanding of the physics of injection, flux-surface closure, and current drive for CHI plasmas
- guidance for existing experiments and extend these results to NSTX-U
- insight to viability of CHI for steady-state operation of ST reactor
- validation exercises

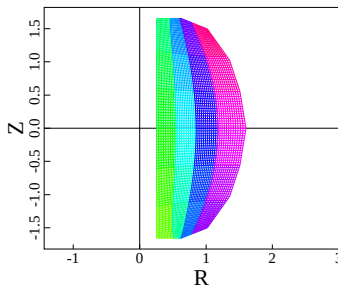
outline

- Computational model
- Simulations results
- Physics of magnetic reconnection and flux closure

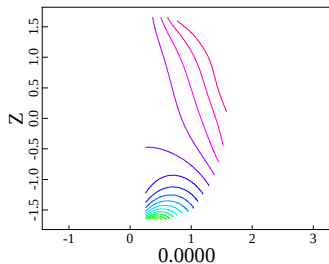
The computational model

- Similar geometry to the experiment
- Fixed boundary fields (including NSTX poloidal coil currents) for a narrow slot of 4cm as in the experiment
- Axisymmetric ($n=0$) with poloidal grid 45×90 fourth or fifth order finite elements
- Voltage is applied across the injector gap (V_{inj}), and no current is allowed on the absorber gap by setting $\Delta B_\phi = 0$ [Bayliss&Sovinec 2010 and Hooper 2011)
- $E \times B$ normal flows at the injector and absorber gaps

Finite Element Mesh



Poloidal flux



0.0000

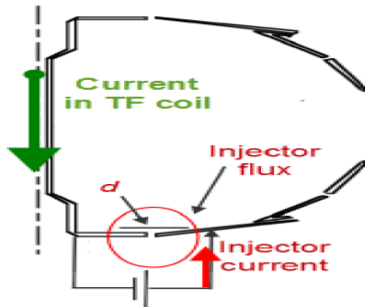
What are the minimum conditions for flux surface to close?

We start with zero pressure model to allow fuller and more detailed determination of flux closure physics

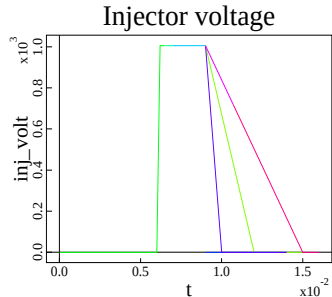
- **The effect of magnetic diffusivity?**

*F. Ebrahimi, Hooper, Sovinec and Raman, PoP letter
October 2013*

The effect of magnetic footprint?
Injector flux?

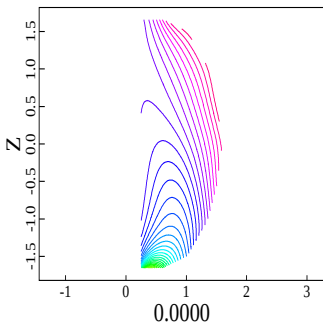


The effect of injector voltage drop?

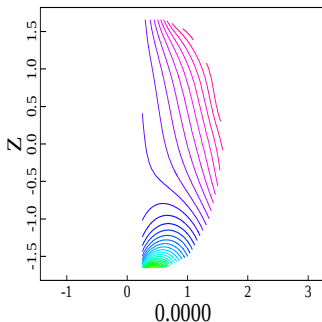


No closed flux surfaces are obtained at high magnetic diffusivity

Poloidal flux



Poloidal flux



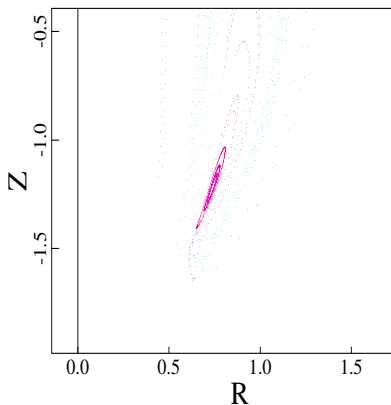
Poloidal flux surfaces (a) $t=9.1\text{ms}$ (b) $t=9.8\text{ms}$, during the sharp decay of the voltage

- $\eta/\mu_0 = 400\text{m}^2/\text{s}$, $T_e \approx 1\text{eV}$.
- Diffusion time is too fast, $\tau_R \approx 1\mu\text{s}$. No closed flux surfaces are obtained.

Magnetic fluxes are only reconnected at low magnetic diffusivity

Poloidal flux evolution

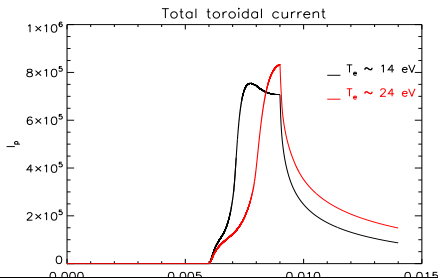
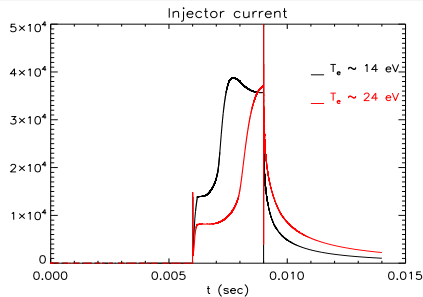
X point formation
 $t=9.015\text{ms}$
Surface of Section



(Loading poloidalflux.mp4)

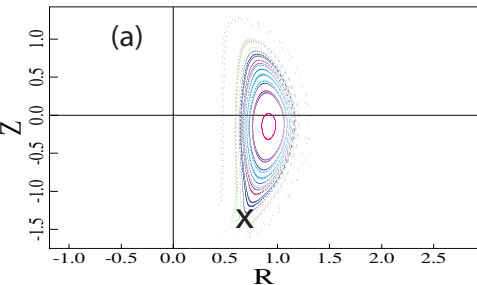
Simulations with magnetic diffusivities similar to those in the experiment produce flux closure

(a)



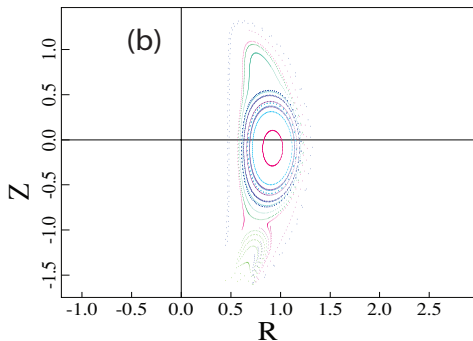
- $\eta/\mu_0 = 8 \text{ m}^2/\text{s}$, ($T_e \approx 14 \text{ eV}$),
 $I_{\text{tor}} = 0.74 \text{ MA}$, $I_{\text{inj}} = 37 \text{ kA}$
Toroidal current in the closed flux surfaces is about 11% of the total plasma current
- $\eta/\mu_0 = 3.5 \text{ m}^2/\text{s}$,
($T_e \approx 24 \text{ eV}$), $I_{\text{tor}} = 0.84 \text{ MA}$,
 $I_{\text{inj}} = 36 \text{ kA}$ Toroidal current in the closed flux surfaces is about 15% of the total plasma current

Surface of Section



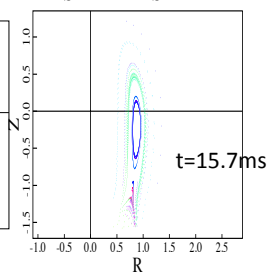
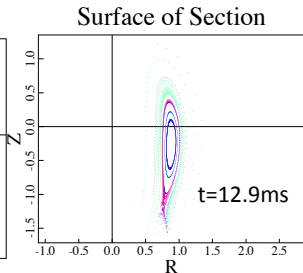
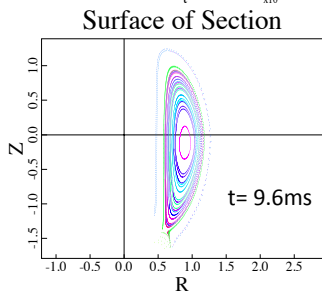
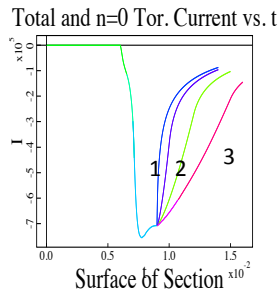
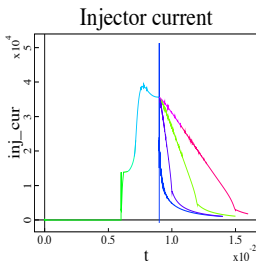
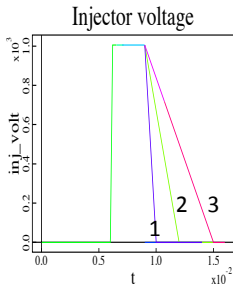
$\Leftarrow (T_e \approx 14\text{eV})$

- The volume of closed flux surfaces increases as the magnetic diffusivity is reduced

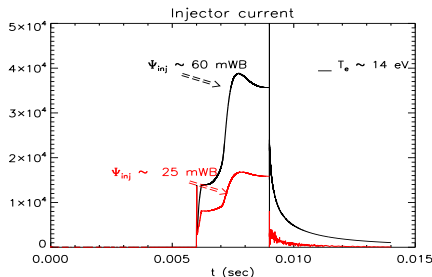
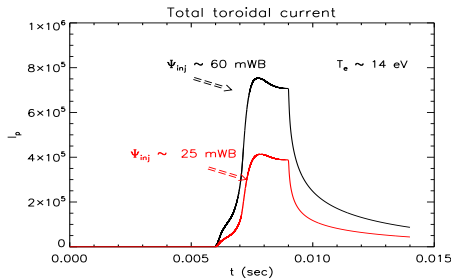


$\Leftarrow (T_e \approx 24\text{eV})$

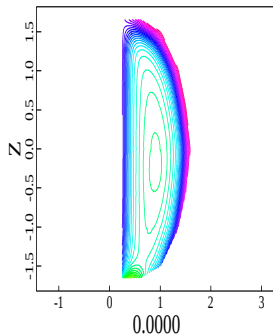
Largest volume of the closed flux surfaces is reduced when the injector voltage does not drop to zero fast enough



I_p increases with more poloidal flux injection

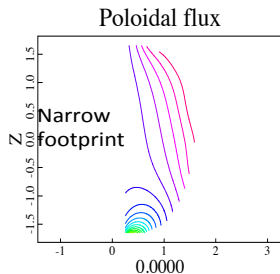


Poloidal flux

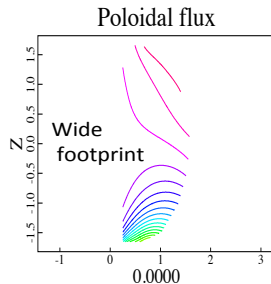


- Coil currents are reduced to give $\Psi_{inj} \approx 25 \text{ mWb}$ and $I_{tor} = 0.34 \text{ MA}$, $I_{inj} = 11 \text{ kA}$
- Plasma current scales with Ψ_{inj}

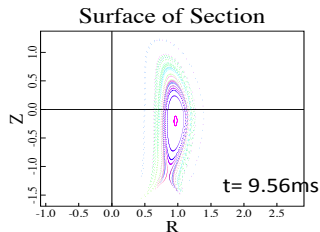
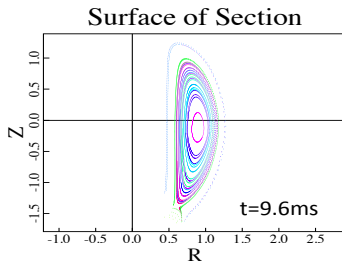
The volume of closed flux surfaces decreases as the injector flux footprints become wide



$I_{\text{closed}}/I_p = 11\%$



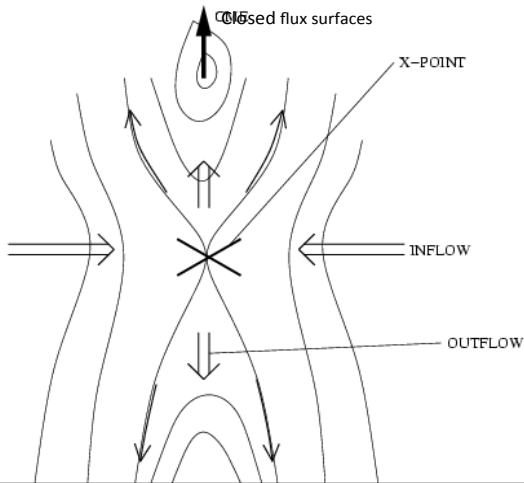
$I_{\text{closed}}/I_p = 4\%$



Physics of X point formation and magnetic reconnection

Are there any signatures of Sweet-Parker reconnection in transient CHI?

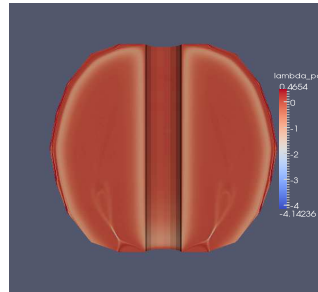
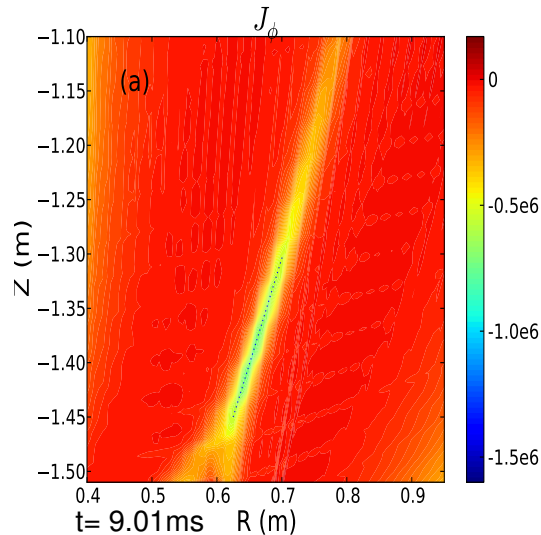
The formation of the X point can be explained through the Lorentz forces and flows at the reconnection site.



- Elongated current sheets?

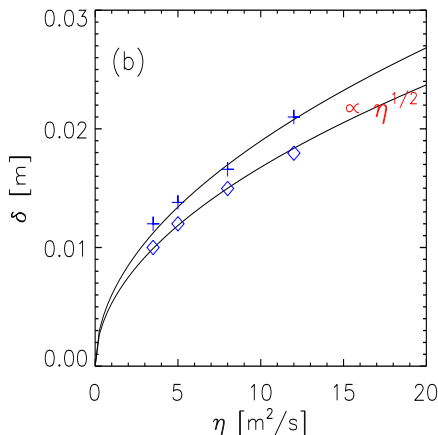
- $\delta/L \sim S^{-1/2}?$
 $\sim V_{in}/V_{out}$
($S = \mu_0 L V_A / \eta$)

An elongated current sheet is formed during the X point formation



- A coordinate transformation to coordinates aligned with the current sheet (shown with dotted black line)
 $t = 9.03 \text{ ms}$

The width of current sheet scales with magnetic diffusivity close to $\eta^{1/2}$



- Simulations for several η
- For reconnection $\tau_{rec} < \tau_R$
- It is confirmed that the reconnecting field (B_z) fits well with a 1-D Harris sheet type profile [$\propto \tanh(R/\delta)$]
- Diamonds and crosses obtained using a fit to a Harris type profile and half width of the vertically averaged profile

$\eta [m^2/s]$	τ_{rec}	$\tau_R = \delta^2/\eta$	τ_A	$\sqrt{(\tau_A \tau_R)}$	$S_{rec} = LV_A/\eta$
$8 \approx 14eV$	$\leq 0.01ms$	$\geq 0.035ms$	0.006ms	0.014ms	~ 7000
$400 \approx 1eV$	no rec	0.001ms	—	—	—

What triggers magnetic reconnection?

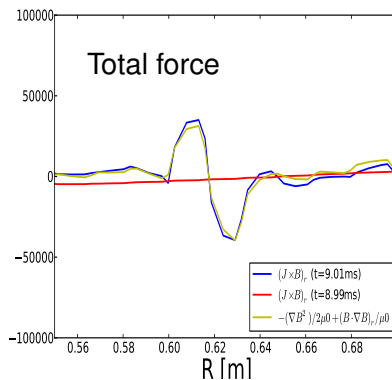
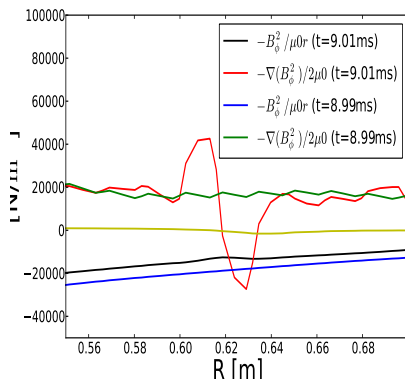
- Total fields consists of a fixed background field and evolving axisymmetric fields ($\mathbf{B} = \mathbf{B}_0 + \tilde{\mathbf{B}}$) during helicity injection.

$$(\mathbf{J} \times \mathbf{B})_r = -\nabla(B_r^2 + B_z^2 + B_\phi^2)/2\mu_0 + (\mathbf{B} \cdot \nabla \mathbf{B})|_r/\mu_0$$

$$\rightarrow \underbrace{-\nabla(B_z^2 + B_\phi^2)/2\mu_0}_{\text{magnetic compression}} - \underbrace{B_\phi^2/r\mu_0 + B_z dB_r/dz/\mu_0}_{\text{magnetic curvature}}$$

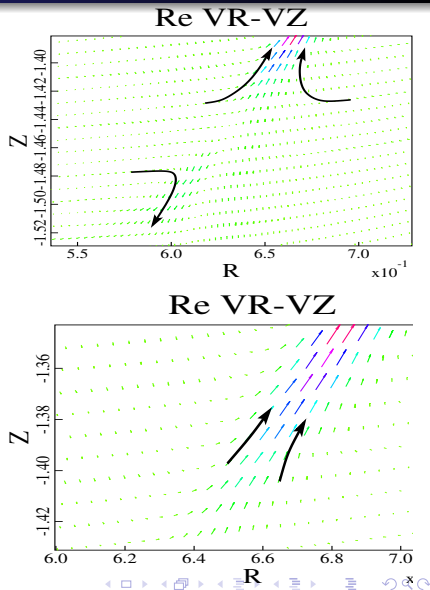
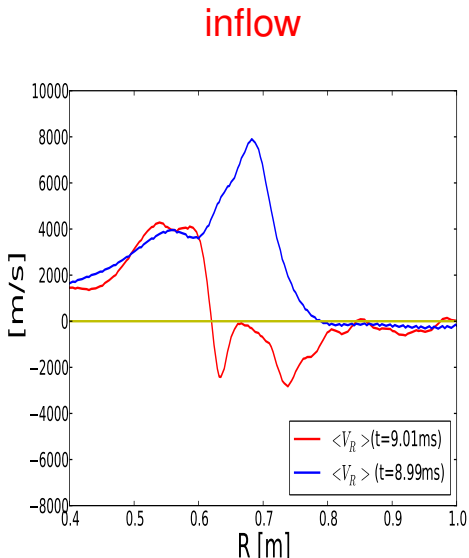
The magnetic compression exerts a bidirectional pinch force that brings the oppositely directed fields together to reconnect

The system is almost in equilibrium before reconnection, and when voltage is turned off and B_ϕ drops around the injector region, magnetic compression exerts a radial force and system is imbalanced

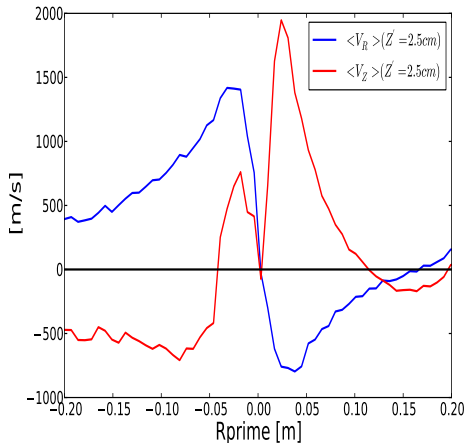


Terms around the injector region ($Z=-1.4\text{m}$)

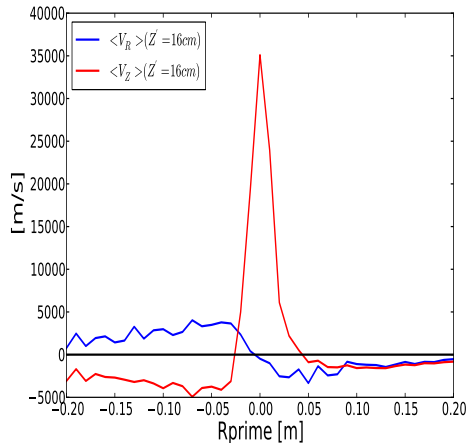
Consistent with the bidirectional pinch force, as the injector voltage is turned off, a radial pinch ExB flow is generated



The Sweet-Parker type structure of inflow and outflow are clearly seen in the coordinate along the elongated current sheet

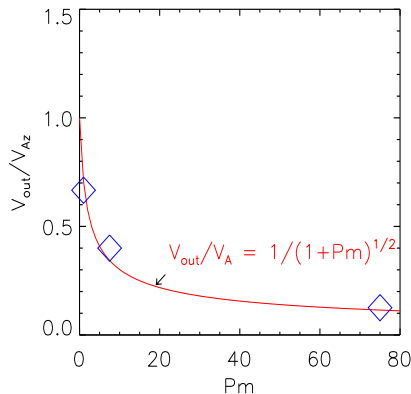


Poloidal flow is zero around X point at ($R = 0.63 \text{ m}$, $Z = -1.445 \text{ m}$)



Maximum outflow along the current sheet at $\text{Pm} = 1$

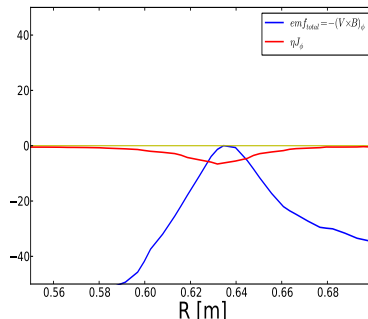
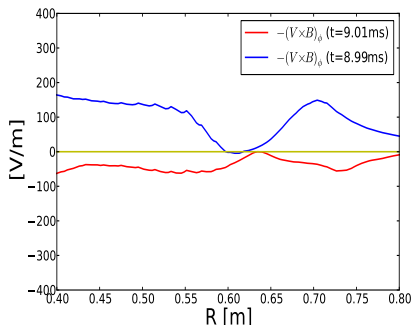
The outflows approach poloidal Alfvén velocities at small magnetic Prandtl numbers



- In Sweet-Parker
 $V_{out} \approx V_A = B_{in}/\sqrt{(\mu_0\rho)}$
- Viscosity modifies the outflows
 $V_{out}/V_A = 1/\sqrt{(1+Pm)}$
- $B_{in} = B_z \approx 0.0118T$ (The reconnecting field)
 $V_{Az} \approx 90km/s$
- The outflows can reach 60km/s ($Pm=1$)

The evolution of poloidal flux that leads to reconnection is caused by an induced toroidal electric field (loop voltage)

$$E_\phi = -(V \times B)_\phi + \eta \mu_0 J_\phi$$



The change of sign of induced toroidal electric field (loop voltage) shows the poloidal flux evolution but not sufficient for the reconnection.

Sweet-Parker signature:

$E_{emf}[out] \approx V_{in} B_0 \approx V_r B_z$ outside reconnection layer ($r \approx 0.63$) and inside $E_{emf}[in] \approx \eta J$

$$\frac{n}{(\Gamma - 1)} \left(\frac{\partial T}{\partial t} + \mathbf{V} \cdot \nabla T \right) = -p \nabla \cdot \mathbf{V} - \nabla \cdot \mathbf{q} + Q$$

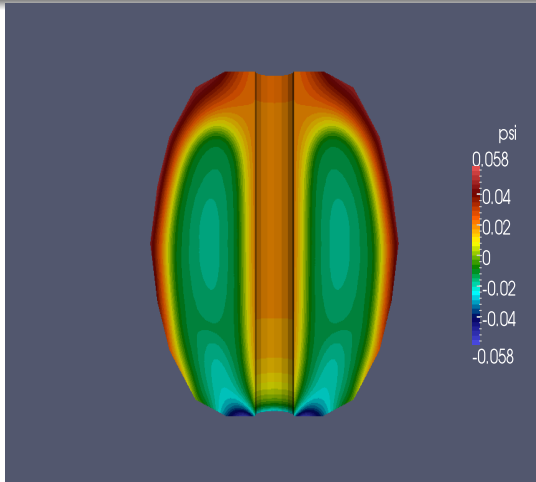
$$p = nkT, \mathbf{q} = -n[(\kappa_{\parallel} - \kappa_{\perp})\hat{b}\hat{b} + \kappa_{\perp}\mathbf{I}] \cdot \nabla T, Q = \eta J^2.$$

- Finite pressure simulations use anisotropic thermal conductions, $\kappa_{\parallel} \propto T_e^{5/2}$ and $\kappa_{\perp} = 2.5m^2/s$, ohmic heating and Spitzer resistivity ($\eta \propto T_e^{-3/2}$), with *time dependent boundary poloidal fields* reported in [E. B. Hooper, C. R. Sovinec, R. Raman, F. Ebrahimi, J. Menard, PoP 2013]

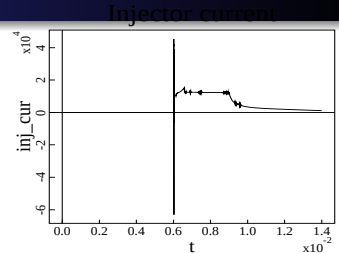
Here:

- Simulations at finite pressure with *fixed boundary fields*

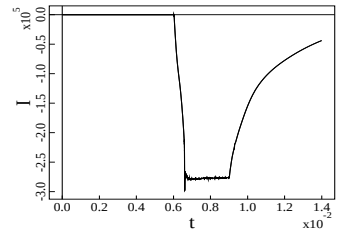
Simulations with temperature evolution also show formation of a fairly large volume of closed flux surfaces on the experimental time scales



Poloidal flux surfaces from simulation with pressure model during the flux closure at $t=9.72\text{ms}$.

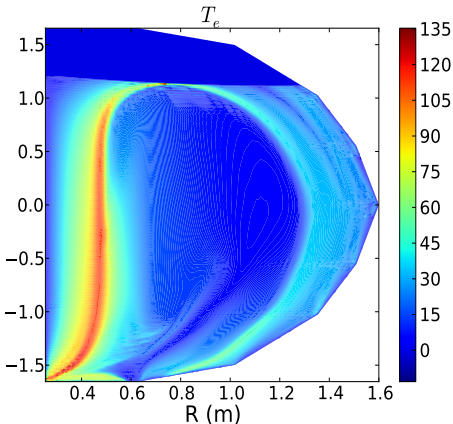


Total and $n=0$ Tor. Current vs. t

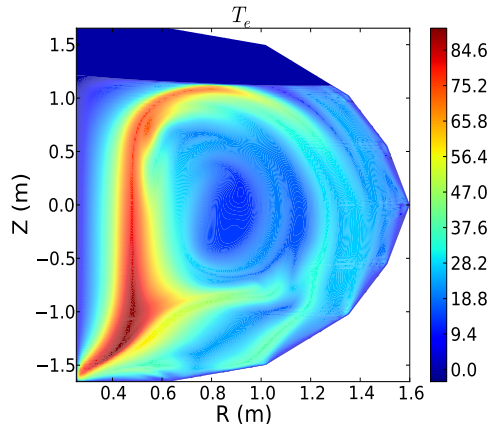


Core plasma temperature (in the closed flux region) does not exceed 30eV

$t=9.08\text{ms}$

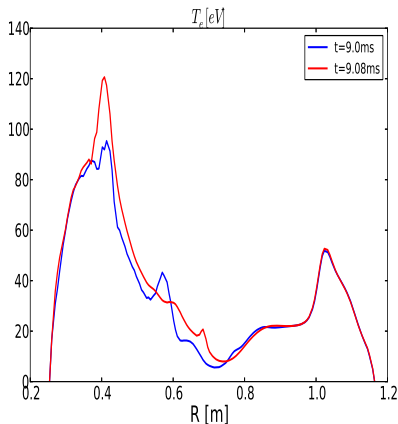


$t=9.87\text{ms}$

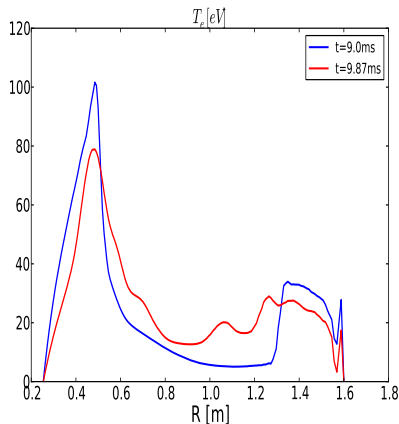


As plasma is heated by ohmic heating, due to temperature-dependent resistivity, at hot (and less resistive) plasma locations, current can easily flow and causes the flux to close

Temperature is equilibrated after the X point formation and reaches about 20eV at the X point



Temperature profiles around the reconnection site ($Z = -1.3$ m), before and after X point formation.



The temperature is equilibrated further in the closed flux surface region as the closure occurs ($Z = 0$)

Summary

- Simulations show that an X point is formed in the injector region, followed by formation of closed flux surfaces within 0.5 ms after the driven injector voltage and injector current begin to rapidly decrease.
- Through direct numerical calculations of the radial forces, we found that as the injector voltage is turned off, a radial bi-directional pinch flow causes the field lines to reconnect.
- Closed flux surfaces during transient CHI can be explained through 2-D Sweet-Parker type reconnection.
- Numerical simulations hint that transient CHI experiments in NSTX may be the first to demonstrate a forced magnetic reconnection process in a large scale fusion device.

Ongoing work:

Could non MHD effects (2-fluid, Hall terms) affect the volume of the closed flux surfaces?

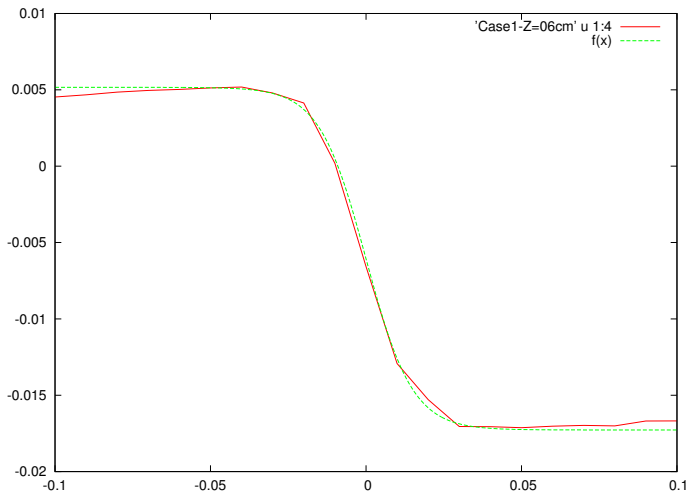
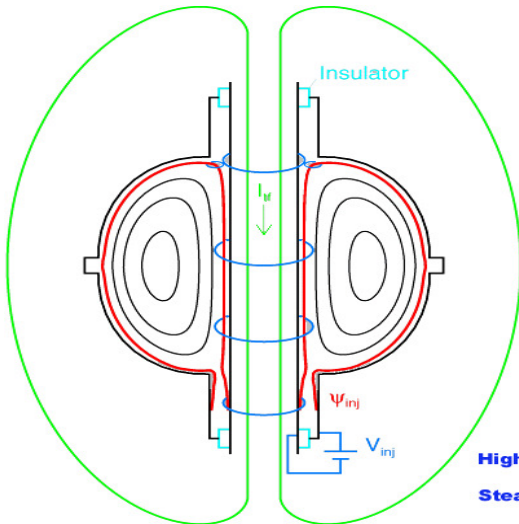


Figure : The profile of evolving reconnecting component, B'_z vs R' for $Z' = 6\text{cm}$ (R' , Z' are the new sheet-aligned coordinates) shown in red, and a fit of $f(x) = a \tanh(x/\delta) + b$ shown in green.

Low aspect ratio tokamaks are formed and sustained through electrostatic helicity injection

NSTX - HIT-II



dc helicity injection has been tested on **CDX**, **HIT** tokamaks, **CTX** spheromak and **NSTX**.

- ⇒ edge current drive
- Current relaxation by fluctuations (driven CHI)
- ⇒ Non-inductive start up (transient CHI)

$$\dot{K} = 2\Psi_{inj} V_{inj}$$

T. R. Jarboe *et al.* (1998)

M. Ono *et al.* (1987)

**High Efficiency
and
Steady State**

Helicity injection simulations are performed using the extended-MHD NIMROD code

- Solves the linear and nonlinear MHD equations

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E} + \kappa_{divb} \nabla \nabla \cdot \mathbf{B}$$

$$\mathbf{E} = -\mathbf{V} \times \mathbf{B} + \eta \mathbf{J} + \frac{1}{ne} \mathbf{J} \times \mathbf{B}$$

$$\mathbf{J} = \nabla \times \mathbf{B}$$

$$\frac{\partial n}{\partial t} + \nabla \cdot (n \mathbf{V}) = \nabla \cdot D \nabla n$$

$$\rho \left(\frac{\partial \mathbf{V}}{\partial t} + \mathbf{V} \cdot \nabla \mathbf{V} \right) = \mathbf{J} \times \mathbf{B} - \nabla P - \nabla \cdot \Pi$$

$$\frac{n}{(\Gamma - 1)} \left(\frac{\partial T_\alpha}{\partial t} + \mathbf{V} \cdot \nabla T_\alpha \right) = -p_\alpha \nabla \cdot \mathbf{V} - \nabla \cdot \mathbf{q}_\alpha + Q$$

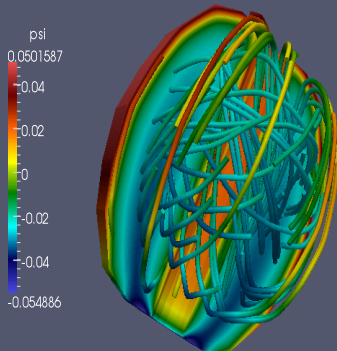
- $\mathbf{q} = -n[(\kappa_{||} - \kappa_{\perp})\hat{b}\hat{b} + \kappa_{\perp}\mathbf{I}] \cdot \nabla T$
- Π is the stress tensor (also includes numerical $\rho\nu\nabla V$)
- κ_{divb} and D are magnetic-divergence and density diffusivities for numerical purposes.

The computational model

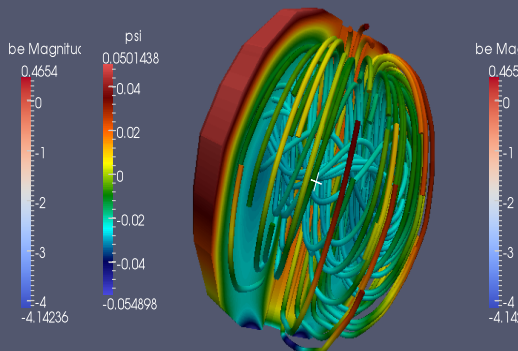
- Voltage is applied across the injector gap (V_{inj}), and no current is allowed on the absorber gap by setting $\Delta B_\phi = 0$. (CHI gap B.C. in HIT-II simulations by Bayliss&Sovinec POP 2010 and been generalized by Hooper)
- $E \times B$ normal flows at the injector and absorber gaps
- Perfectly conducting no-slip boundary conditions except in the gaps
- A layer of large resistivity and viscosity at the top and bottom used $(1 + (\sqrt{(dvac - 1)}|y/y_{max}|^{dexp})^2, dvac = 30, dexp = 20$
- Kinematic viscosity = $1 - 60m^2/s$, magnetic diffusivity = $3 - 400m^2/s$

Topology of the evolving field lines before and after flux closure

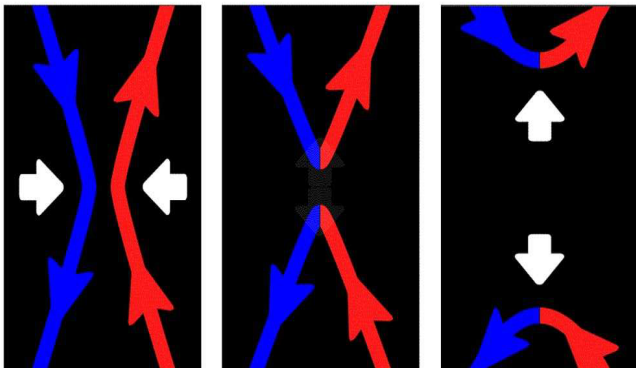
Before



After

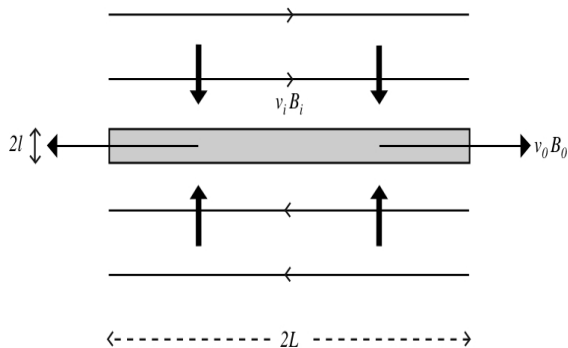


Magnetic reconnection is the rearrangement of magnetic field topology of plasmas



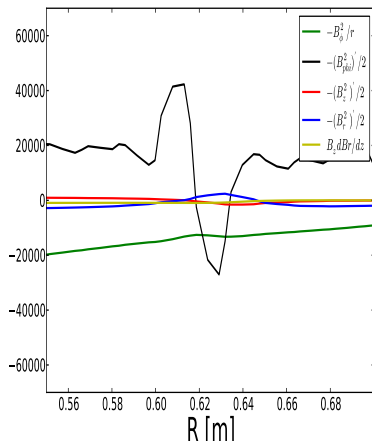
Credits: Center for Visual computing, Univ. of California Riverside

In the local Sweet-Parker model the plasma inflow is related to the outflow from the diffusion region

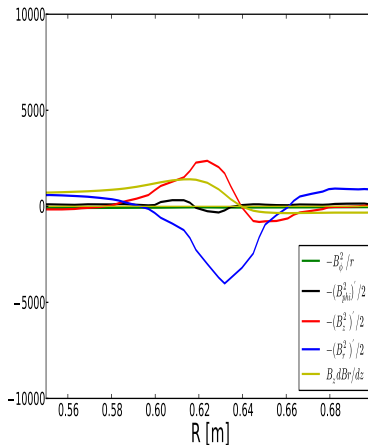


- mass conservation
 $V_{in}L \approx V_{out}\ell$
- energy conservation
 $V_{out} \approx V_A = B_{in}/\sqrt{(\mu_0\rho)}$
- matching ideal E outside the layer with the resistive E in the layer
 $V_{in}B_{in} \approx \eta J \rightarrow V_{in} \approx \eta/\mu_0\ell$
 (using $J \approx B_{in}/\mu_0 l$)
- $V_{in}/V_{out} = S^{-1/2} = \ell/L$
 ($S = \mu_0 L V_A / \eta$)

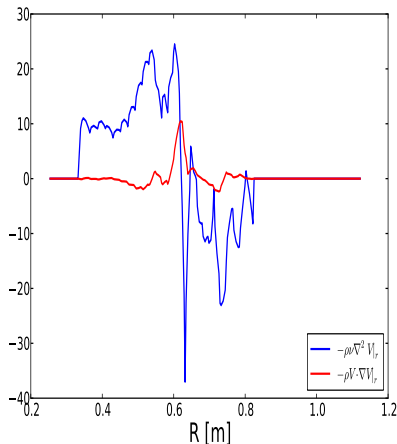
Linear terms including vacuum fields



Nonlinear terms with evolving fields



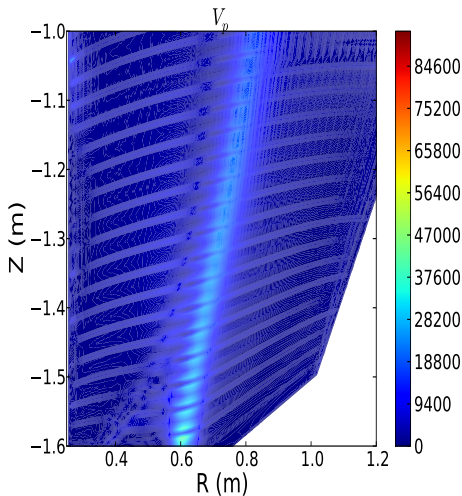
The only contributing force during reconnection is $\mathbf{J} \times \mathbf{B}$ force



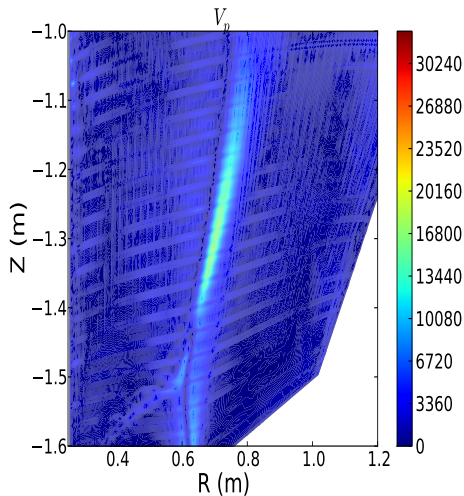
- Viscous $-\rho\nu\nabla^2 V|_r = -\rho\nu(1/r d/dr(rd/dr V_r) + d^2/dz^2 V_r - V_r/r^2)$ and the inertia $(\rho\mathbf{V} \cdot \nabla V|_r)$ terms in the momentum equations during reconnection.
- These forces are very small.

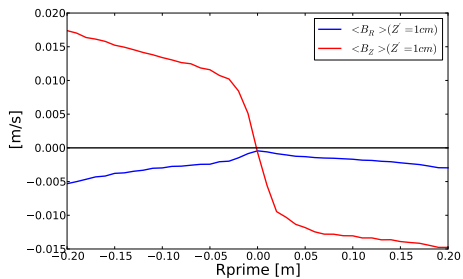
Poloidal flows before and after the reconnection show X point formation

before



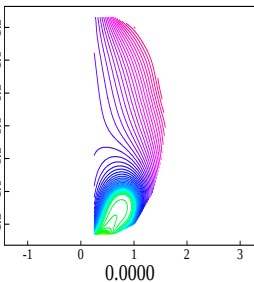
after



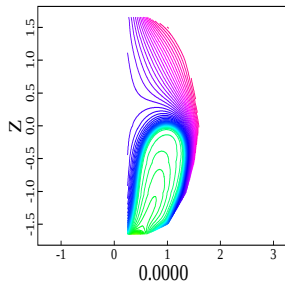


Time evolution of poloidal flux surfaces

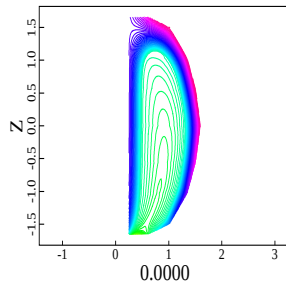
$t=6.12\text{ms}$
Poloidal flux



$t=6.3\text{ms}$
Poloidal flux

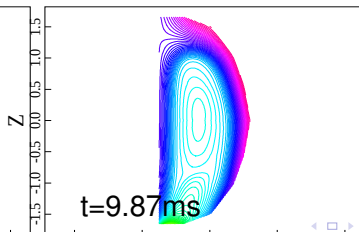
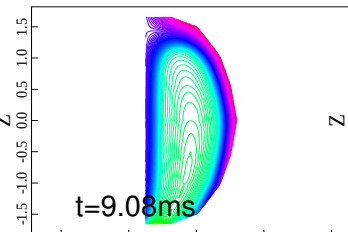


$t=8.63\text{ms}$
Poloidal flux



Poloidal flux

Poloidal flux

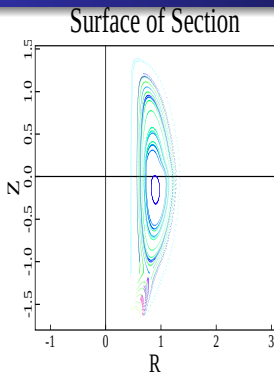


Ongoing project:

Could non MHD effects (2-fluid, Hall terms) affect the volume of the closed flux surfaces?

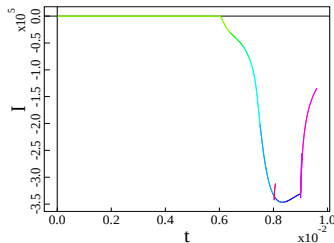
$$\mathbf{E} = -\mathbf{V} \times \mathbf{B} + \eta \mathbf{J} + \frac{1}{ne} \mathbf{J} \times \mathbf{B}$$

Closed flux surfaces form during the late injection phase with the Hall term

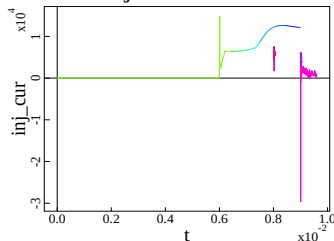


- $\eta/\mu_0 = 8m^2/s$, $T_e \approx 14eV$,
 $I_{tor} = 0.34MA$, $I_{inj} = 11kA$,
- Hall term is turned on at $t=8ms$
(shown in red line)
- Reconnection occurs but numerical problem appear later in time

Total and n=0 Tor. Current vs. t



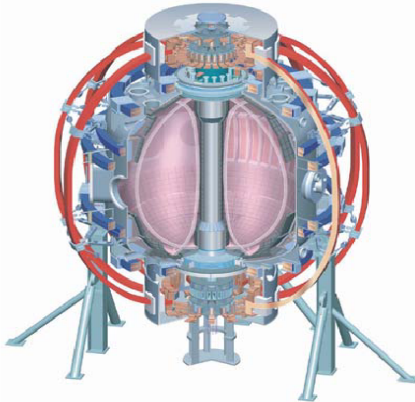
Injector current



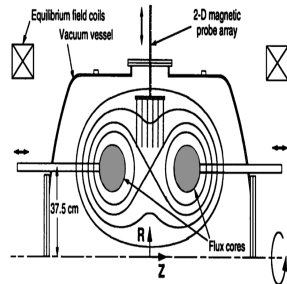
Navigation icons: back, forward, search, etc.

TCHI experiments in NSTX may be the first to demonstrate a forced magnetic reconnection process in a large scale device

NSTX



MRX



Device	$n[m^{-3}]$	T[eV]	L[m]	$B_{rec}[G]$	S_{rec}	S
NSTX	4×10^{18}	15-30	1	10-15	10^4	5×10^5
MRX	5×10^{19}	5-20	0.25	350	240	—