Supported by

Office of Science

# Control of asymmetric magnetic perturbation in tokamaks by computation of ideal perturbed equilibria


**Jong-kyu Park,**
**Jonathan E. Menard, Allen H. Boozer[a], Michael J. Schaffer[b]**

*Princeton Plasma Physics Laboratory, Princeton, NJ08543*
*[a]Department of Applied Physics and Applied Mathematics, Columbia University, New York, NY10027*
*[b]General Atomics, San Diego, CA92186*

# Motivation

- A small error field $\delta\overrightarrow{B^x}$ can cause various deleterious effects on toroidal plasmas even when $|\delta\overrightarrow{B^x}|/|\vec{B}| \approx 10^{-4}$ ,compared with the equilibrium field $\vec{B}$

- The small error field is unavoidable due to defects in primary magnets

- Perturbation theory is natural to calculating the effects by small deviations from an equilibrium

- Perturbed equations $\vec{f}_d + \vec{\nabla} P = \vec{J} \times \vec{B}$ with fixed $P(\psi), q(\psi)$ give well-defined perturbed equilibria and can be solved by minimizing perturbed potential energy $\delta W$

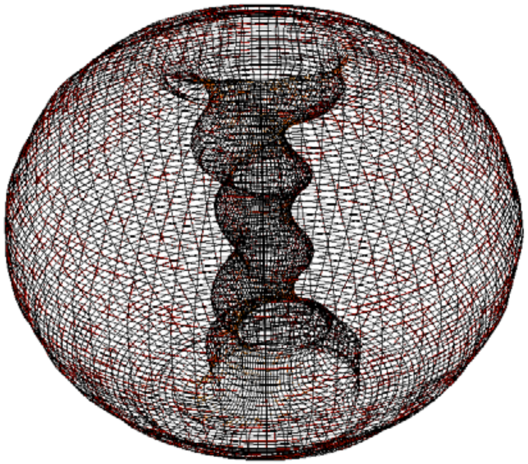
- IPEC finds 3D perturbed equilibria from axisymmetric equilibria and they are computationally efficient and experimentally useful for the control of small perturbations

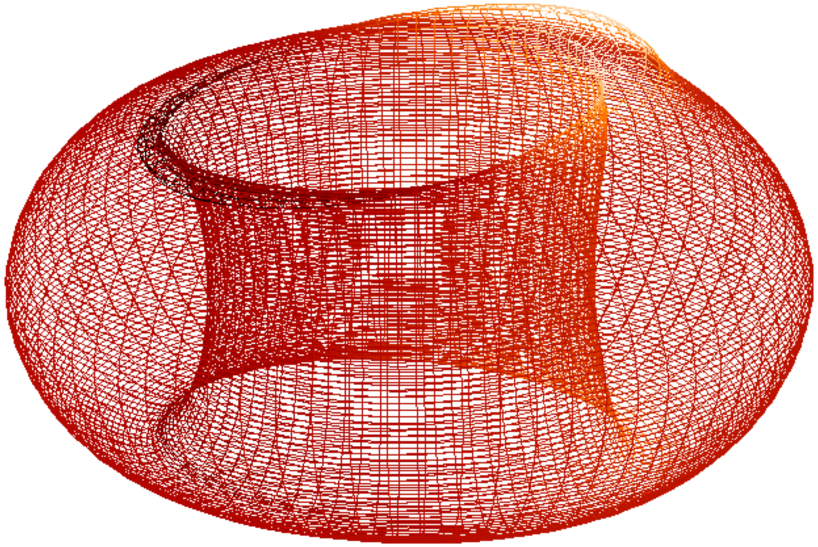
# Reference

- DCON: *A. H. Glasser, et al., Bull. Am. Phys. Soc. 42, 1848(1997)*
- VACUUM: *M. S. Chance, Phys. Plasmas 4, 2161(1997)*
- Plasma Response: *A. H. Boozer, Phys. Rev. Lett., 86, 5059(2001)*
- Perturbed Equilibrium: *A. H. Boozer, et al., Phys. Plasmas 13, 102501(2006)*
- IPEC: *J.-K. Park, et al., Phys. Plasmas 14, 052110(2007)*
- LM application: *J.-K. Park, et al., Phys. Rev. Lett. 99, 195003 (2007)*
- Error correction in ITER: *J.-K. Park, et al., Nucl. Fusion, submitted (2007)*

# Welcome

For a copy of this poster, please write your name and address





# I. IPEC (Ideal Perturbed Equilibrium Code)

- The code finds a perturbed equilibrium $\vec{f}_{ideal}(\vec{\xi}) = \vec{0} = \vec{\nabla}\delta p + \delta\vec{J} \times \vec{B} + \vec{J} \times \delta\vec{B}$ from an axisymmetric equilibrium $\vec{\nabla}P = \vec{J} \times \vec{B}$ given an external perturbation $\delta\overrightarrow{B^{x}}$

- The code uses DCON and VACUUM stability code to minimize the perturbed energy $\delta W = -\frac{1}{2}\int dx^{3}\vec{f}_{ideal}(\vec{\xi})\cdot\vec{\xi}$ with a given total field $\delta\vec{B}$ on the plasma boundary

- The code constructs an interface between the total field $\delta\vec{B}$ and the external field $\delta\overrightarrow{B^{x}}$ by a surface current $\vec{K}$ that is required to support a perturbed equilibrium

- The code gives various relations of resonant fields $\left(\delta\vec{B}\cdot\hat{n}\right)_{mn}$ on rational surfaces $q = m/n$ or variation of field strength $\delta B$ to external error fields specified on the plasma boundary $\delta\vec{B}\cdot\hat{n}_{b}$

# I.I. Computation of perturbed equilibria

- Ideal perturbed force balance equation

  Three-component equations of plasma displacement $\vec{\xi}$

$$\vec{f}_{ideal}(\vec{\xi}) = \vec{0} = \vec{\nabla}\delta p + \delta\vec{J}\times\vec{B} + \vec{J}\times\delta\vec{B}$$

$$\vec{\nabla}\delta p = -\vec{\xi}\cdot\vec{\nabla}P - \gamma P(\vec{\nabla}\cdot\vec{\xi}) \qquad \delta\vec{B} = \vec{\nabla}\times(\vec{\xi}\times\vec{B}) \qquad \delta\vec{J} = \vec{\nabla}\times\vec{\nabla}\times(\vec{\xi}\times\vec{B})/\mu_0$$

  Can be obtained by the minimization of the perturbed potential in ideal MHD

$$\delta W = -\frac{1}{2}\int dx^3\, \vec{f}_{ideal}(\vec{\xi})\cdot\vec{\xi}$$

- Euler-Lagrange equation for $\vec{\xi}\cdot\vec{\nabla}\psi$ is solved in DCON

$$(\overleftrightarrow{F}\cdot\vec{\Xi}_\psi' + \overleftrightarrow{K}\cdot\vec{\Xi}_\psi)' - (\overleftrightarrow{K}^{\dagger}\cdot\vec{\Xi}_\psi' + \overleftrightarrow{G}\cdot\vec{\Xi}_\psi) = \vec{0}$$

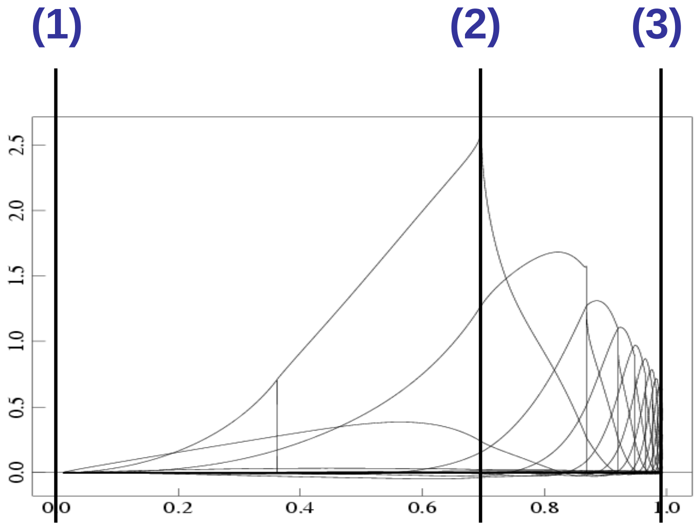
$$\vec{\Xi}_\psi \equiv \left\{\xi^\psi_{mn}(\psi)\right\}$$

  Boundary conditions are

  **(1)** $\vec{\Xi}_\psi = \vec{0}$

  **(2)** $\textit{Continuous, but } (\delta\vec{B}\cdot\hat{n})_{mn} = 0$

  **(3)** $\textit{Given } (\vec{\xi}\cdot\hat{n}_b) \textit{ or } (\delta\vec{B}\cdot\hat{n}_b)$



# I.II. Boundary condition on the plasma boundary

- DCON gives M neighboring perturbed equilibria with the associated total field on the plasma boundary

$$\vec{\xi}_i(\psi,\theta,\varphi),\ \left(\vec{\xi}_i\cdot\hat{\boldsymbol{n}}_b\right)(\theta,\varphi),\ \delta\vec{\boldsymbol{B}}(\psi,\theta,\varphi)=\vec{\nabla}\times(\vec{\xi}\times\vec{\boldsymbol{B}}),\ \ldots\ldots \Longrightarrow \delta\vec{\boldsymbol{B}}\cdot\hat{\boldsymbol{n}}_b,\ \hat{\boldsymbol{n}}_b\times\delta\vec{\boldsymbol{B}}$$

- The normal field is continuous across the boundary and gives the unique vacuum field outside

$$\delta\vec{\boldsymbol{B}}\cdot\hat{\boldsymbol{n}}_b \Longrightarrow \hat{\boldsymbol{n}}_b\times\delta\vec{\boldsymbol{B}}^v$$

- If an wall at infinity is taken as a boundary condition, then the jump of the tangential field on the boundary is precisely the representative surface current on the control surface

$$\mu_0\vec{\boldsymbol{K}}=\hat{\boldsymbol{n}}_b\times\delta\vec{\boldsymbol{B}}-\hat{\boldsymbol{n}}_b\times\delta\vec{\boldsymbol{B}}^v$$

- The surface current gives the external field

$$\mu_0\vec{\boldsymbol{K}}=\vec{\nabla}\times\delta\vec{\boldsymbol{B}}^x \Longrightarrow \delta\vec{\boldsymbol{B}}^x\cdot\hat{\boldsymbol{n}}_b$$

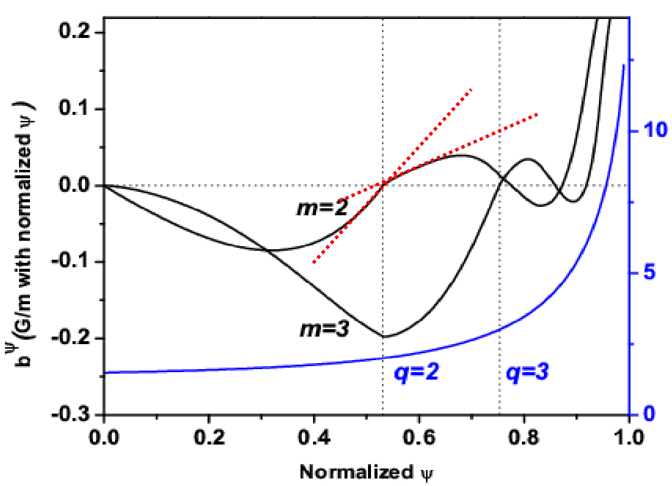
$$\left\{\begin{aligned}(\delta\vec{\boldsymbol{B}}\cdot\hat{\boldsymbol{n}}_b)&=\hat{\Lambda}\left[\vec{\boldsymbol{K}}\right]\\(\delta\vec{\boldsymbol{B}}^x\cdot\hat{\boldsymbol{n}}_b)&=\hat{\boldsymbol{L}}\left[\vec{\boldsymbol{K}}\right]\\(\delta\vec{\boldsymbol{B}}\cdot\hat{\boldsymbol{n}}_b)&=\hat{\boldsymbol{P}}\left[(\delta\vec{\boldsymbol{B}}^x\cdot\hat{\boldsymbol{n}}_b)\right]\end{aligned}\right.$$

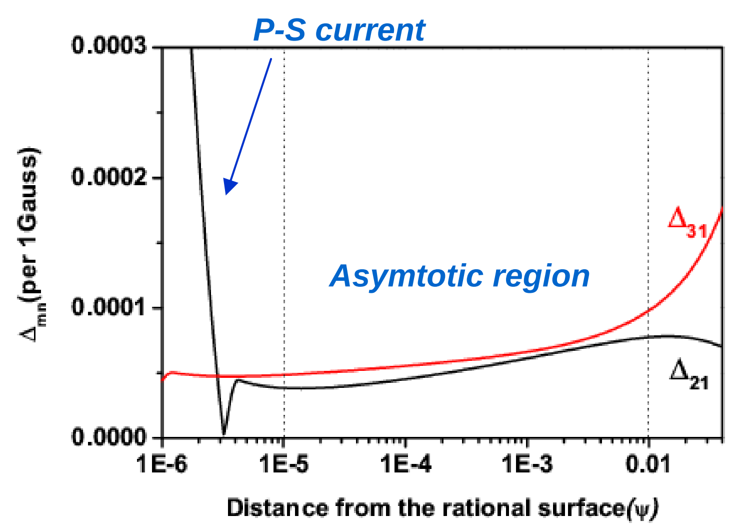
# I.III. Boundary conditions at rational surfaces

- Ideal MHD constraint does not allow islands, so produces a singular current to preserve magnetic topology - $\left(\delta\vec{B}\cdot\vec{\nabla}\psi\right)_{mn}=0 \quad at \quad q=m/n$

$$\Delta_{mn}=\left[\frac{\partial}{\partial\psi}\left(\frac{\delta\vec{B}\cdot\vec{\nabla}\psi}{\vec{B}\cdot\vec{\nabla}\varphi}\right)\right]_{mn} \Rightarrow \vec{j}_s=\frac{\Delta_{mn}ime^{i(m\theta-n\varphi)}}{\mu_0 n^2\left(\oint dSB^2/\left|\vec{\nabla}\psi\right|^3\right)}\delta(\psi-\psi_{mn})\vec{B} \Rightarrow \left(\delta\vec{B}\cdot\hat{n}\right)_{mn}$$

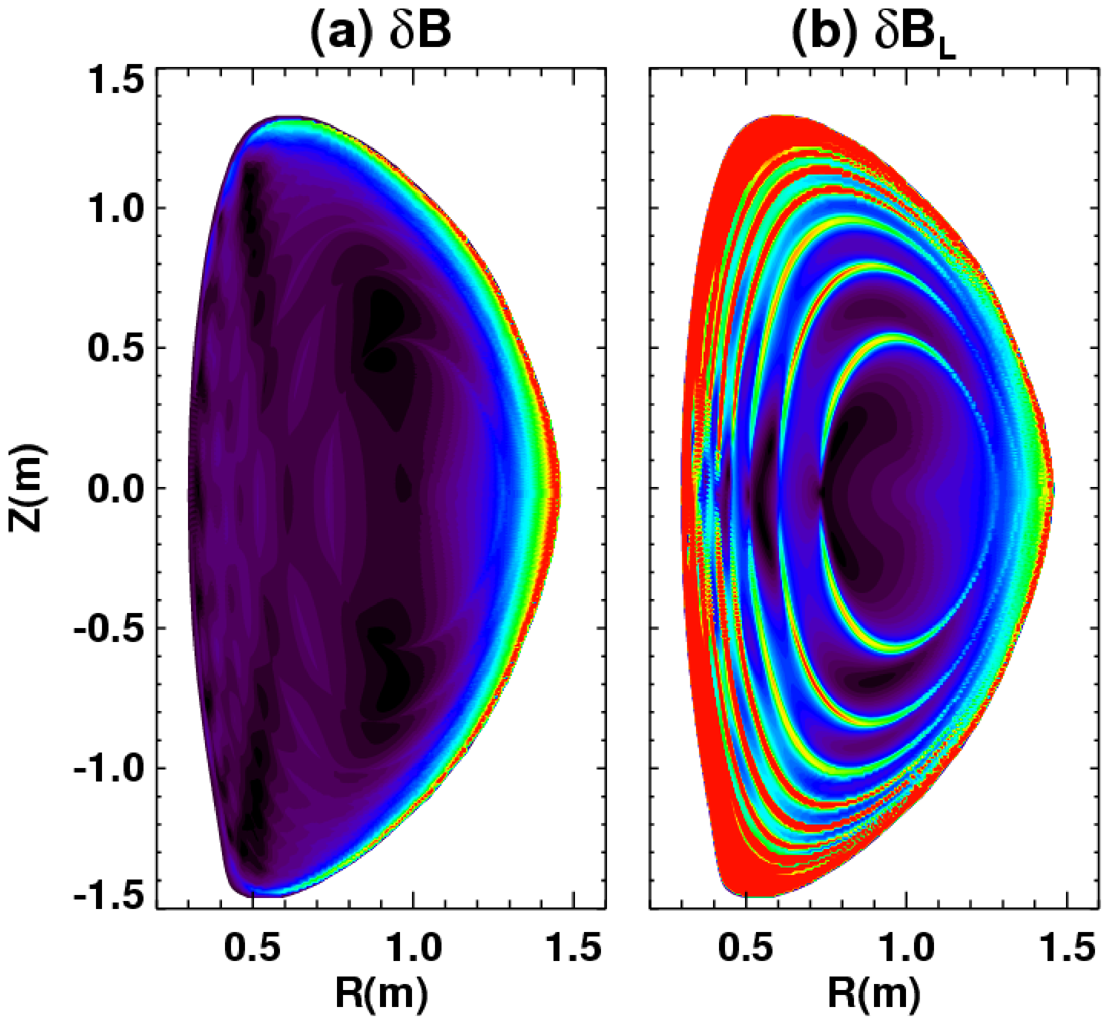
***Jump of tangential fields*** — ***Parallel singular current preventing magnetic islands from opening*** — ***Total resonant normal field driving magnetic islands***



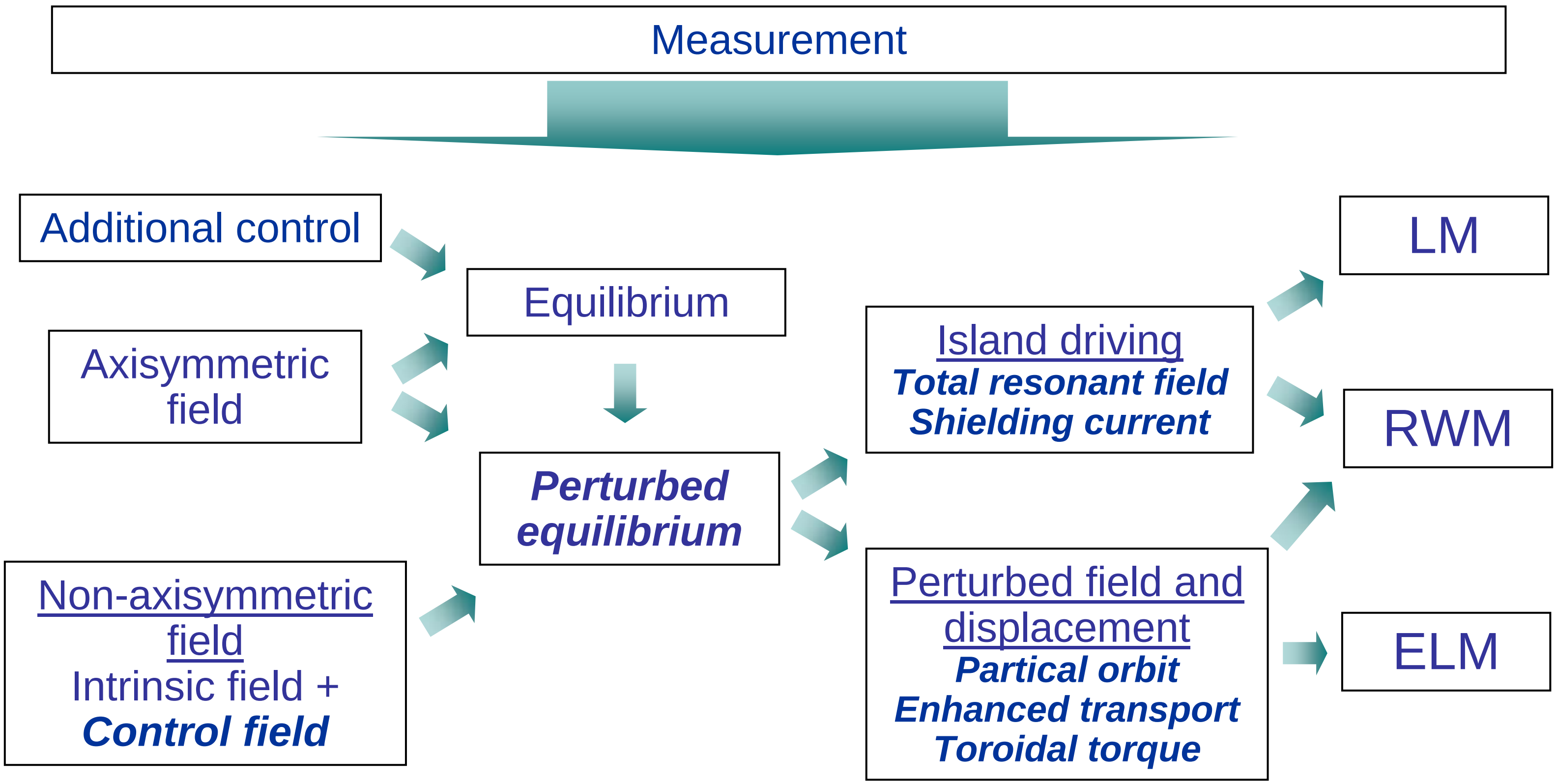
# I.IV. Ideal perturbed plasma

**#124800 at t=600ms**



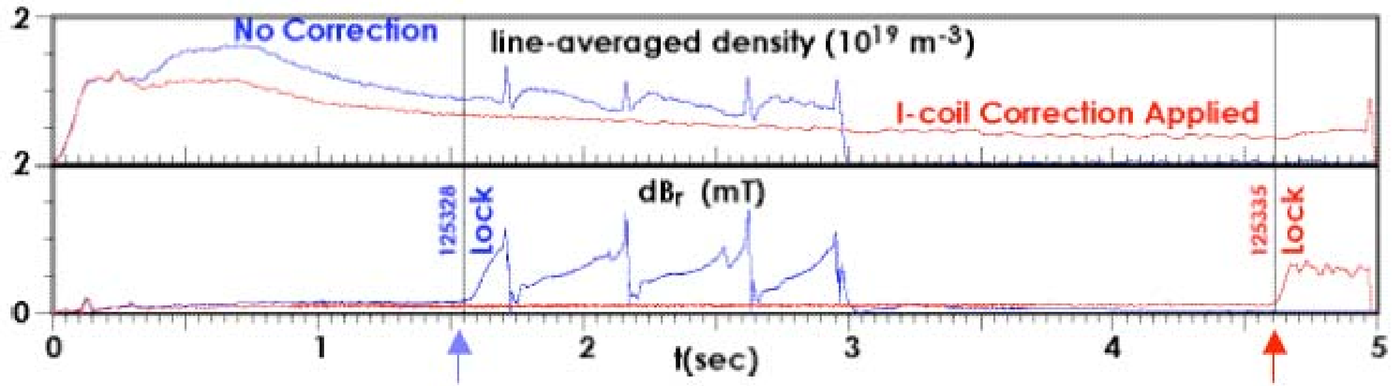
# I.V. 3D equilibrium reconstruction in tokamaks

- IPEC can be used for three-dimensional equilibrium reconstruction in tokamaks when non-axisymmetric external field and current are specified



# II. Locked Mode (LM) application

- Plasma locks when an error field is beyond a critical magnitude. This locked mode should be mitigated by correcting intrinsic error field

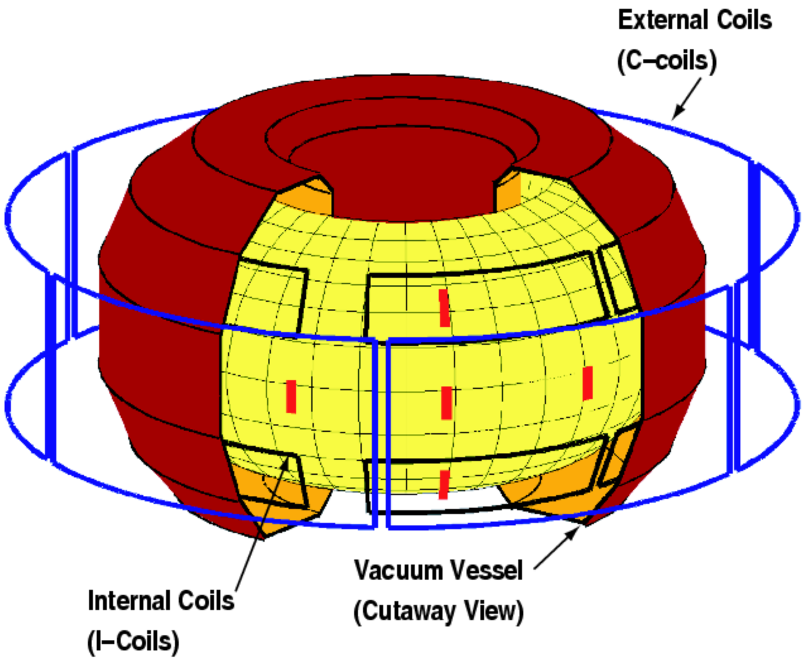



- The total resonant field driving islands $\left(\delta\vec{B}\cdot\hat{n}\right)_{mn}$ explained locked modes of tokamaks (DIII-D and NSTX), which were paradoxical when using external field $\left(\delta\vec{B}^{x}\cdot\hat{n}\right)_{mn}$ based on a cylindrical theory

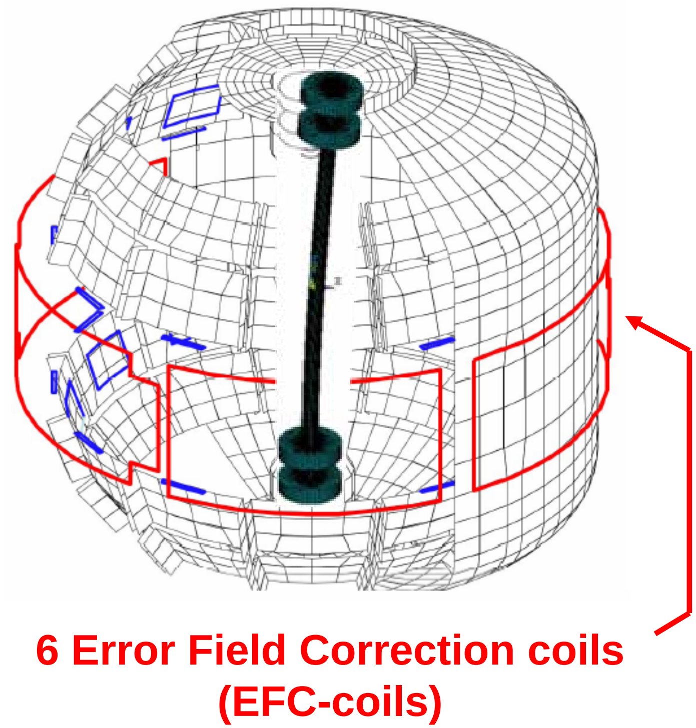
- IPEC shows that the strong poloidal harmonic coupling occurs in tokamak plasmas, so the total resonant field is necessary to explain locked modes

# II.I. Error field correction coil

- NSTX uses 6 EFC-coils to compensate n=1,2,3 intrinsic error field
- DIII-D uses 6 C-coils similar to EFC-coils and 12 I-coils for more flexible control of poloidal harmonics in external field




*DIII-D*




*NSTX*

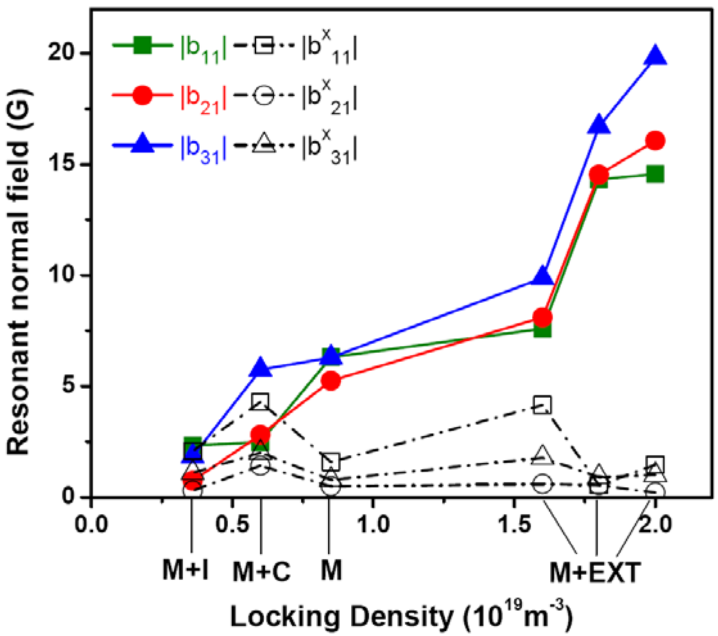
# II.II. Locking driven by total resonant field

- Previous approximation using external (vacuum) resonant field $\left(\delta\vec{B}^{x}\cdot\hat{n}\right)_{mn}$ on rational surfaces has often failed to explain plasma locking

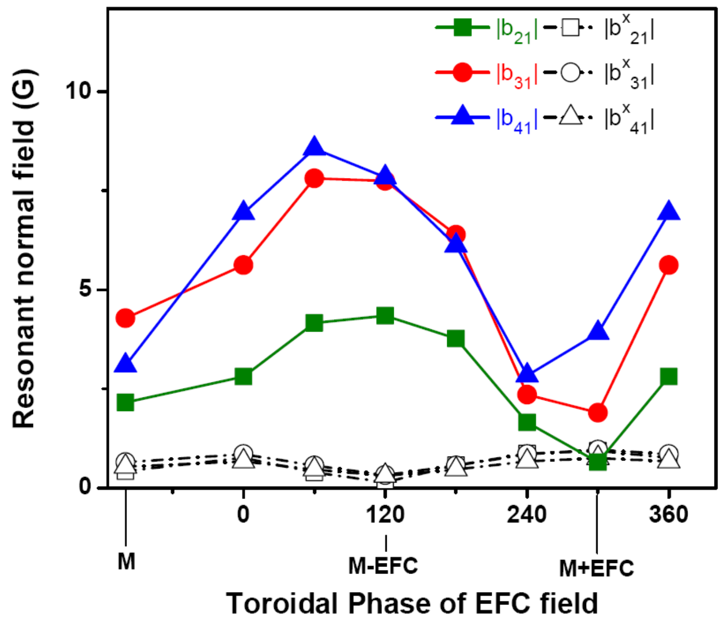
- IPEC using total resonant field $\left(\delta\vec{B}\cdot\hat{n}\right)_{mn}$ explains the paradoxical "optimized correction" results in DIII-D and NSTX tokamaks

$$\left(\delta\vec{B}\cdot\hat{n}\right)_{mn}^{critical}(Gauss)\cong n_e(10^{18}m^{-3})$$

**DIII-D**

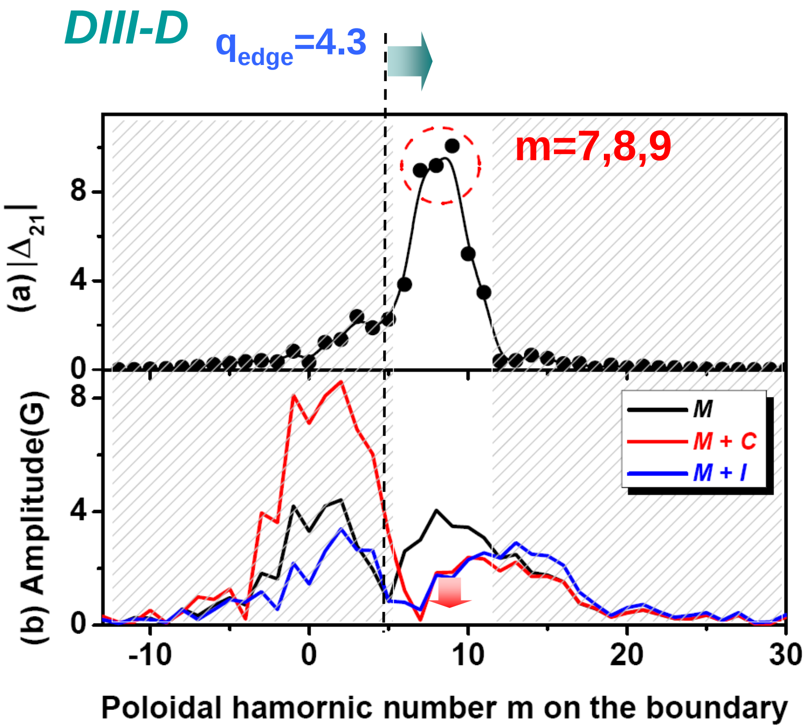


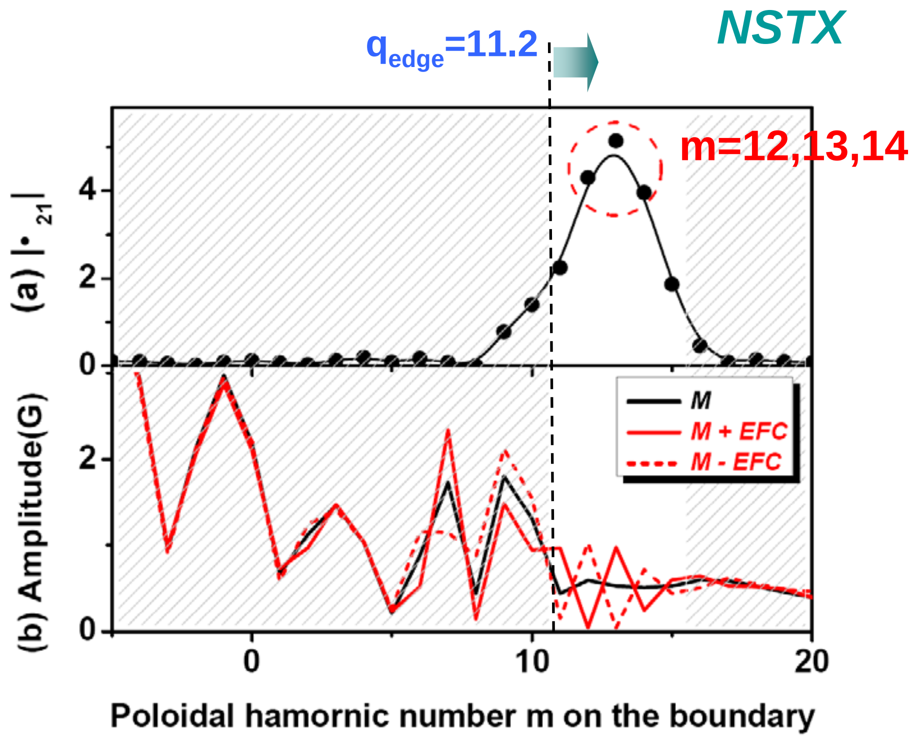

**NSTX**



# II.III. Strong poloidal harmonic coupling

- The “measure” of plasma locking $\Delta_{mn}, \vec{j}_s, (\delta\vec{B}\cdot\hat{n})_{mn}$ by individual poloidal harmonics of external field on the boundary $\left(\delta\vec{B}^{x}\cdot\hat{n}_b\right) = 1\,Gauss\; e^{i(m\theta-\varphi)}$ gives “dominant poloidal harmonics” on flux coordinates
- The shift of the peak towards higher harmonics and broad coupling of poloidal harmonics are found



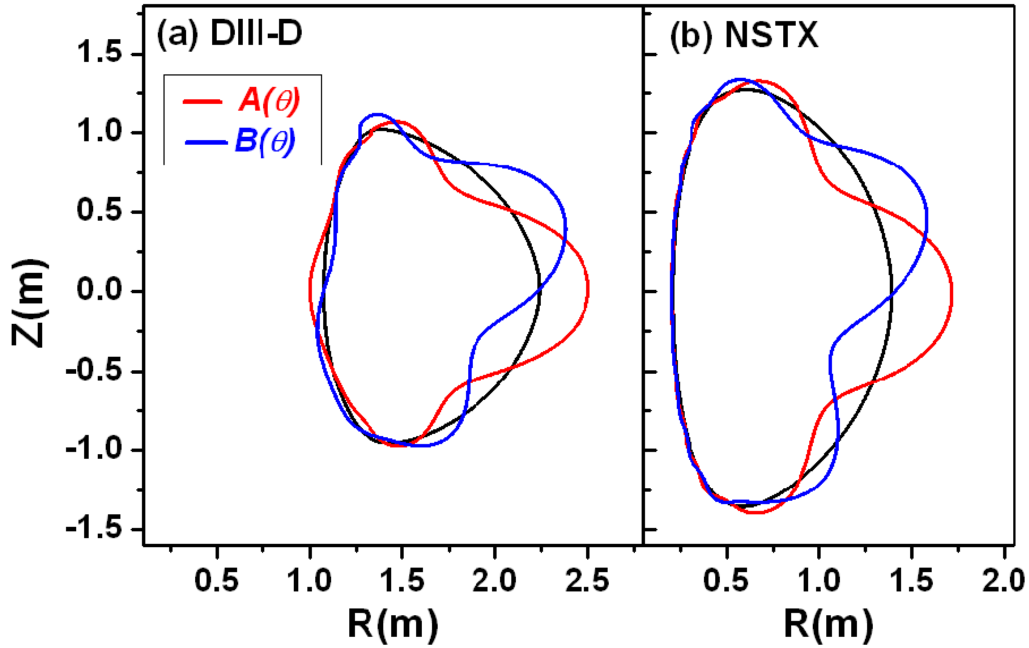



# III. Error Field (EF) control application

- New approach to error field correction is necessary in tokamaks
- The most important external field can be defined on the plasma boundary, and the important field is highly localized in the outboard
- Decomposition of the external field by the new ranks provides the most efficient way to mitigate error field effect

**The dominant components observed in real space is given by:**

$$\left(\delta\vec{B}^{x}\cdot\hat{n}_{b}\right)=A(\theta)\cos\phi+B(\theta)\sin\phi$$



# III.I. The most important external field in ITER

- The $i^{th}$ important external field can be defined by $i^{th}$ singular eigenvector of coupling matrix $\overleftrightarrow{\mathbb{C}}$

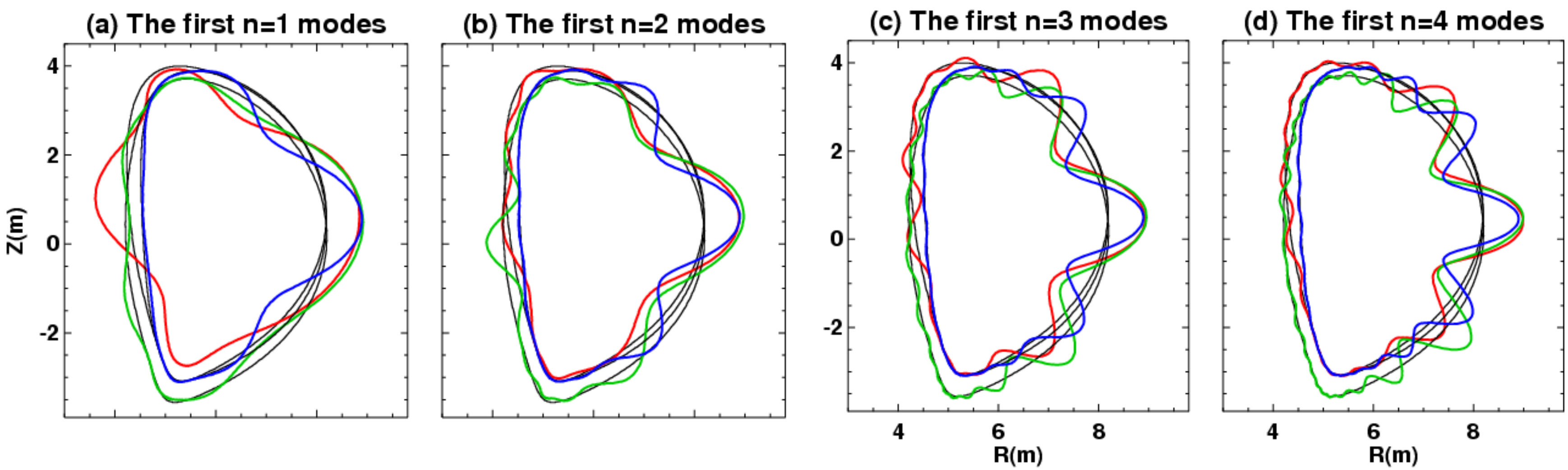
$$\left(\delta\vec{B}\cdot\hat{n}\right)_{mn}$$

$$\left(\delta\vec{B}^{x}\cdot\hat{n}_{b}\right)(\theta,\varphi)=Re\left(\sum_{m}\Phi^{x}_{mn}w(\theta)e^{i(m\theta-n\varphi)}\right) \quad \Rightarrow \quad \vec{B}=\overleftrightarrow{\mathbb{C}}\cdot\vec{\Phi}^{x}$$

- The most important external field, or the first mode is highly localized for each toroidal harmonic n. This is not sensitive to plasma configuration

*ITER*

**Equilibria** Scen2_bn18 Scen3a_bn22 Scen4_bn23



# III.II. Combined error mitigation in ITER

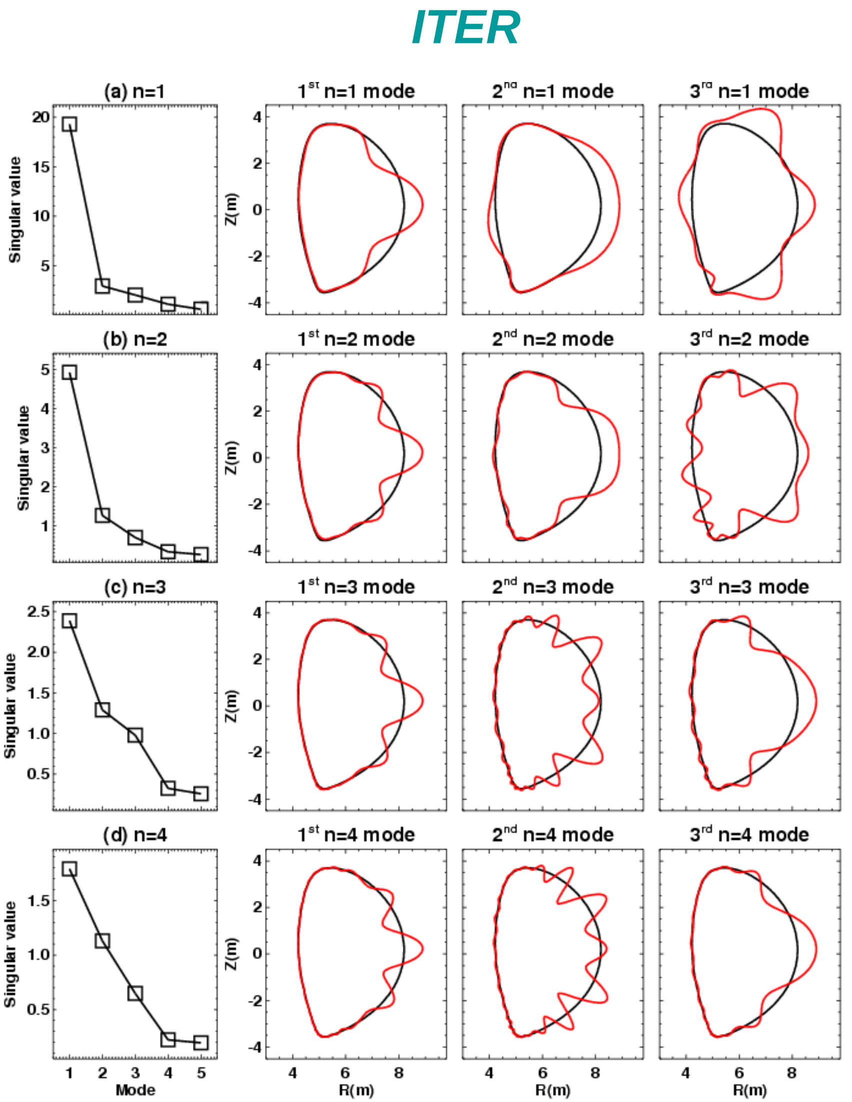
- When plasma evolves between possible scenarios, then combined coupling matrix can be used to identify $i^{th}$ important external field

$$\vec{B}^{combined} = \overleftrightarrow{\mathbb{C}}^{combined} \cdot \vec{\Phi}^{x}$$

- The combined B vector includes total resonant fields on rational surfaces in all possible scenarios

- The first mode is very dominant, and highly localized

- In practice, correction coils have to minimize the first mode

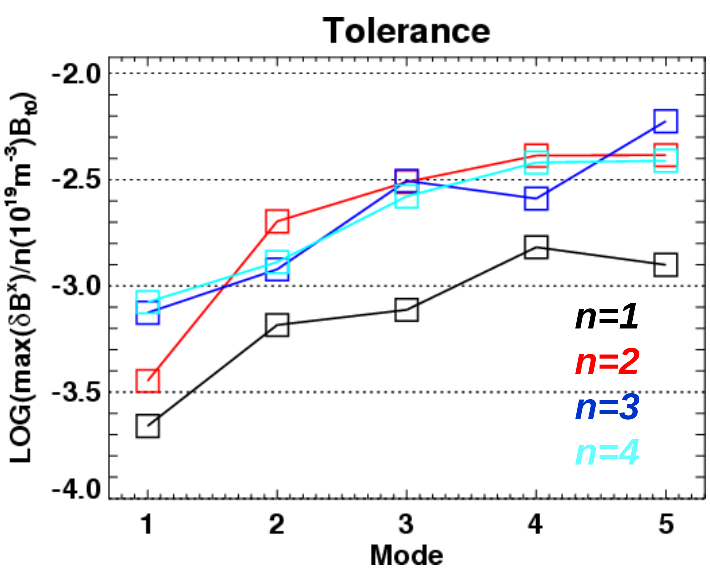


# III.III. New guideline of error mitigation in ITER

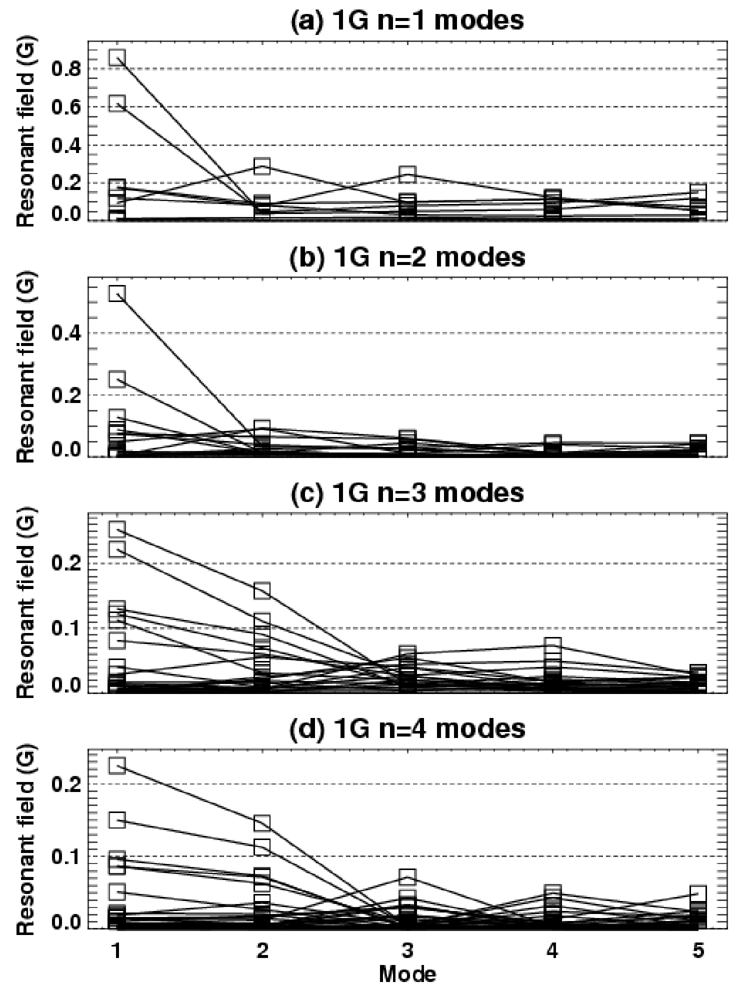
- The tolerances can be guided by all the total resonant fields in ITER scenarios by applying $i^{th}$ important mode (combined) with 1G maximum amplitude.

using $\left(\delta\vec{B}\cdot\hat{n}\right)_{mn}^{critical}(Gauss)\cong n_e(10^{18}m^{-3})$

*Maximally allowed field (normalized) to avoid locking*




*Resonant field on all rational surfaces by 1G maximum amplitude of modes*



# IV. RWM and ELM application

- Resistive Wall Mode (RWM) needs precise description of plasma response to external currents, which can be given by IPEC
- Non-ambipolar transport by external torque causes toroidal rotation damping, which is necessary to describe more precise plasma response
- Torque computations in theory and experiment show self-shielding effect of plasma by amplified viscous force
- Edge Localized Mode (ELM) can be suppressed by non-axisymmetric field, which has to increase edge transport, but not affect core
- Perturbed field on perturbed flux surfaces can be used to calculate enhanced transport in edge

# IV.I. Plasma response to external field

- Plasma response to external field is generally determined by energy and torque required to produce perturbations

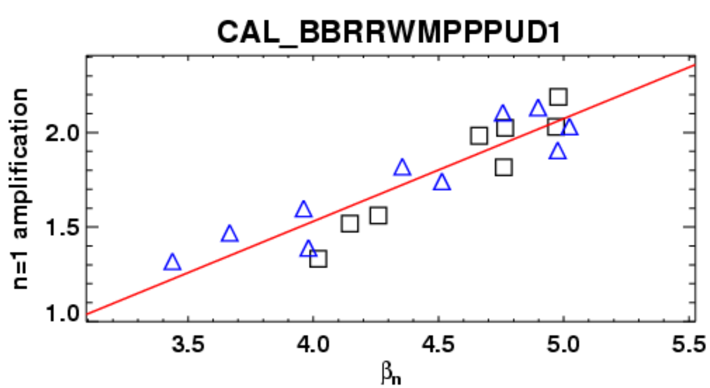
$$\delta W = \int \vec{j} \cdot \vec{A} dx^3 = \frac{1}{4}\left(\vec{I}^{\dagger} \cdot \vec{\Phi} + \vec{\Phi}^{\dagger} \cdot \vec{I}\right)$$

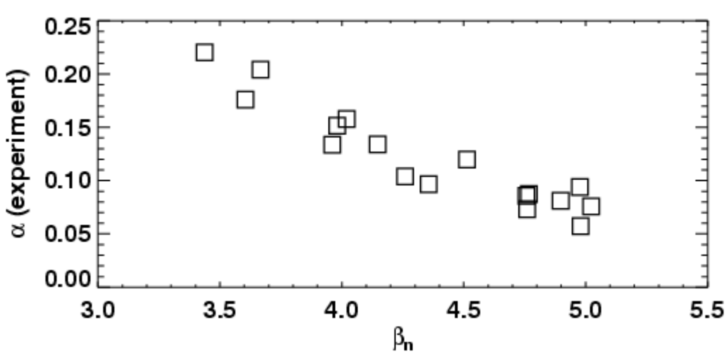
$$\tau_{\varphi} = \int \left(\vec{j} \times \vec{B}\right) \cdot \frac{\partial \vec{x}}{\partial \varphi} dx^3 = i\frac{n}{2}\left(\vec{I}^{\dagger} \cdot \vec{\Phi} - \vec{\Phi}^{\dagger} \cdot \vec{I}\right) \longrightarrow \vec{\Phi} = \vec{P} \cdot \vec{\Phi}^{x} \longrightarrow \vec{\Phi} \approx -\frac{1}{s - i\alpha}\vec{\Phi}^{x}$$

$$\vec{\Phi}^{x} = \overleftrightarrow{L} \cdot \vec{I}$$

- Torque parameter $\alpha$ can be estimated by measuring amplification of applied field in experiment and by calculating $s$ in IPEC, assuming $\alpha << 1$

$$\frac{\Phi_w}{\Phi_w^x} = -\frac{\gamma_w}{v_g + i\omega_r} = \frac{(1-c)\left\{(d-s)^2 + \alpha^2\right\}}{\left\{s(d-s) - \alpha^2\right\} - id\alpha} \quad ,where \;\; d = c/(1-c)$$



# IV.II. Torque by neoclassical theory

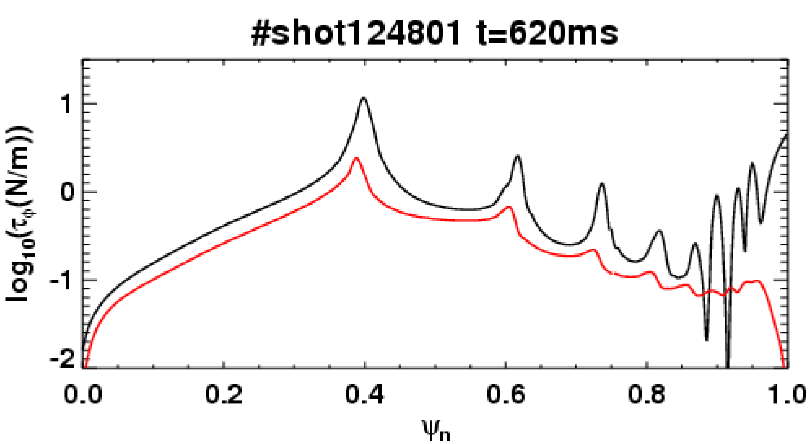
- Torque parameter can be calculated by

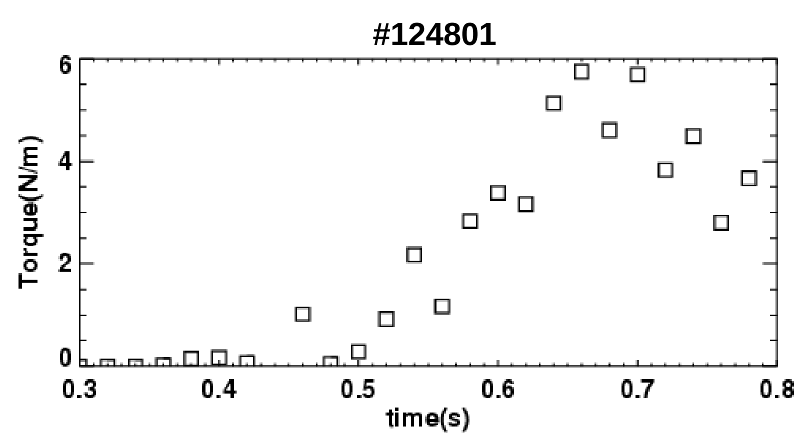
$$\tau_\varphi = \int d\psi \langle R \rangle \left\langle \frac{dV}{d\psi} \right\rangle \left\langle C_{eq1} \frac{p_i}{V_{Ti}} \Omega_\phi q \sum_{n,m\neq 0} \left( n^2 \left|\delta B_{nm}\right|^2 \frac{\mu_{ps1}}{\frac{2\sqrt{\pi}}{3} \frac{\nu_i}{V_{Ti}/R_0 q} + \mu_{ps1} \left| m - nq \right|} \right) + C_{eq2} \frac{p_i}{\nu_i} \Omega_\phi \sum_{n,m\neq 0} \left( n^2 \left|\delta B_{nm}\right|^2 W_{nm} \right) \right\rangle$$

- The variation of field strength is calculated on perturbed flux surfaces

$$B\delta B_L = \left(\vec{B}\cdot\delta\vec{B}\right)_L = \left(\vec{B}\cdot\delta\vec{B}\right) + B(\vec{\xi}\cdot\vec{\nabla}B) \gg B\delta B = \left(\vec{B}\cdot\delta\vec{B}\right) \gg B\delta B^x = \left(\vec{B}\cdot\vec{b}^x\right)$$

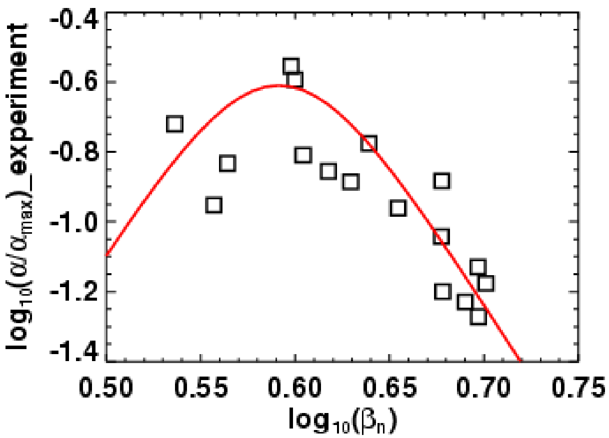
- Torque can not be larger than the maximum, $\oint R(\delta\vec{B}^p \cdot \hat{n}_b)(\delta\vec{B}^x \cdot \hat{n}_b) da$
- Theory and experiment gives different $\alpha / \alpha_{max}$

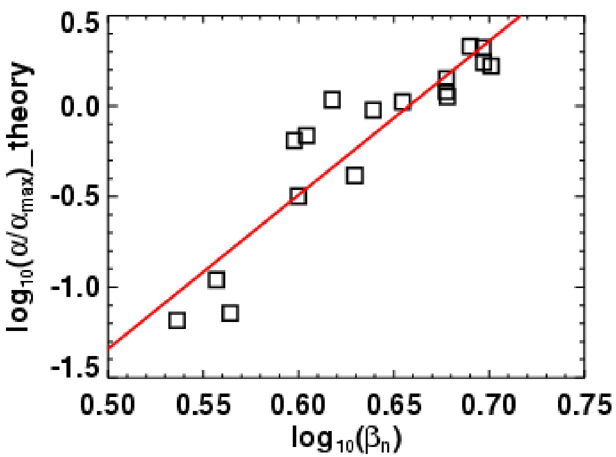


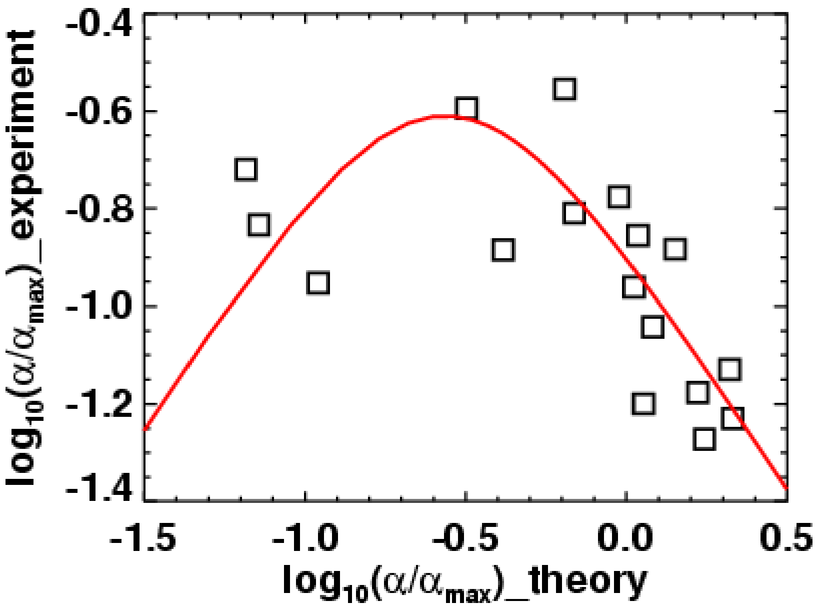
# IV.III. Self-shielding effect

- Anti-correlation between theory and experiment in high density regime indicates plasma self-shielding effect, that is, shielding of external perturbation by amplified viscous force




$$\alpha_{exp} / \alpha_{max} = \frac{\boldsymbol{a}\left(\alpha_{the} / \alpha_{max}\right)}{1 + \boldsymbol{b}\left(\alpha_{the} / \alpha_{max}\right)^2}$$

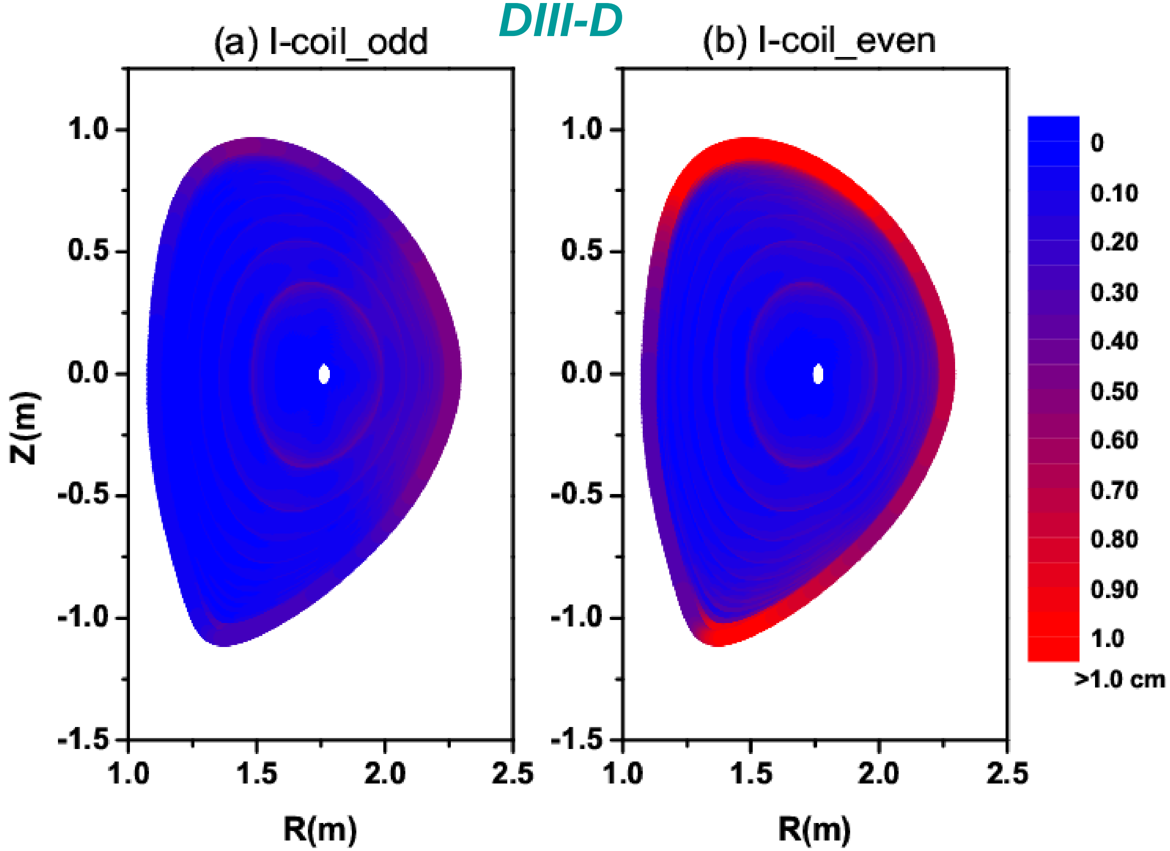


# IV.IV ELM suppression by external field

- The control of Edge Localized Mode (ELM) can greatly change plasma performance in tokamaks
- An external magnetic perturbation can suppress ELM if the perturbation induces stochastic field lines only at the edge without affecting core region

***Predicted maximum normal displacement on a poloidal plane***

***DIII-D***



# Summary

I. IPEC (Ideal Perturbed Equilibrium Code) has been developed modifying DCON and VACUUM, and constructing interface between - external and total field.
This code can be used to reconstruct perturbed equilibria in tokamaks

II. Perturbed equilibria and total resonant field explain Locked Mode (LM) in NSTX and DIII-D, and show strong poloidal harmonic coupling

III. Locked Mode (LM) in tokamaks can be effectively mitigated by resolving $i^{th}$ important mode minimizing total resonant field on rational surfaces, as shown in ITER examples

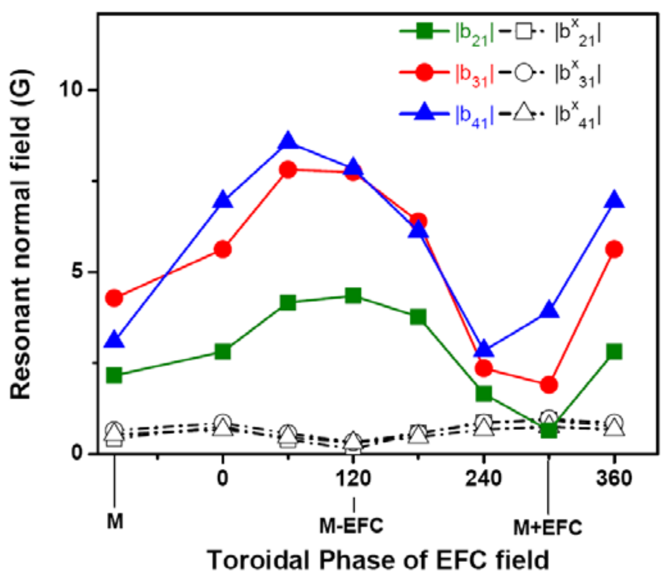
IV. Resistive Wall Mode (RWM) and Edge Localized Mode (ELM) can be described by plasma response and perturbed structure provided by IPEC.
Calculation of perturbed energy and torque in theory and experiment shows self-shielding effect of plasma by amplified viscous force

# Analysis of error fields has improved significantly recently with the development of IPEC - Ideal Perturbed Equilibrium Code

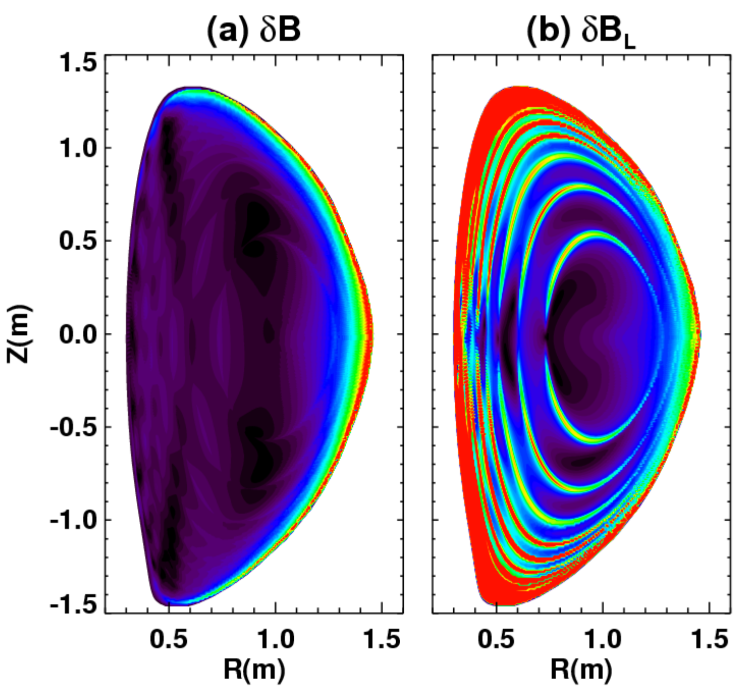
NSTX

- The code finds a perturbed equilibrium $\vec{f}_{ideal}(\vec{\xi}) = \vec{0} = \vec{\nabla}\delta p + \delta\vec{J} \times \vec{B} + \vec{J} \times \delta\vec{B}$ from an axisymmetric equilibrium $\vec{\nabla}P = \vec{J} \times \vec{B}$ given an external perturbation $\delta\overrightarrow{B^x}$
- The code gives various relations of resonant fields $\left(\delta\vec{B}\cdot\hat{n}\right)_{mn}$ on rational surfaces or variation of field strength $\delta B$ to external error fields specified on the plasma boundary $\delta\vec{B}\cdot\hat{n}_b$

**_Resonant field_**




**_Field strength_**




- IPEC: *J.-K. Park, et al., Phys. Plasmas 14, 052110(2007)*
- LM application: *J.-K. Park, et al., Phys. Rev. Lett. 99, (2007)*

***See POSTER TP8.080 at this conference***