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Nonlinear Simulations of NBI-driven GAE modes in NSTX

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Abstract

Hybrid 3D code HYM is used to investigate beam ion effects on MHD modes in a NSTX, aiming at simulations of NSTX shots where chirping frequency GAE/CAE modes have been observed. The thermal plasma is modeled using the MHD equations, and full-orbit delta-f kinetic description is used for the beam ions. The simulations show that for large neutral beam injection velocities and strong anisotropy in the pitch-angle distribution, many Alfven modes are excited. Unstable GAEs modes for $2<n<7$ and weakly unstable CAE for $n>7$ are observed. Scaling of the growth rate of GAE mode with beam ion density is stronger than linear due to significant modification of plasma equilibrium profiles. Profile modification is due to self-consistent beam ion effects, and it has indirect effect on the stability. It was demonstrated that phase velocity of the unstable GAE mode has opposite sign compared to the beam injection velocity, and the resonant particles satisfy Doppler-shifted cyclotron resonant conditions. Dependence of the growth rate on dissipation parameters is studied. Nonlinear simulations show that the GAE instability saturates at low amplitude. *This work supported by U.S. DOE Contract # DE-AC02-76CH03073*

Motivation

• Multiple sub-cyclotron frequency modes are observed in NSTX during NBI injection.

• CAE and GAE modes are predicted to be driven unstable by super Alfvenic NBI ions with $V_b \sim 3V_A$ (80 keV) through the Doppler shifted cyclotron resonance.

• Strong anisotropy in the fast-ion pitch-angle distribution provides the energy source for these instabilities.

• Both CAE and GAE modes are observed.

• Numerical simulations are needed to include: self-consistent anisotropic equilibrium, FLR effects, thermal ion and nonlinear effects.

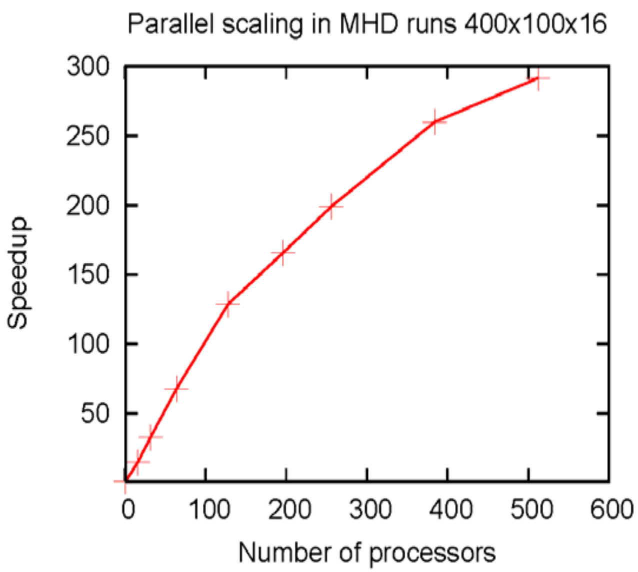
HYM – Parallel Hybrid/MHD Code

HYM code developed at PPPL and used to investigate kinetic effects on MHD modes in toroidal geometry (FRCs and NSTX)

- 3-D nonlinear.
- Three different physical models:
  - Resistive MHD & Hall-MHD.
  - Hybrid (fluid electrons, particle ions).
  - MHD/particle (one fluid thermal plasma, + energetic particle ions)[1]
- Full-orbit kinetic ions.
- For particles: delta-f / full-f numerical scheme.
- Parallel (3D domain decomposition, MPI)[2].

[1]Beam ions: $V_0 > V_A$, large Larmor radius $\rho_i / L \sim 0.1\text{-}0.3$.
High-frequency modes: $\omega \sim \omega_{ci}$

[2]Simulations are performed at NERSC.



N_{procs}

New MPI version of HYM shows good parallel scaling up to 500 processors for production-size jobs, and allows high-resolution nonlinear simulations.

Self-consistent anisotropic equilibrium the energetic NBI ions in NSTX

Grad-Shafranov equation for two-component plasma: MHD plasma (bulk) and fast ions

$$\frac{\partial^2\psi}{\partial z^2} + R\frac{\partial}{\partial R}\left(\frac{1}{R}\frac{\partial\psi}{\partial R}\right) = -R^2 p' - HH' - GH' + RJ_{i\phi}$$

$$\mathbf{B} = \nabla\phi \times \nabla\psi + h\nabla\phi$$

$$h(R,z) = H(\psi) + G(R,z)$$

$$\mathbf{J}_{ip} = \nabla G \times \nabla\phi \qquad , G - \text{poloidal stream function}$$

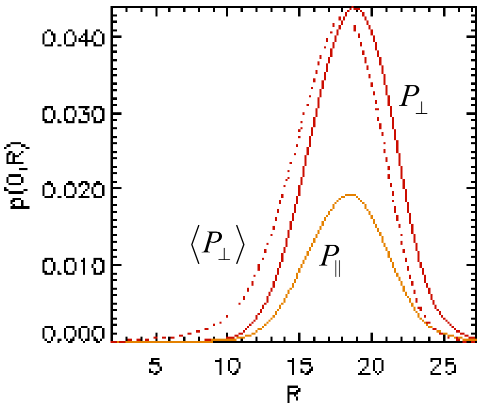
Self-consistent MHD + fast ions coupling scheme

Background plasma - fluid: $$\rho\frac{d\mathbf{V}}{dt} = -\nabla p + (\mathbf{j} - \mathbf{j}_i) \times \mathbf{B} - n_i(\mathbf{E} - \eta\mathbf{j})$$

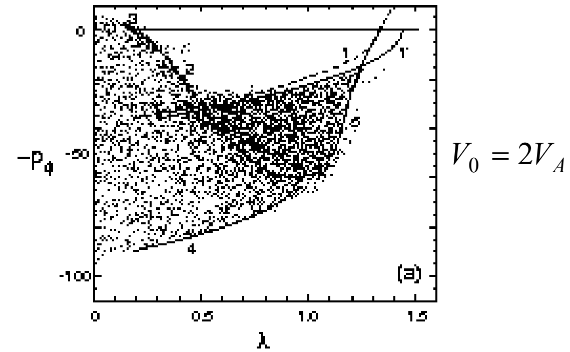
Fast ions – delta-f scheme: $F_0 = F_0(\varepsilon, \mu, p_\varphi)$, where μ is calculated up to 1st order in ρ_i /L; realistic F_0 to match the distribution functions computed from the TRANSP code.

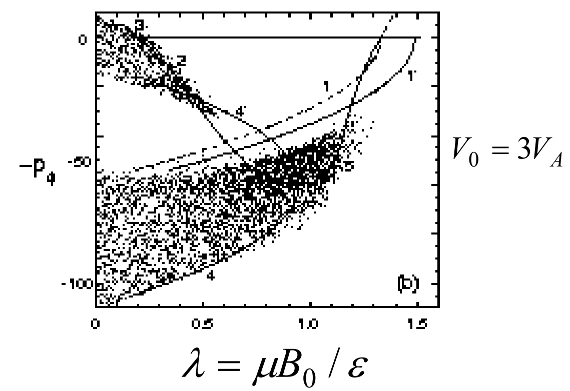
The prompt-loss condition, anisotropy, the large Larmor radius of the beam ions and the strong pitch-angle scattering at low energies have been included in order to match the distribution functions computed from the TRANSP code.

Energetic NBI ions significantly modify equilibrium profiles in NSTX



Beam ion pressure profiles (major radius)





Strong modifications of equilibrium profiles due to beam ions: more peaked current profile, anisotropic pressure, increase in Shafranov shift and reduction of q_{axis} → indirect effect on stability.

3D simulations of energetic ion-driven instabilities in NSTX

Linearized delta-f simulations

- Low-n simulations (n=1-6) show instability of Global Alfven Eigenmode (GAE) with large $k_{||}$, and significant compressional component $\delta B_{||} \approx 1/3\, \delta B_{\perp}$.

  For *n*=4 and m=-2, $\gamma = 0.005\omega_{ci}$ and $\omega = 0.3\omega_{ci}$ for n_b=3%
  $\gamma = 0.016\omega_{ci}$ and $\omega = 0.3\omega_{ci}$ for n_b=5% (E=80keV)

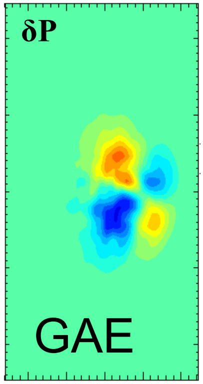
- Hybrid simulation with *n*=8 show weakly unstable CAE mode with *m*=8-10, $\omega = 0.4\omega_{ci}$, $\gamma \approx 0.001\omega_{ci}$, and $\delta B_{||} > \delta B_{\perp}$

- GAE modes are more unstable than CAE (agrees with analytical calculations) with $\gamma/\omega \sim n_b/n_0$.

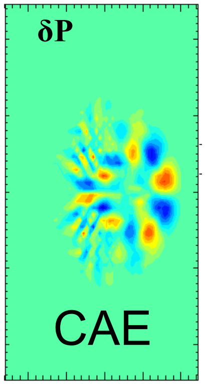
- Main damping mechanism for GAE is continuum damping (modeled in HYM with artificial viscosity): $\gamma_d/\omega \sim (r/r_{res})^{2m+\delta}$

- Simulations with different *n* show that most unstable modes have (*n*-*m*)~6-7, ie same $k_{||}$.

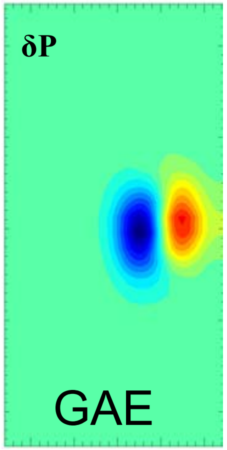
- Modes with larger-*m* have smaller radial extent.



n=4 (m=-2)

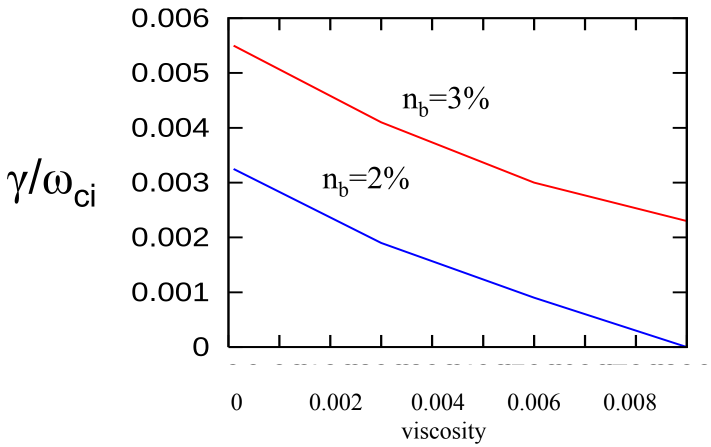


n=8



n=6(m=-1)

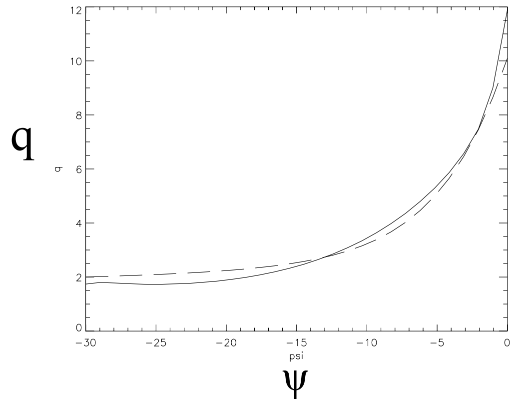
Dependence of the linear growth rate on damping



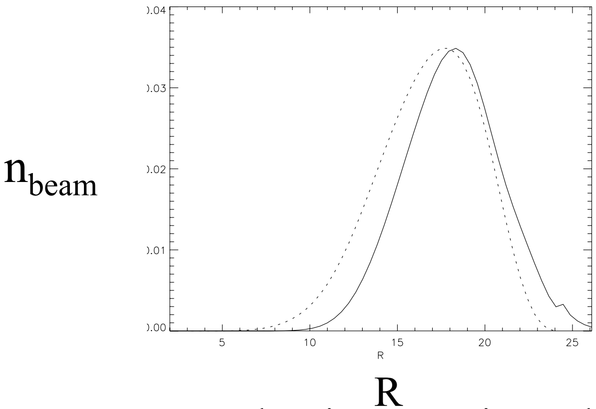
Linear growth rate of the n=4 GAE mode vs normalized viscosity.

Main damping mechanism for GAE is continuum damping (modeled in HYM with artificial viscosity): $\gamma_d/\omega \sim (r/r_{res})^{2m+\delta}$

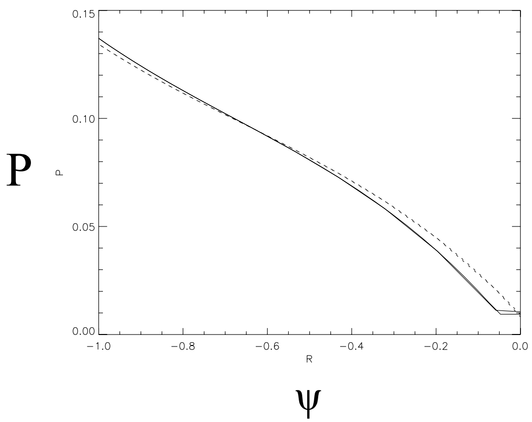
Plasma parameters and profiles are matched to NSTX shot #114147 (TRANSP)



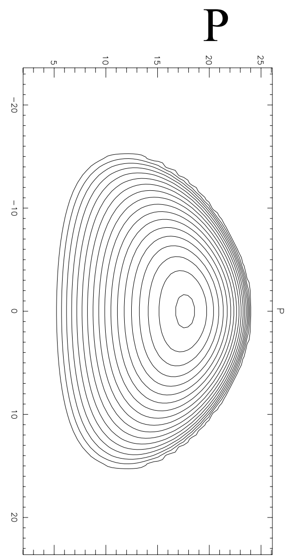
q-profile vs poloidal flux

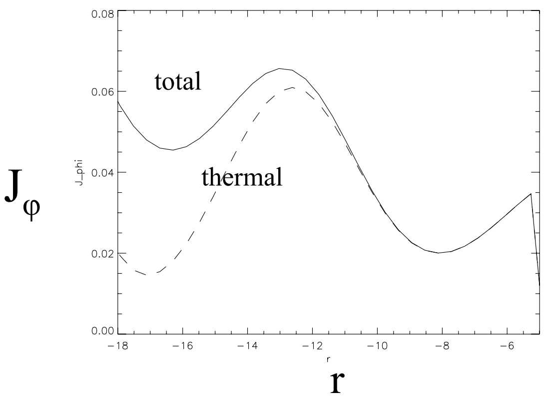


Beam density vs major radius



Pressure profile vs poloidal flux



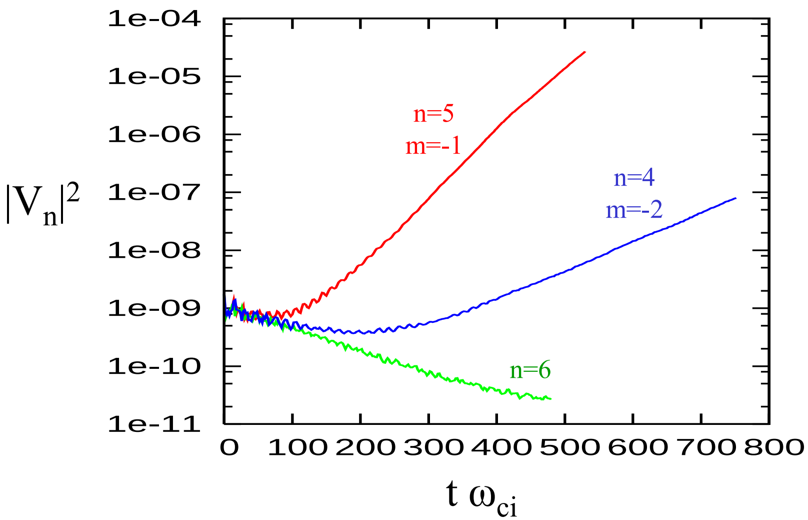


Toroidal current vs minor radius

3D simulations of energetic ion-driven instabilities in NSTX

shot #114147

- Growth rates of unstable modes are very sensitive to details of distribution function (pitch-angle); equilibrium profiles are not.
- Most unstable mode toroidal number shifts to larger n for larger q_0: n=5, m=-1



γ_4= 0.005ω_{ci} and ω= 0.3ω_{ci}
γ_5= 0.014ω_{ci} ω= 0.3ω_{ci}

$$k_{\|} = \frac{\omega_{ci} - |\omega|}{v_{\|}}$$

Observed features agree with that of GAE mode, which exists just below the lower edge of the Alfven continuum:

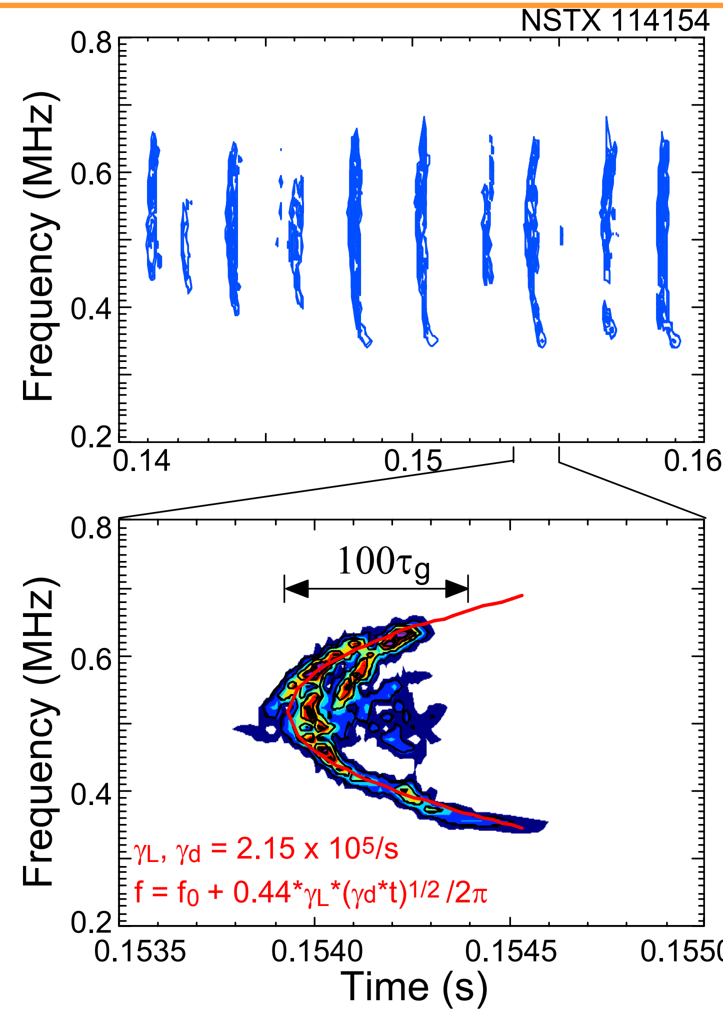
- For each *n,* several *m* are unstable with large $k_{||}$ and $nm<0$.
- Localized near magnetic axis.
- Large $\delta B_\perp$ component.

Most unstable mode in HYM simulations compares well with experimental results for NSTX 114147

• Measured toroidal number n=5 with frequency ω=480kHz (plasma frame) agrees with simulation results ω=403-530kHz.

• Wave propagates opposite to the beam injection direction ie $\omega/k_{||} < 0$ (GAE mode).

• Linear growth rate is inferred from frequency chirping: $\gamma \sim 2.15 \times 10^5$ 1/s [E. Fredrickson'05 , Berk, IAEA'06] compares well with numerically calculated $\gamma \approx 1.54 \times 10^5$ 1/s.

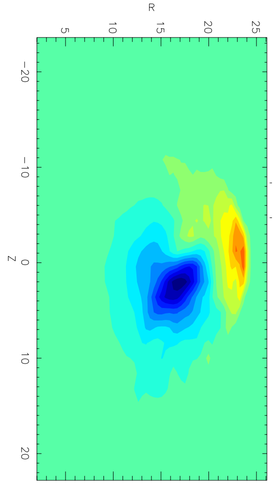


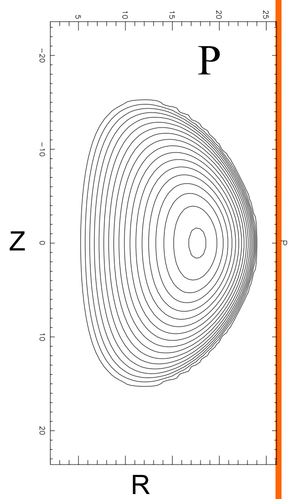
Up and down chirp observed during early NBI on NSTX.[E. Fredrickson'05].

Mode structure (equilibrium #114147)

n=5 m=-1

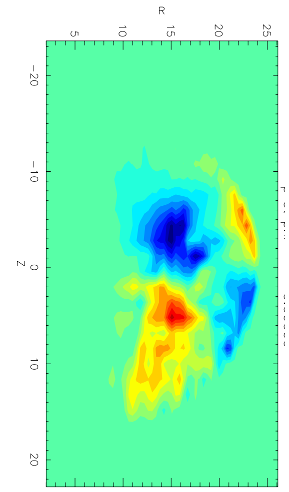
δP



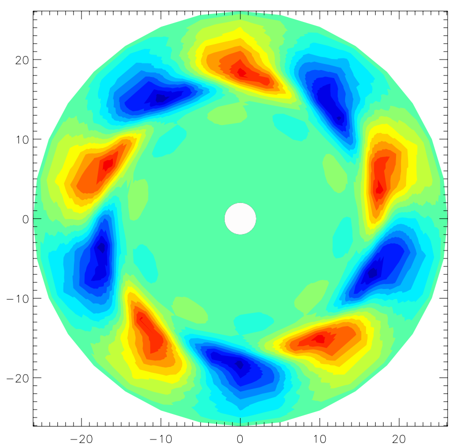


n=4 m=-2

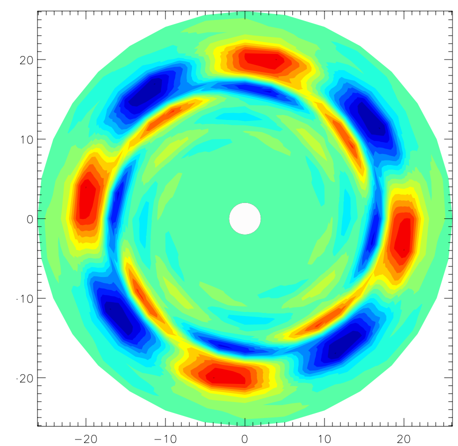
δP



δV_R

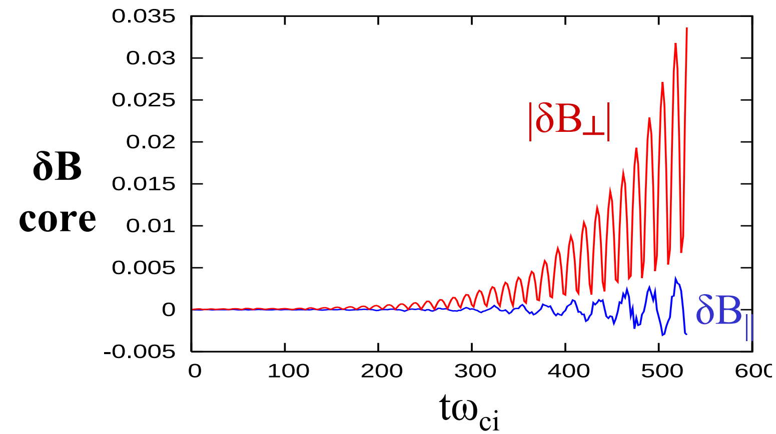


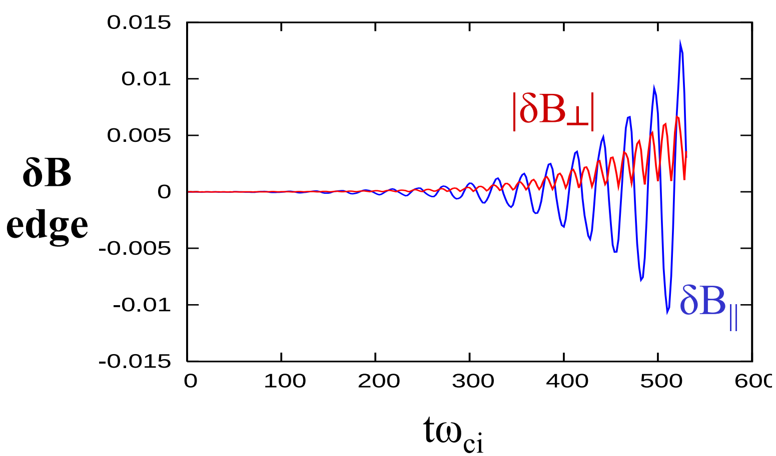
δV_R



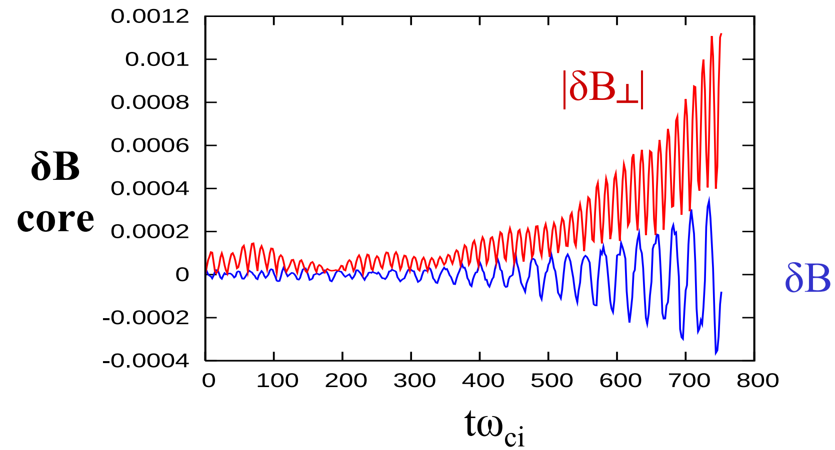
Magnetic mode structure shows significant compressional component at the edge

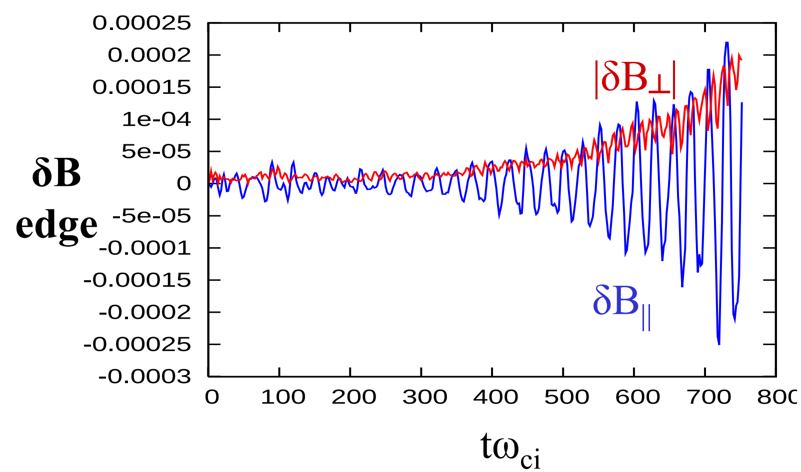
n=5 m=-1





n=4 m=-2



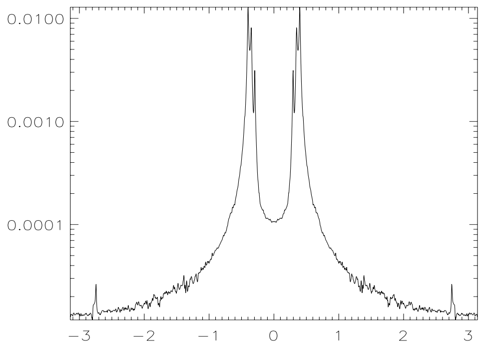


At peak amplitude $\delta B_{\|} \approx$ 1/5-1/3 $\delta B_{\perp}$, but at the edge there is significant compressional component $\delta B_{\|} \sim \delta B_{\perp}$.

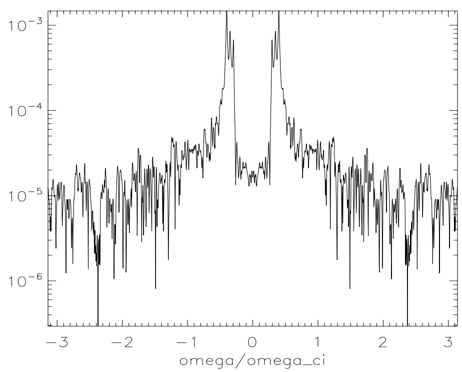
Magnetic spectrum shows at least three modes with comparable amplitudes

Core

δB_R

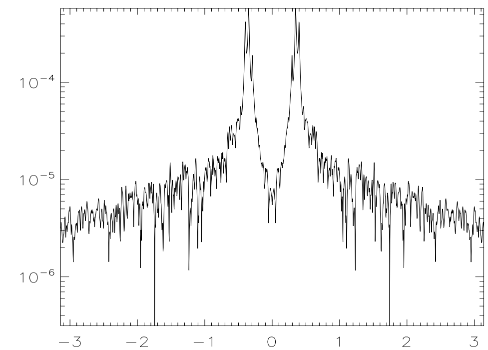


$\delta B_\parallel$

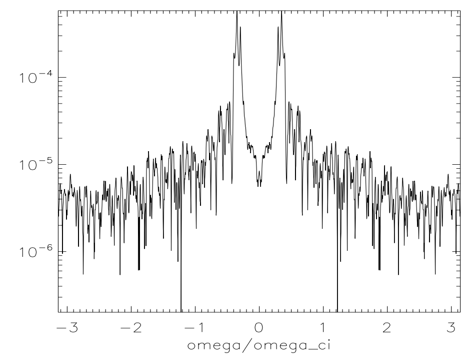


Edge

δB_R



$\delta B_\parallel$



Several modes present with $\Delta\omega \sim 1/R_0$: ω=0.3; 0.35; 0.4 ω_{ci}.
Higher frequency mode (higher poloidal mode number m) dominates in the core, while lower frequency mode has maximum amplitude at the edge (lower m).

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Resonant condition for GAE instability

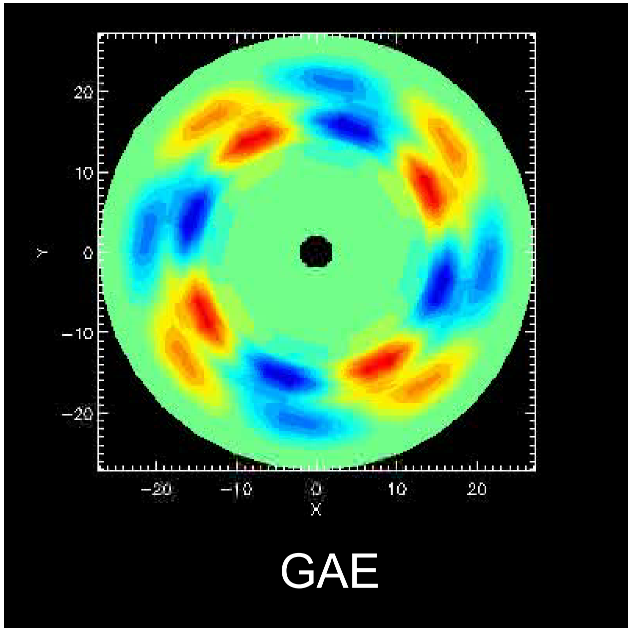
Resonant beam ions satisfy condition:

$$\omega - k_{\|}V_{\|} - \omega_{ci} \approx 0$$

$$\omega \approx k_{\|}V_{\|}$$

In the simulations: $V_{\|} > 0$ and $\omega/k_{\|} < 0$.

For $\omega = 0.3\text{-}0.4\omega_{ci}$ and $k_{\|}V_A \sim 0.3$, the resonant particles will have $V_{\|} \sim 2V_A \rightarrow$ significant fraction of beam ions can be in resonance.

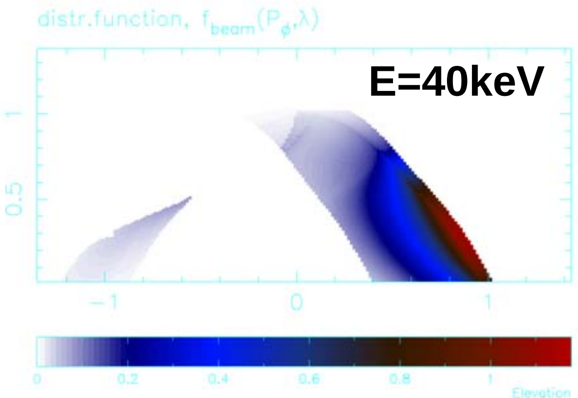


Contour plot of perturbed fluid pressure at equatorial plane.

Location of resonant regions in phase-space

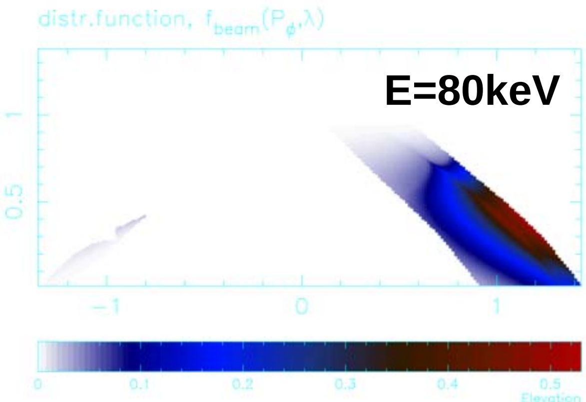
NOVA calculations

$\lambda = \mu B_0 / \varepsilon$



p_φ

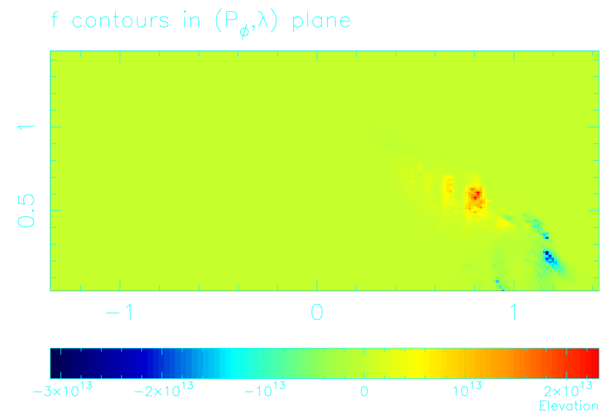
λ



p_φ

Equilibrium distribution for energetic ions.

λ

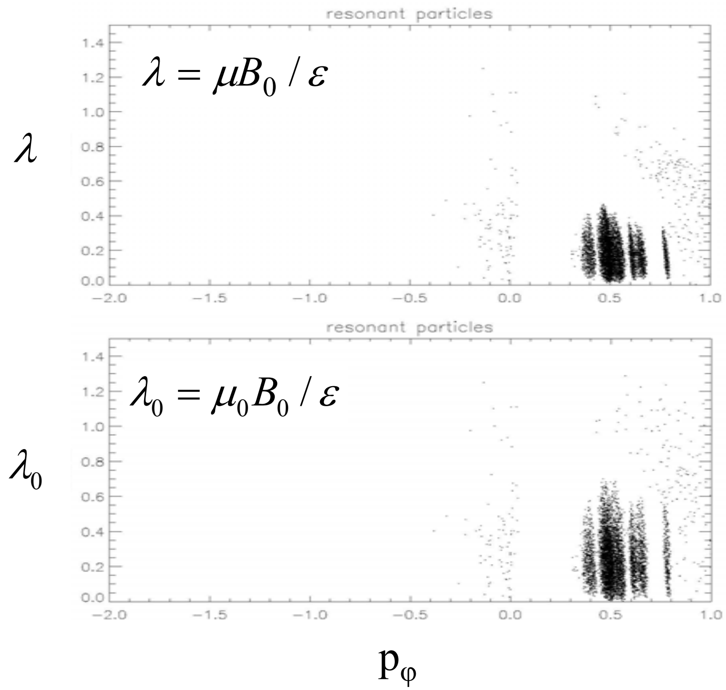


p_φ

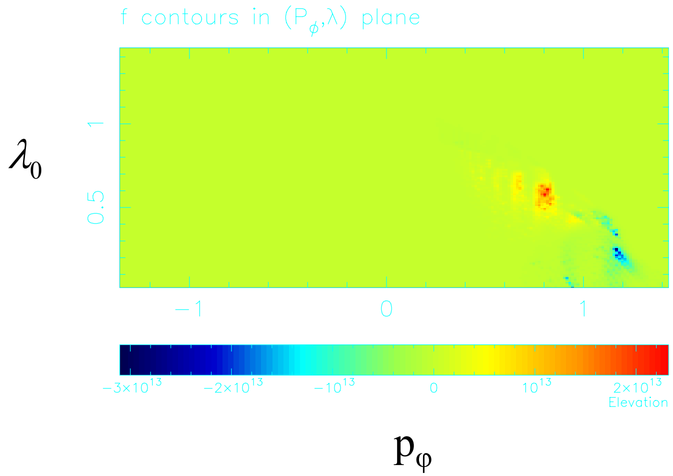
Contour plot of perturbed distribution function.

Comparison between HYM and NOVA calculations

HYM calculations



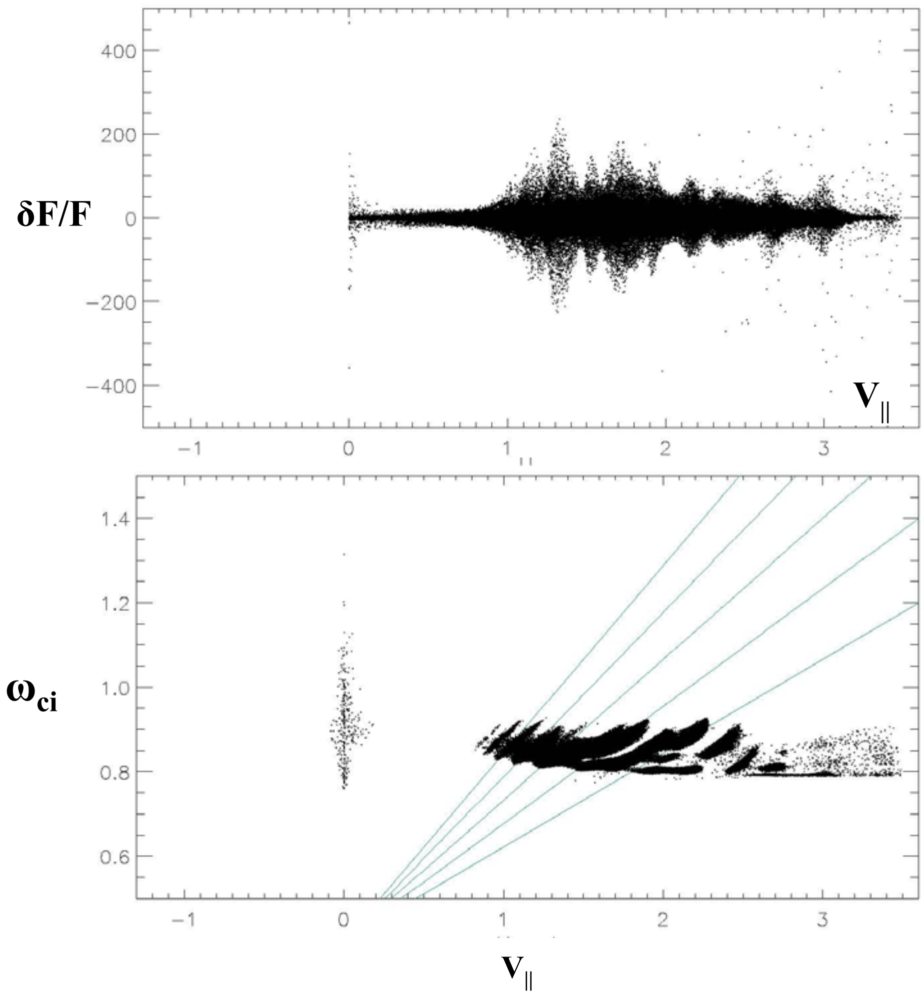
NOVA



Location of resonant regions in phase-space.

Note that lowest-order approximation for magnetic moment results in the over-estimated values of pitch-angle parameter λ.

Multiple resonances are seen in particle phase space



Orbit-averaged parallel velocity vs orbit-averaged cyclotron frequency for strongly resonant particles.

Resonant condition:

$$\omega - l\overline{\omega_{ci}} - \overline{k_\parallel v_\parallel} - \overline{k_\perp v_D} = 0$$

$$\omega_{ci} \sim B_0(1 + \varepsilon \cos\theta)$$

$$k_\perp v_D \sim \sin(\psi + \theta)$$

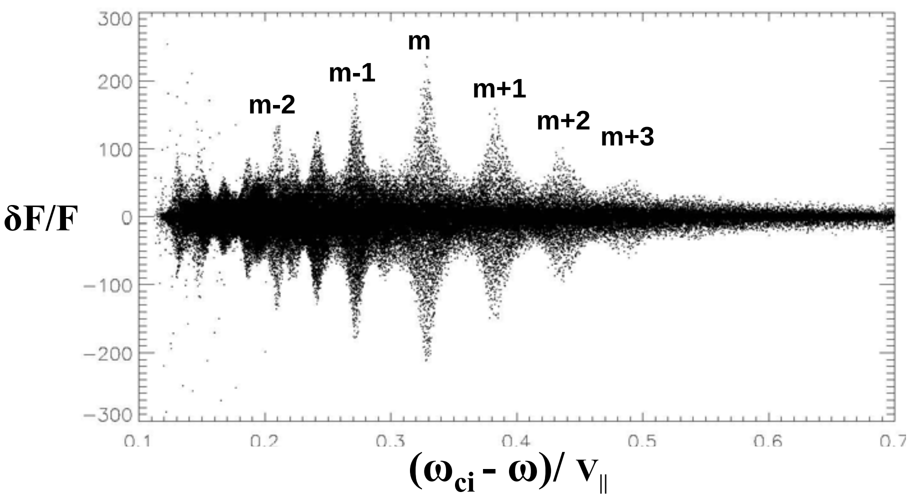
$$\omega - l\overline{\omega_{ci}} - (n - m/q)\overline{v_\parallel}/R - (p + r)\overline{v_\parallel}/qR = 0$$

Straight lines correspond to approximate relation:

$$\overline{\omega_{ci}} - \omega = \overline{\mathrm{v}_\parallel}\,(nq + m')/q_0 R_0$$

for ω=0.4 and m'=1-5.

Multiple resonances are seen for each mode (poloidal mode number)

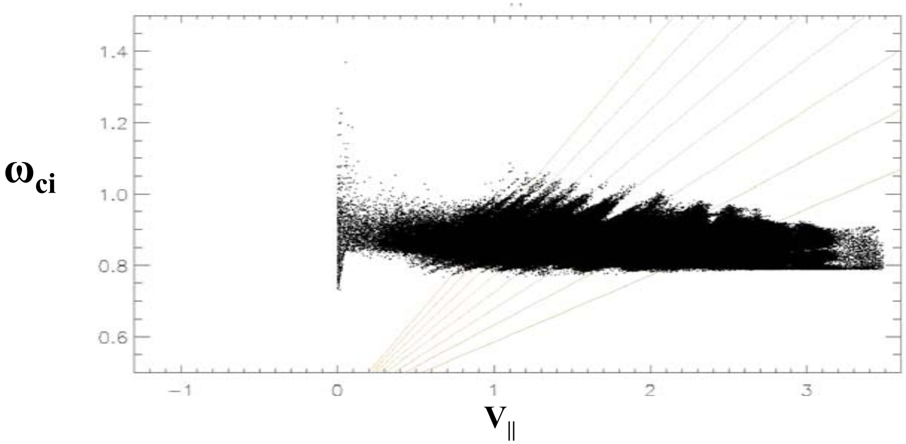


Resonances are shown for one mode with $\omega=0.4\ \omega_{ci}$ (m=3). Resonant interaction is seen for m; m+/-1; m+/-2 etc

$$\omega - l\overline{\omega_{ci}} - (nq-m)\overline{v_{\parallel}}/qR - (p+r)\overline{v_{\parallel}}/qR = 0$$

p, r are integers

corresponds to : $m \rightarrow m' = m-(p+r)$



Orbit-averaged parallel velocity vs orbit-averaged cyclotron frequency for particles with large weights (δF/F).

Straight lines correspond to approximate relation:

$$\overline{\omega_{ci}} - \omega = \overline{v_{\parallel}}\,(nq+m')/q_0 R_0$$

for m'=0-7.

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Nonlinear simulations (GAE instability)

3D nonlinear hybrid simulations for given toroidal mode number

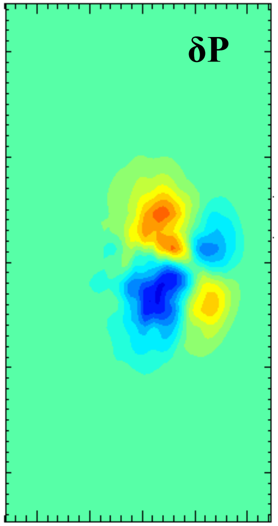
- Nonlinear results (n_b=5%) show saturation of instability at relatively low amplitudes:

  $\delta P \sim 0.8 \cdot 10^{-3}$
  $\delta V_R \sim 6 \cdot 10^{-3}$
  $\delta V_Z \sim 6 \cdot 10^{-3}$
  $\delta V_\varphi \sim 10^{-3}$

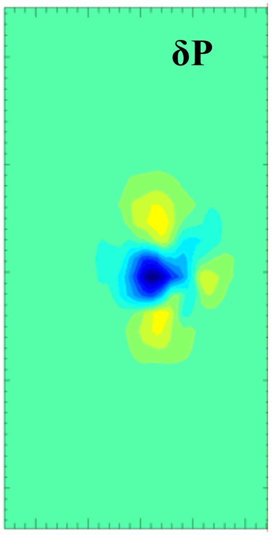
- Most unstable mode changes nonlinearly from n=4 m=-2 mode to (4,-3) mode.

- Poloidal mode structure is complicated: several modes with different poloidal mode number have comparable amplitudes.

- Initial fully nonlinear simulations (many-*n*) show dominant *n*=3 instability.



Linear mode structure, n=4, m=-2



Nonlinear mode structure n=4, m=-3 (at saturation)

Conclusions

- Nonlinear 3D HYM code has been modified for ST geometry.
- Grad-Shafranov solver for general case of the two component plasma: MHD plasma with $p=p(\psi)$, plus energetic ions with realistic (anisotropic) distribution.
- Simulations show that for large injection velocities, and strong anisotropy in the pitch-angle distributiom, many Alfven modes can be excited: GAE modes for $2<n<7$, and CAE modes for larger *n.*
- Instabilities are excited via Doppler-shifted resonance with NBI ions.
- GAE instability saturates at low amplitude, for given-n several poloidal modes are excited with comparable amplitudes in nonlinear regime.