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Modeling of Blob Formation in NSTX Edge Turbulence

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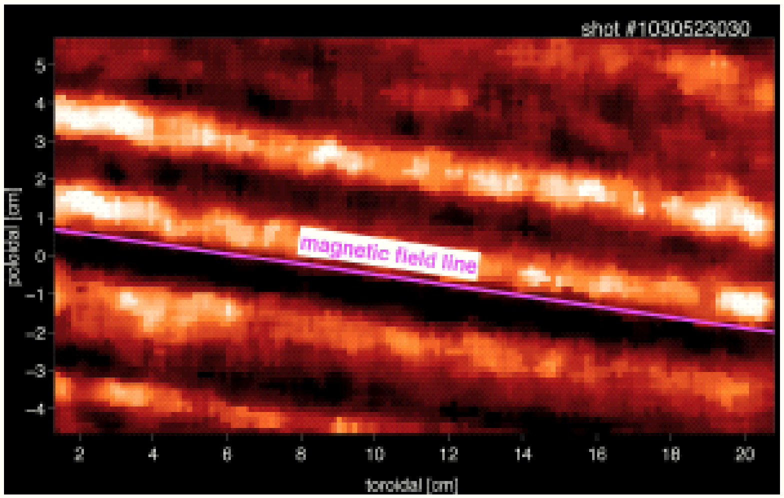
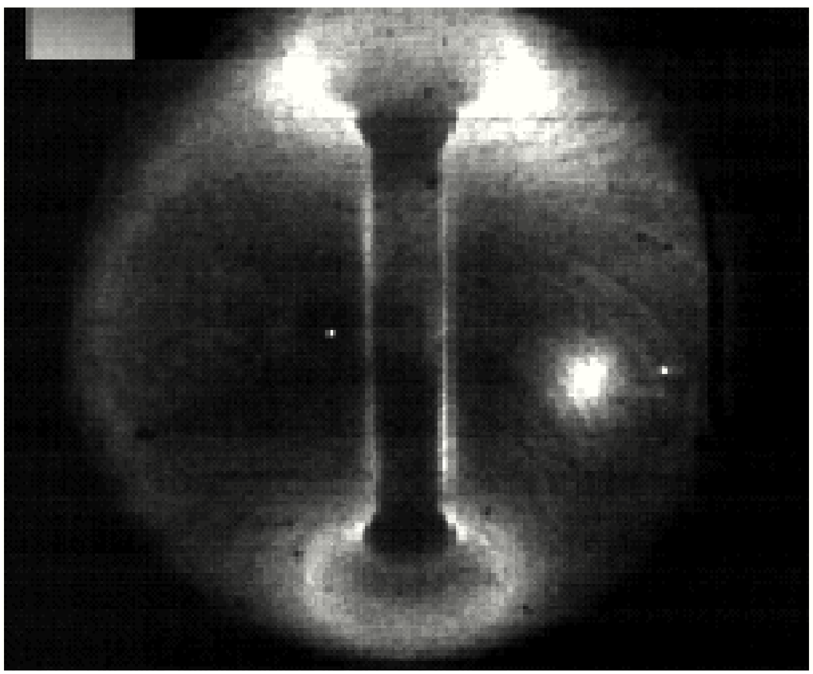
The objective of this work is to obtain a 2D, one-field model that realistically models tokamak edge turbulence but is also amenable to analytic treatment.

Outline

- Tokamak edge turbulence is 2D. Or is it?
- Disparities between parallel and perpendicular frequencies give us some fairly small parameters.
- Energy flow is controlled by the phase shift between n_e and ϕ, which is controlled by competition between parallel and perpendicular physics.
- The near X-point region's magnetic geometry strongly constrains parallel coupling. A gyrofluid formulation in field-line-following coordinates allows a simple, accurate model through this region.
- Effective parallel resistivity provides the dominant dissipation mechanism.
- We may split the state variables into one which experiences rapid parallel coupling ("*b*") and one which experiences almost no direct parallel coupling ("*a*").
- The parallel envelope of *a* is constrained by energy balance.
- What are the next steps?

Data shows edge turbulence is nearly 2D.

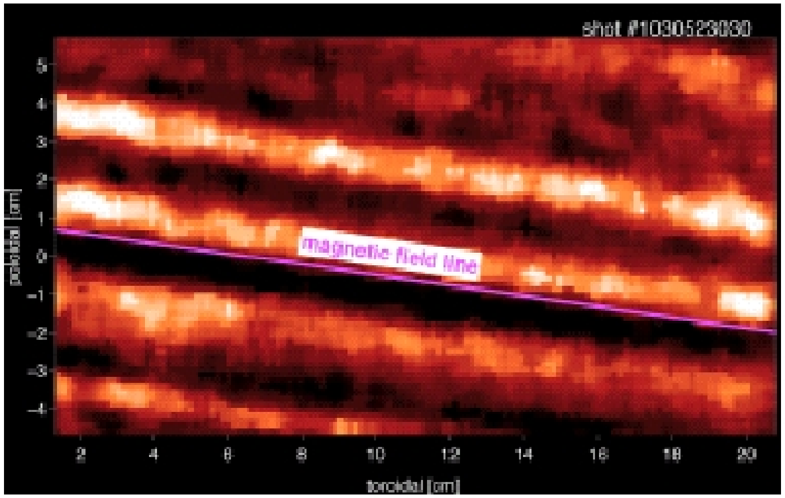




- Images of emission from turbulent density and temperature fluctuations in the edge show elongated filaments.
- Correlation lengths are long along **B**, but short perpendicular to **B**.
- In fluid turbulence, 2D structure has important implications, such as inverse energy cascade.

S.J. Zweben. TTF April 2004. IPELS 2003.

Electrons experience a rapid parallel coupling but ion gyrocenters don't...



Inside the separatrix, most flux surfaces are covered by a single field line.

$k_{||}\!=\!0$ generally not allowed,

except for zonal modes

Electron parallel force balance exerts a rapid, controlling influence.

Seeks $\nabla_{||}(\phi\!-\!n_e) = 0$

Seems to indicate flutelike structure

However, ions' parallel transit time is very long relative to turbulent timescales $\longrightarrow$ *no* effective direct parallel coupling for ion gyrocenters.

...as is seen by comparing parallel rates near the NSTX separatrix.

- Edge physics controlled by geometric scale ratio:
  - $k_{\|}\sim 1/R$, $\nabla n_{e0}\sim n_{e0}/L_p$, ratio $L_p/R \lesssim 1/20$ in NSTX
- Drift turbulence scales as $\omega_* \sim (k_y\rho_s)\, c_s/L_p$
  - In this document, frequencies normalized against c_s/L_p
  - Parallel effects penalized by geometric factor $\hat{\epsilon} \doteq (R/L_p)^2$

induction: $\hat{\beta} \doteq \left(\frac{c_s/L_p}{k_{\|} v_A}\right)^2 = \frac{1}{2}\beta_e \hat{\epsilon} \sim 0.075$

ion inertia: $\hat{\epsilon} \doteq \left(\frac{c_s/L_p}{k_{\|} c_s}\right)^2 = \left(\frac{R}{L_p}\right)^2 \sim 400$

e^- inertia: $\hat{\mu} \doteq \left(\frac{c_s/L_p}{k_{\|} v_{te}}\right)^2 = \frac{m_e}{m_i}\hat{\epsilon} \sim 0.13$

$k_\perp \rho_s \sim 1/10$ at outboard midplane

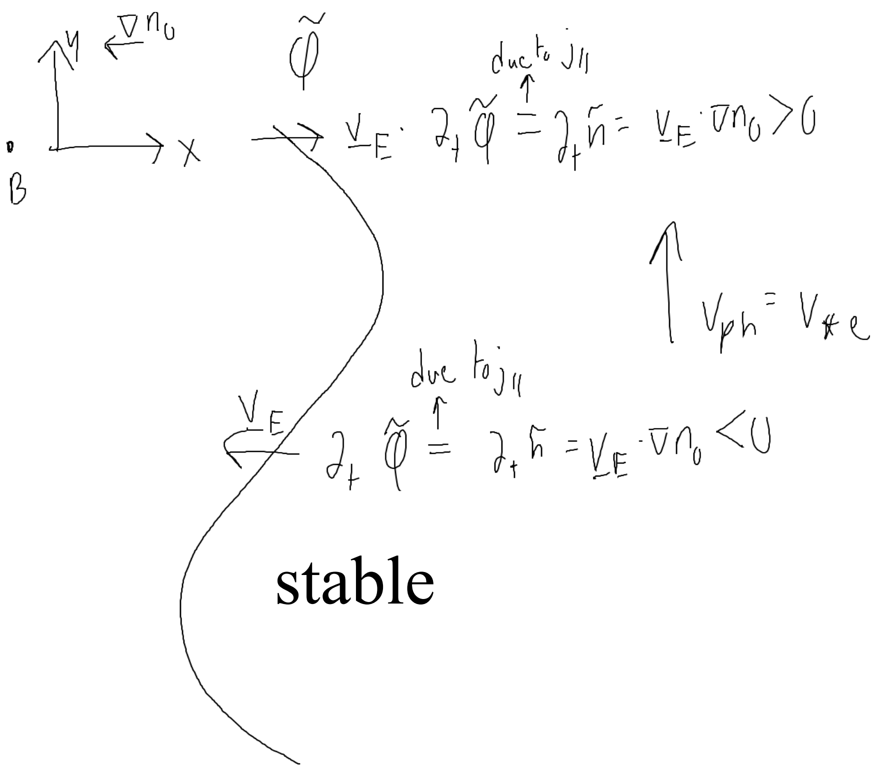
$\|$ resistive diffusion: $\frac{c_s/L_p}{\eta_{\|}^{res}} = C\,(k_\perp \rho_s)^2$ $\quad C \doteq 0.51 \frac{\nu_e L_p}{c_s}\frac{m_e}{m_i}\hat{\epsilon} \sim 0.3$

resistive damping rate

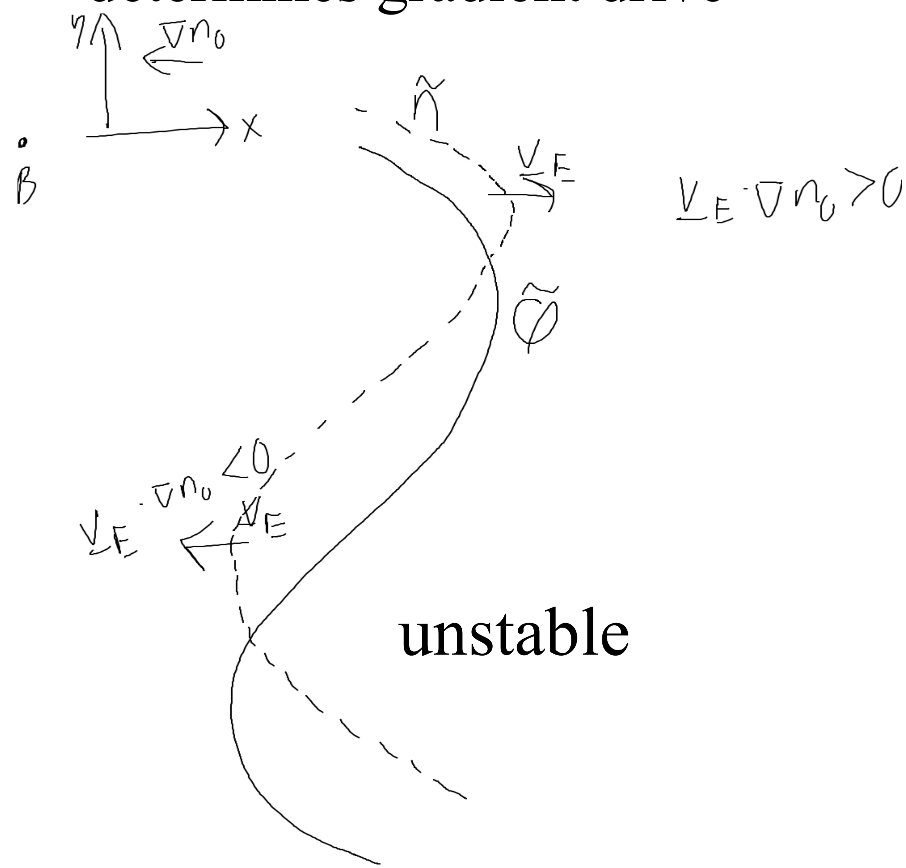
Although small, nonadiabatic fluctuations control the energy balance.

With $k_{||} \neq 0$ and rapid parallel electron response, fluctuations with $\phi \neq n_e$ tend to be small, but are very important because:

- Parallel resistivity $\eta j_{||}$ is the dominant dissipative channel
- Phase shift due to $\phi - n_e$ determines gradient drive



stable



unstable

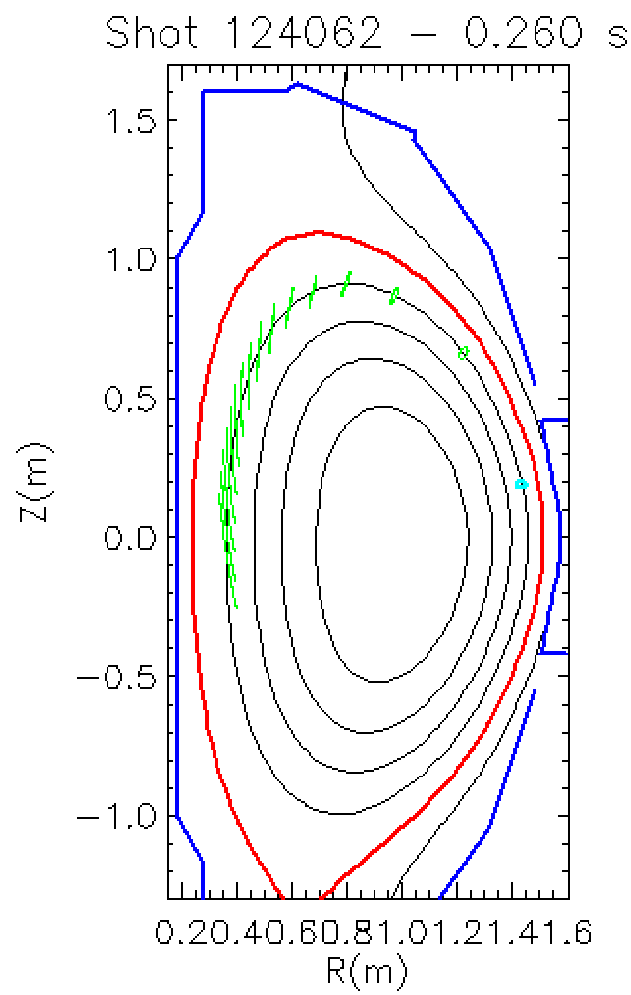
Perpendicular physics drives ϕ and n_e apart but parallel electron motion brings them back together.

- Polarization nonlinearity and curvature drive stir up nonadiabatic fluctuations ($\phi \neq n_e$).
- Ohm's Law dissipates them.
  - If perpendicular scale smaller than resistive skin depth, fluctuations damp due to parallel electron diffusion, either:
    - $D_{||} \sim v_{te}{}^2/\nu_e$ (for $k_\perp \rho_s \gtrsim 1$)
    - effective $D_{||} \sim v_{te}{}^2/\nu_e (k_\perp \rho_s)^2$ ($k_\perp \rho_s \lesssim 1$), due to parallel electron diffusion combined with low-$k_\perp$ Poisson equation
  - If perpendicular scale larger than resistive skin depth, nonadiabatic fluctuations propagate in parallel direction with the Alfvén speed.
- Balance of nonadiabatic drive and damping sets turbulent drive

$$\underbrace{\hat{\beta}\partial_t \tilde{A}_{||}}_{\text{induction}} + \underbrace{\hat{\mu} d_t \tilde{j}_{||}}_{e^- \text{ inertia}} + \underbrace{C \tilde{j}_{||}}_{\| \text{ resistivity}} = \underbrace{\nabla_{||}\left(p_{e0} + \tilde{p}_e - \tilde{\phi}\right)}_{\text{Sourced by } \perp \text{ drifts}}$$

X-point magnetic structure limits parallel coupling.

- Curvature effects and rapid parallel coupling suggestive of ballooning transform
- However, near-separatrix magnetic geometry dominated by X-point
  - Flutelike perturbations strongly constrained
- Extended parallel correlation ($k_{||} < 1/Rq$) extremely unlikely

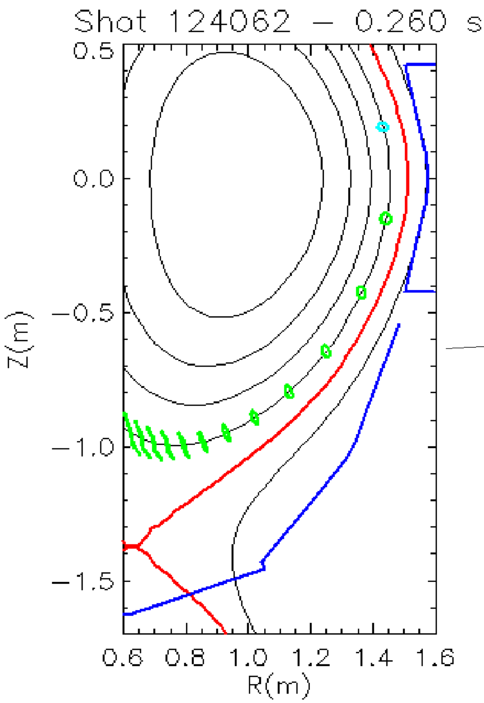


See J. R. Myra and D. A. Dippolito, Phys. Plasmas **12**, 029511 (2005)

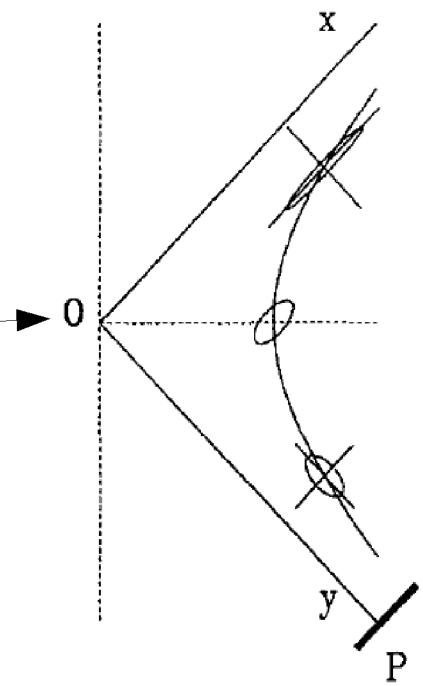
R. J. Maqueda, private communication.

A simple explicit model approximates X-point magnetic mapping.

- By Taylor-expanding $\mathbf{B}_{pol}$ about the poloidal field null, obtain approximate magnetic mapping of flux tubes→exponential contraction parallel to one separatrix and expansion relative to the other. See D. Farina et al, Nucl. Fusion **33**, 1315 (1993).
- Perpendicular wave number enhanced by up to d/Δ, where d is "X-point region length" (~50cm on NSTX) and Δ is radial separation of perturbation and separatrix at outboard midplane ($\lesssim$5 cm for layer of interest), so $k_\perp$ enhanced by $\gtrsim$10x through X-point region.



R. J. Maqueda, private communication.



D. Farina et al, Nucl. Fusion **33**, 1315 (1993)

With field-line-following coordinates, all model magnetic geometry effects are contained in a few simple operators.

- Transform to field-line-following perpendicular variables.
- Let x, y be field-line labels equal to radial and poloidal positions at the outboard midplane region, which is taken to be shearless, and let z measure toroidal distance.
- Drift frequencies proportional to $\nabla n_e \times \nabla \phi \cdot \hat{z}$ (equiv. $\{n_e, \phi\}$), thus unaffected by the area-preserving field deformation, thus ind. of z.
- Curvature ($\mathcal{K}$), Laplacian (∇^2), and gyroaveraging (Γ_0) operators become z-dependent due to the X-point field structure.

$$\partial_t n_e + \omega_n \partial_y \varphi + \{\varphi, n_e\} = \partial_z j_\parallel - \hat{\beta}\left\{A_\parallel, j_\parallel\right\} + \mathcal{K}(z)(\varphi - n_e)$$

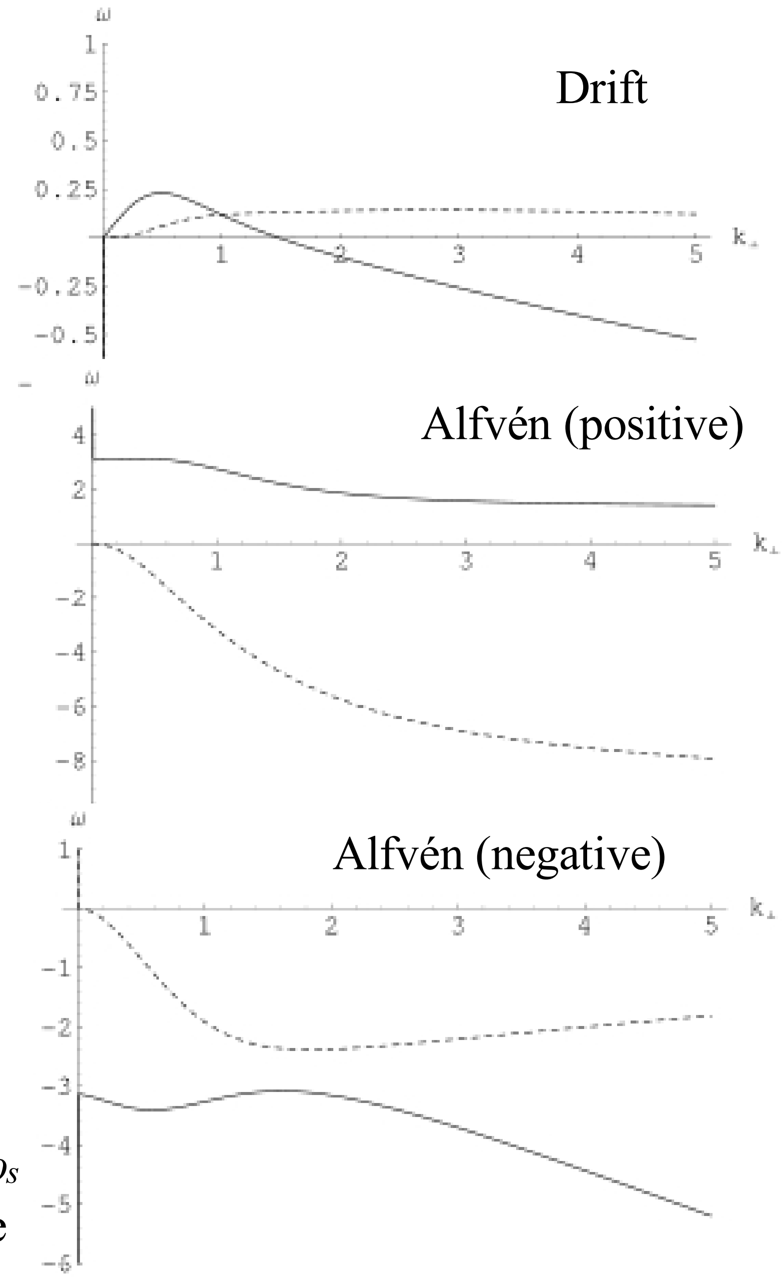
$$\partial_t n_i + \omega_n \partial_y \Gamma_0^{1/2}(z)\varphi + \left\{\Gamma_0^{1/2}(z)\varphi, n_i\right\} = \mathcal{K}(z)\left(\Gamma_0^{-1/2}(z)(\tau_i n_e + \varphi)\right)$$

$$\hat{\beta}\partial_t A_\parallel + \hat{\mu}\left(\partial_t j_\parallel + \{\varphi, j_\parallel\}\right) + C j_\parallel = \partial_z (n_e - \varphi) - \hat{\beta}\left\{A_\parallel, n_e - \varphi\right\} - \hat{\beta}\omega_n \partial_y A_\parallel$$

$$\Gamma_0^{1/2}(z)\, n_i + \frac{\Gamma_0(z) - 1}{\tau_i}\varphi = n_e \qquad -\nabla_\perp^2(z)\, A_\parallel = j_\parallel$$

Drift wave is unstable, but Alfvén waves are strongly damped.

- Neglecting the parallel variation of the magnetics, wave behavior can be determined with a homogeneous approximation (i.e. a $k_{||}$)
- Gyrofluid formulation robust for arbitrary perpendicular wave number.
- Parallel resistivity and parallel electron Landau damping are the only dissipation included in these linear equations.
- Drift wave unstable for all $k_\perp$ (but would be stabilized by curvature-driven phase mixing for modestly large $k_\perp$)
- Alfvén waves stable for all $k_\perp$, heavily damped by parallel resistivity and electron Landau damping for $k_\perp \rho_s > 1$ ($\eta_A \sim O(c_s/L_p)$)



Frequencies in $\omega L_\perp / c_s$, wave numbers in $k_\perp \rho_s$
Solid line is real frequency, dashed is growth rate

Other dissipation mechanisms are weaker than parallel resistivity.

- Through the X-point region, $k_\perp$ grows exponentially, with a total enhancement $\gtrsim$10x (see X-point magnetic model slide).
- With Alfvén waves driven at the outboard midplane and damping into the X-point region, the dissipation that matters most is the one that acts at lowest $k_\perp$, since that is what the wave "sees first."
- At what scales can various dissipation mechanisms' damping rates compete with the resistive and electron Landau damping we calculated for the Alfvén waves, $\eta_A \sim c_S/L_P$ (which is active for $k_\perp \rho_s \gtrsim 1$)?
  - Ion-ion collisions: $k_\perp \rho_i \gtrsim 20$
  - Electron-ion collisions: $k_\perp \rho_i \gtrsim 40$
  - Curvature-driven phase-mixing: $k_\perp \rho_i \gtrsim 15$
  - Nonlinear $E \times B$ phase-mixing: $k_\perp \rho_i \sim 1$?, not more nor less?
    - Since $k_\perp$ varies rapidly with z, this dissipation may act over only a short parallel length, likely causing limited damping.

Linear decomposition to adiabatic/nonadiabatic variables clarifies nature of parallel coupling.

$$\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} \frac{1}{(1+\hat\rho)} & \frac{\hat\rho}{(1+\hat\rho)} \\ -1 & 1 \end{bmatrix} \begin{bmatrix} n_e \\ \varphi \end{bmatrix} \qquad \begin{bmatrix} n_e \\ \varphi \end{bmatrix} = \begin{bmatrix} 1 & -\frac{\hat\rho}{1+\hat\rho} \\ 1 & \frac{1}{1+\hat\rho} \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix}$$

- Polarization effects included in a nonnegative z-dependent operator $\hat\rho\,(z) \doteq \dfrac{1-\Gamma_0\,(z)}{\tau_i} \underset{\tau_i\to 0}{\longrightarrow} -\nabla_\perp^2\,(z)$

$$\partial_t\,(1+\hat\rho)\,a + \omega_n\partial_y\Gamma_0\left(a+\frac{1}{1+\hat\rho}b\right) + \Gamma_0^{1/2}\left(\left\{\Gamma_0^{1/2}a, \Gamma_0^{-1/2}\left(1+\frac{1}{\tau_i}\right)a\right\} + \left\{\frac{\Gamma_0^{1/2}}{1+\hat\rho}b, \Gamma_0^{-1/2}\,(1+\hat\rho)\,a\right\}\right) - \mathcal{K}\left((1+\tau_i)\,a + \frac{\Gamma_0}{1+\hat\rho}b\right) = 0$$

$$\partial_t b - (1+\tau_i)\,\omega_n\partial_y\left(a+\frac{1}{1+\hat\rho}b\right) + \mathcal{K}\left(-\frac{1+\tau_i}{\hat\rho}a + \left(1+\frac{1+\tau_i}{1+\hat\rho}\right)b\right)\cdots$$

$$= -\frac{1+\hat\rho}{\hat\rho}\left(\partial_z j_\parallel - \hat\beta\left\{A_\parallel, j_\parallel\right\}\right) - \left[\frac{1}{\hat\rho}\Gamma_0^{1/2}\left(\left\{\Gamma_0^{1/2}a, \Gamma_0^{-1/2}\left(1+\frac{1}{\tau_i}\right)a\right\} + \left\{\frac{\Gamma_0^{1/2}}{1+\hat\rho}b, \Gamma_0^{-1/2}\,(1+\hat\rho)\,a\right\}\right) + \frac{1+\hat\rho}{\hat\rho}\{a,b\} + \frac{1+\hat\rho}{\hat\rho}\left\{b, \frac{\hat\rho}{1+\hat\rho}b\right\}\right]$$

$$\hat\beta\partial_t A_\parallel + \hat\mu\left(\partial_t j_\parallel + \left\{a+\frac{1}{1+\hat\rho}b, j_\parallel\right\}\right) + Cj_\parallel - \hat\beta\left\{A_\parallel, b\right\} + \hat\beta\omega_n\partial_y A_\parallel = -\partial_z b$$

- The transformation neatly separates out the parallel coupling, since the adiabatic variable a has does not excite or respond to parallel current.
- While b's equation has a complicated form, experience with an analogous cold-ion, $k_{||}=1/Rq$ model suggests that b will be amenable to approximate treatment. We expect it to work in this case too since, neglecting the nonlinear electron inertia term, the order unity variables a and $j_{||}$ are now coupled only via the expected-small variable b.

Energy balance must hold for *a* in each perpendicular plane...

- Since *a*'s equation contains no parallel coupling term, *a*'s evolution must satisfy energy balance separately in each perpendicular plane.

- Formally employing periodicity or spatial homogeneity in *x* and *y*, it is possible to construct an energy equation for *a* by multiplying *a*'s evolution equation by *a* and integrating over the periodic box in *x* and *y* (integral denoted as $\langle \cdot \rangle$)

$$\partial_t E_a(z,t) = \underbrace{\omega_n \left\langle \frac{b}{1+\hat{\rho}} \partial_y \Gamma_0 a \right\rangle}_{\text{gradient drive}} - \underbrace{\left\langle \frac{b}{1+\hat{\rho}} \Gamma_0^{1/2} \left\{ \Gamma_0^{-1/2} \left(1 + \frac{1}{\tau_i}\right) a, \Gamma_0^{1/2} a \right\} \right\rangle}_{\text{nonlinear polarization drift transfer to } b} - \underbrace{\left\langle \frac{b}{1+\hat{\rho}} \mathcal{K}\left(\Gamma_0 a\right) \right\rangle}_{\text{curv. transfer to } b}$$

The "adiabatic energy" $E_a(z)$ is a nonnegative functional of *a* that may be written in Fourier space as

$$E_a = \frac{1}{2} \sum_{ij} \left(1 + \hat{\rho}\left(\mathsf{k}_{ij}, z\right)\right) \left|\hat{a}_{ij}\right|^2$$

and that reduces in the cold-ion limit to a Hasegawa-Mima-like energy

$$\tau_i \to 0: \quad E_a \to \frac{1}{2} \sum_{ij} \left(1 + k_\perp^2\right) \left|\hat{a}_{ij}\right|^2 = \frac{1}{2} \left\langle a^2 + \left|\nabla_\perp a\right|^2 \right\rangle$$

...which constrains the parallel extent of *a*.

•By comparison of the turbulent drive (ω_n) and the curvature transfer to nonadiabatic fluctuations ($\mathcal{K} \approx \omega_B \exp\left(z/L_s\right)\partial_y$, with L_s the X-point magnetic shearing length), we find *a*'s energy loss to *b* through curvature coupling surpasses the gradient drive when

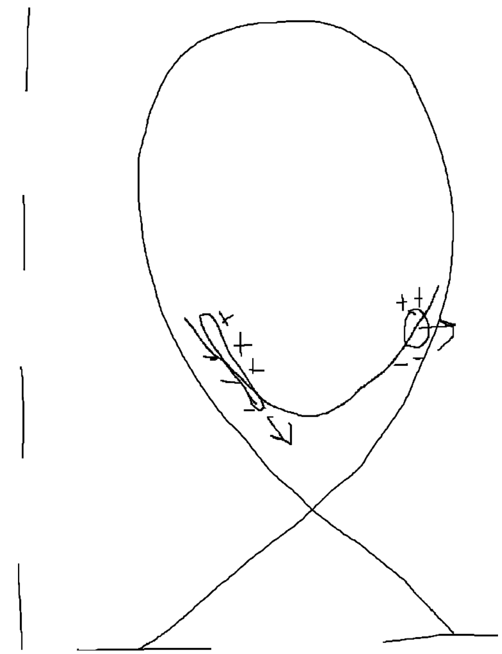
$$\frac{\omega_n \left\langle \frac{b}{1+\hat{\rho}} \partial_y \Gamma_0 a \right\rangle}{\left\langle \frac{b}{1+\hat{\rho}} \mathcal{K}\left(\Gamma_0 a\right) \right\rangle} \approx \frac{\omega_n \left\langle \frac{b}{1+\hat{\rho}} \partial_y \Gamma_0 a \right\rangle}{\left\langle \frac{b}{1+\hat{\rho}} \omega_B e^{z/L_s} \partial_y \Gamma_0 a \right\rangle} = \frac{\omega_n}{\omega_B e^{z/L_s}} = \frac{\omega_n}{\omega_B} e^{-z/L_s} < 1 \qquad \frac{\omega_n}{\omega_B} = \frac{R}{2L_p} \sim 10$$

•Beyond this *z* (towards the end of the X-point region), adiabatic fluctuations may only survive by absorbing energy from *b*, acting as part of the dissipation mechanism rather than a driver

- •Outgoing parallel current acts as an energetic sink for *b*
- •Strong shear does not allow efficient coupling to fluctuations one poloidal transit down the field line

•Heuristic, dubious energy arguments suggest that *a*'s parallel envelope behaves as:

non-rigorous: $a_{\|} \sim \Gamma_0^{1/2}\left(\omega_n - \omega_B e^{z/L_s}\right)$



Although curvature coupling remains favorable for a flutelike density perturbation through the X-point region, eventually more energy is lost through curvature coupling than gained from the density gradient.

How get from here to the desired closure?

- An approximate dynamical treatment of the competition of *a*'s tendency to develop nontrivial parallel structure with *b*'s tendency to encourage flutelike structure should yield a parallel envelope for *a*.
  - In some ranges of perpendicular wave vector, significant nontrivial parallel structure may be inevitable—this will likely cause both enhanced dissipation and, through the resulting n_e,ϕ phase shift, gradient drive.
- With a parallel form for *a* and using the approximation of rapid parallel timescales for *b*, we may calculate an approximate closure for *b* as a function of *a*, as well as a parallel envelope for *b*.
  - Perpendicular "sourcing" of *b* from *a* is balanced against dissipation and "end losses" to the inboard midplane.
- *A priori* estimates, such as the smallness of perpendicular collisional effects, should then be verified for the full closure.

Summary

- Rapid parallel electron motion and X-point geometry provide strong constraints on the parallel form of turbulent fluctuations in the tokamak edge.
- Careful decomposition of fluctuations exposes the structure of the parallel coupling.
- Energy balance limits the extent of adiabatic fluctuations a in the parallel direction.
- Specific steps are planned to obtain an approximate closure for b in terms of a, to yield a 2D, 1-field model that realistically models the tokamak edge but is also amenable to analytic treatment.

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