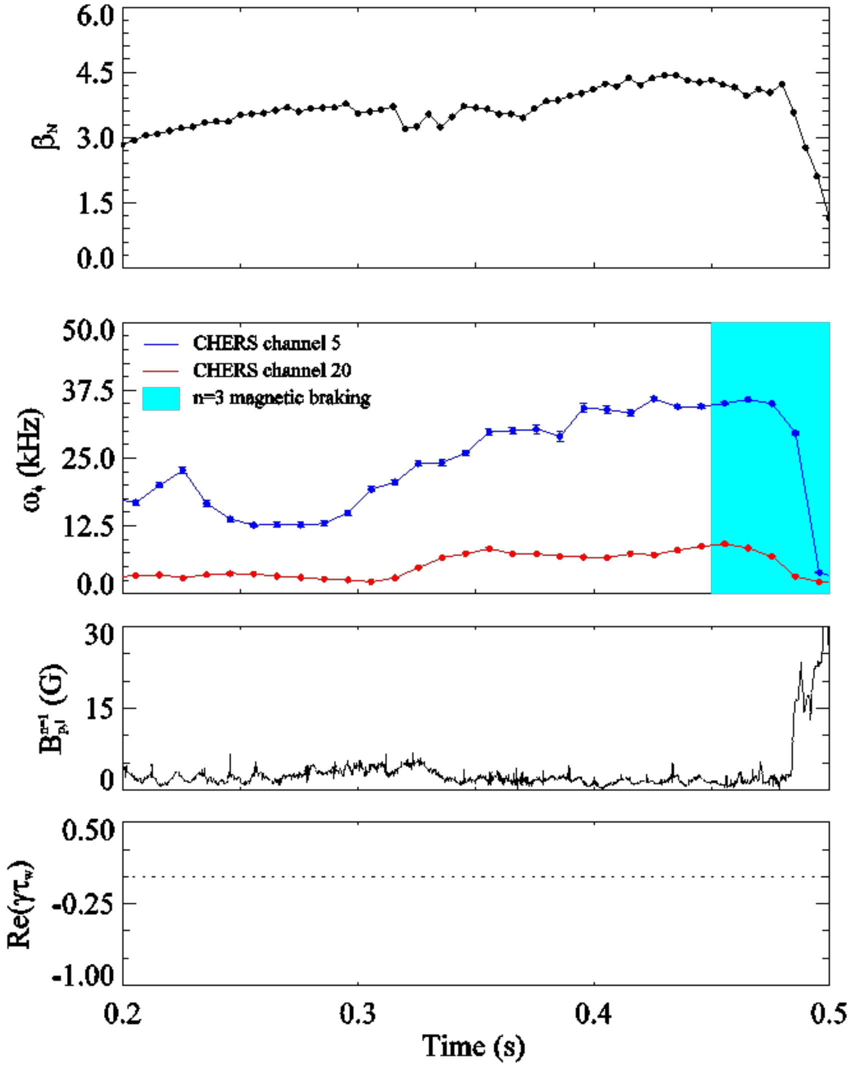
Resistive Wall Mode stabilization in NSTX may be explained by kinetic theory

- Motivation
  - Stable operation of future reactors requires stabilization of the Resistive Wall Mode (RWM).
  - The relationship between plasma rotation and RWM stability in NSTX is more complex than simple models suggest.
- Outline
  - Recently a theory of kinetic stabilization of the RWM was proposed, that has the potential to explain the more complex relationship.
  - Kinetic theory matches NSTX experimental stability evolution.
  - The MISK code has been benchmarked with MARS-K.
  - A DIII-D case indicates the importance of hot ions.

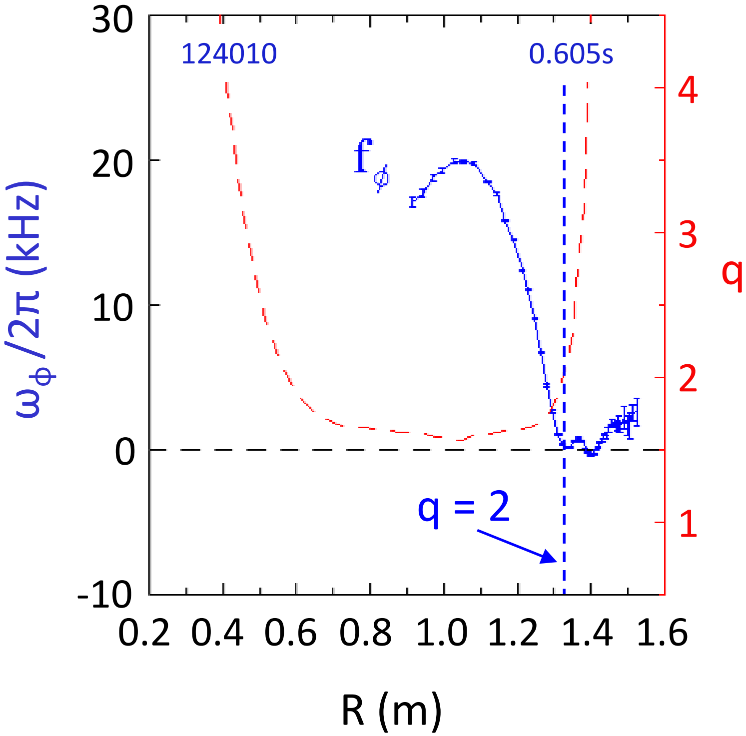
RWM can be experimentally unstable in NSTX

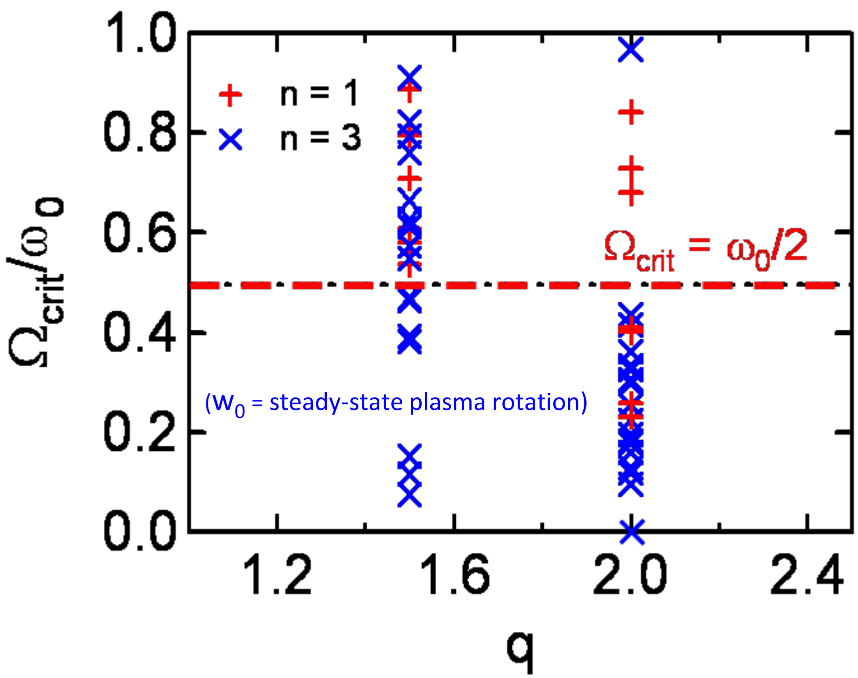


- RWM observed in NSTX on magnetic diagnostics at the time of β and ω_ϕ collapse.
  - Does kinetic theory predict that the RWM growth rate becomes positive at this time?
  - We will calculate $\gamma\tau_w$, the normalized kinetic growth rate, with the MISK code.

Non-resonant magnetic braking is used to probe RWM stabilization physics

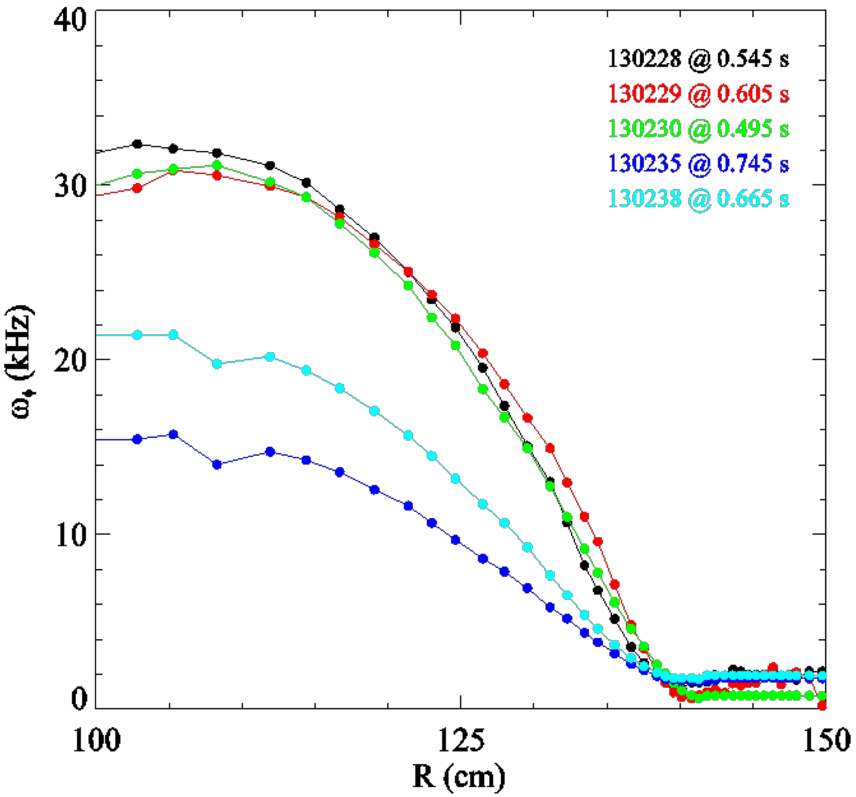
- Scalar plasma rotation at q = 2 inadequate to describe stability.
  - Marginal stability, $\beta_N > \beta_N^{no\text{-}wall}$, with $\omega_\phi^{q=2} = 0$
- Ω_{crit} doesn't follow simple $\omega_0/2$ rotation bifurcation relation.





(Sontag, NF, 2007)

The relationship between rotation and RWM stability is not straightforward in NSTX



Examples of plasma rotation profiles at the time of RWM instability.

- RWM observed in NSTX with a variety of plasma rotation profiles.
  - Including with w = 0 at the q = 2 surface.
  - This does not agree with “simple” theories that predict a “critical” rotation.
  - What can kinetic theory tell us about the relationship between rotation and stability?

The Modifications to Ideal Stability by Kinetic Effects (MISK) code

- Written by Bo Hu, University of Rochester
  - Hu, Betti, PRL, 2004 and Hu, Betti, and Manickam, POP, 2005
- Uses a perturbative calculation, with marginal stability eigenfunction from the PEST code.
- Calculation of δW_K includes the effects of:
  - Trapped Ions
  - Trapped Electrons
  - Circulating Ions
  - Alfven Layers
  - Hot Ions

$$\gamma_K \tau_w = -\frac{\delta W_\infty + \delta W_K}{\delta W_b + \delta W_K}$$

PEST MISK

(Hu, Betti, and Manickam, PoP, 2005)

The Kinetic Approach to δW

$$mn\frac{d\mathbf{u}}{dt} = \mathbf{j}\times\mathbf{B} - \nabla\cdot\mathbf{P}$$ starting with a momentum equation…

$$\omega^2\frac{1}{2}\int mn|\boldsymbol{\xi}|^2 d\mathbf{V} = \frac{1}{2}\int \boldsymbol{\xi}^*\cdot\left(\tilde{\mathbf{j}}\times\mathbf{B} + \mathbf{j}\times\tilde{\mathbf{B}} - \nabla\tilde{p}_F - \nabla\tilde{p}_K\right)d\mathbf{V} \qquad \omega^2 K = \delta W_F + \delta W_K$$

…splitting into fluid and kinetic pressures $$\delta W_K = -\frac{1}{2}\int \boldsymbol{\xi}^*\cdot\nabla\tilde{p}_K d\mathbf{V}$$

Volume

Pitch Angle

Energy

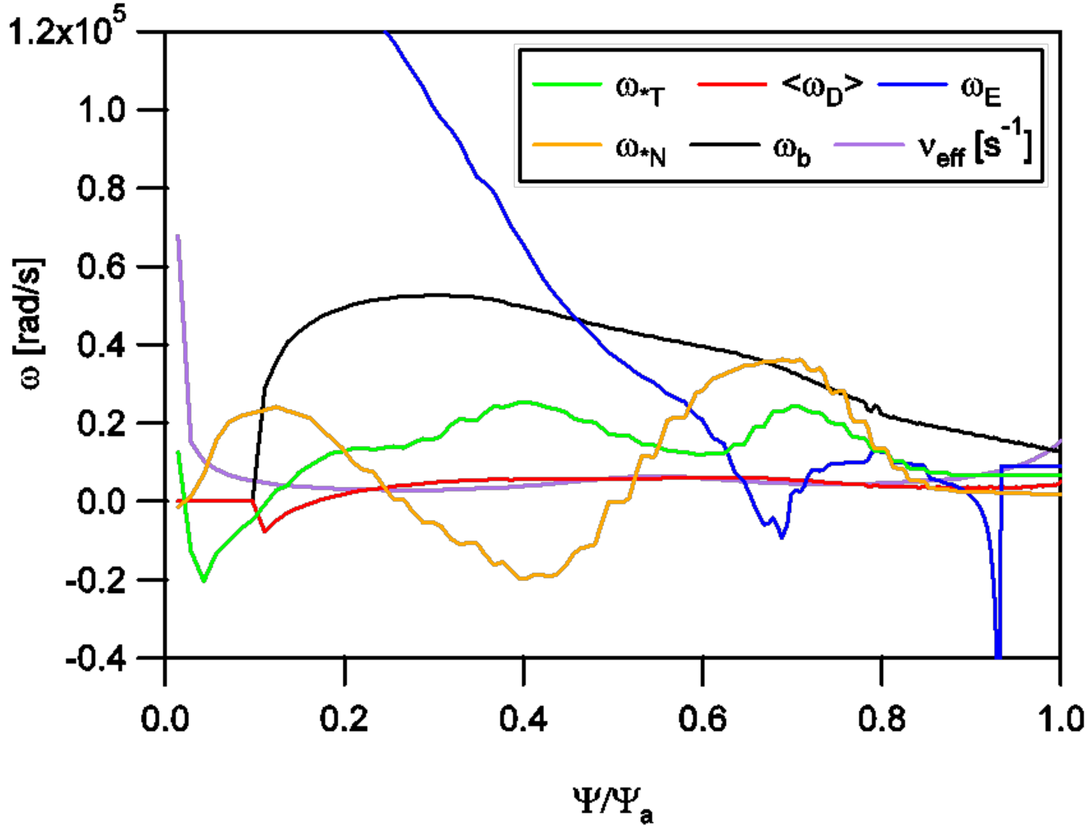
For trapped ions:

$$\delta W_K^{ti} = \int_0^{\Psi_a}\frac{d\Psi}{B_0}\left(\frac{p}{1+\frac{T_e}{T_i}}\right)\frac{\sqrt{\pi}}{2}\sum_{l=-\infty}^{\infty}\int_{B_0/B_{max}}^{B_0/B_{min}} d\Lambda\hat{\tau}_b$$
$$\times\int_0^\infty\left[\frac{\omega_{*N} + \left(\hat{\varepsilon}-\frac{3}{2}\right)\omega_{*T} + \omega_E - \omega - i\gamma}{\langle\omega_D\rangle + l\omega_b - i\nu_{\text{eff}} + \omega_E - \omega - i\gamma}\right]\hat{\varepsilon}^{5/2}e^{-\hat{\varepsilon}}d\hat{\varepsilon}$$
$$\times\left\langle\left(2 - 3\frac{\Lambda}{B_0/B}\right)(\boldsymbol{\kappa}\cdot\boldsymbol{\xi}_\perp) - \left(\frac{\Lambda}{B_0/B}\right)(\nabla\cdot\boldsymbol{\xi}_\perp)\right\rangle^2$$

(Hu, Betti, and Manickam, PoP, 2006)

Stabilization arises from the resonance of various plasma frequencies

Frequency resonance term: $$\delta W_K \propto \int \left[\frac{\omega_{*N} + \left(\hat{\varepsilon} - \frac{3}{2}\right)\omega_{*T} + \omega_E - \omega - i\gamma}{\langle \omega_D \rangle + l\omega_b - i\nu_{\mathrm{eff}} + \omega_E - \omega - i\gamma} \right] \hat{\varepsilon}^{\frac{5}{2}} e^{-\hat{\varepsilon}} d\hat{\varepsilon}$$



The collision frequency shown is for thermal ions, the bounce and precession drift frequencies are for thermal ions and zero pitch angle.

The dispersion relation can be rewritten in a convenient form for making stability diagrams

The kinetic contribution has a real and imaginary part, so:

$$Re(\gamma_K \tau_w) = -\frac{\delta W_\infty \delta W_b + (Im(\delta W_K))^2 + Re(\delta W_K)(\delta W_\infty + \delta W_b + Re(\delta W_K))}{(\delta W_b + Re(\delta W_K))^2 + (Im(\delta W_K))^2}$$

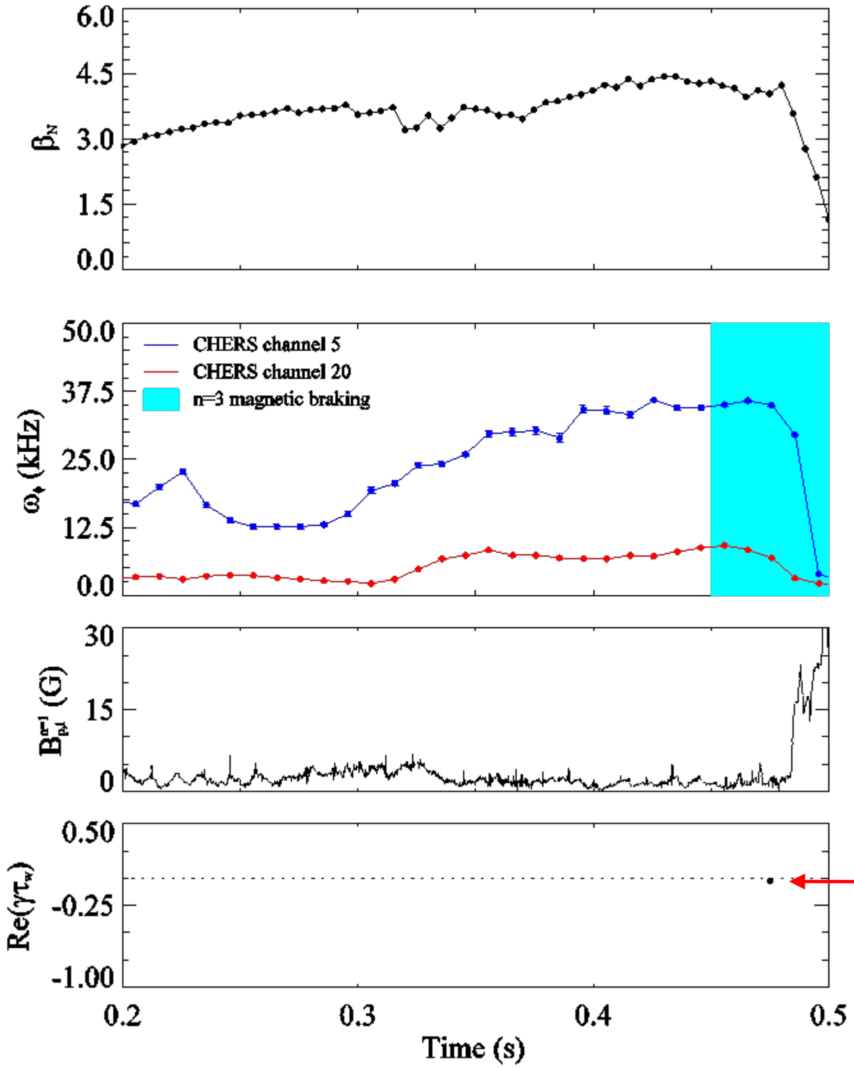
$$\boxed{\left(Re\left(\delta W_K\right) - a\right)^2 + \left(Im\left(\delta W_K\right)\right)^2 = r^2}$$

On a plot of $Im(\delta W_K)$ vs. $Re(\delta W_K)$, contours of constant $Re(\gamma\tau_w)$ form circles with offset a and radius r.

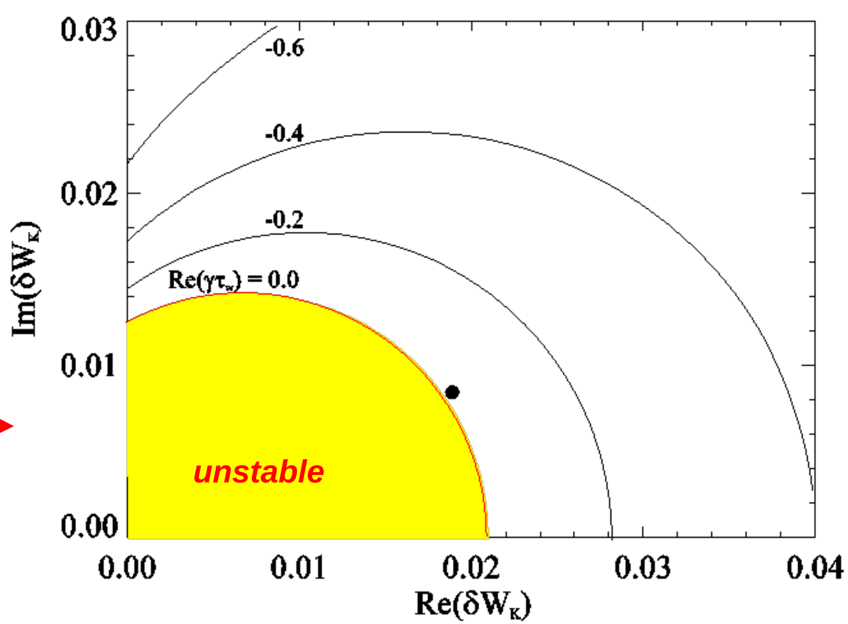
$$a = \frac{1}{2}\left(\delta W_b + \delta W_\infty\right) + \frac{1}{2}\left(\delta W_b - \delta W_\infty\right)\frac{Re(\gamma\tau_w)}{1 + Re(\gamma\tau_w)}$$

$$r = \frac{1}{2}\left(\delta W_b - \delta W_\infty\right)\frac{1}{1 + Re(\gamma\tau_w)}$$

Example calculation: NSTX shot 121083 @ 0.475s



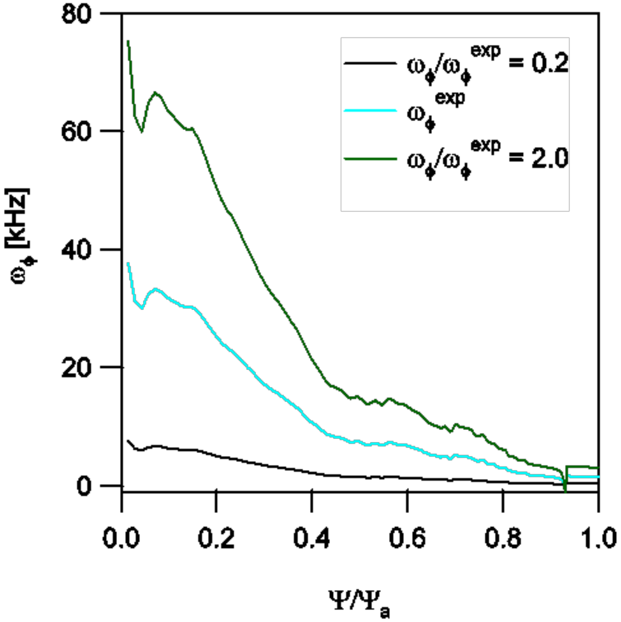
$$\delta W_\infty = -2.09 \times 10^{-2} \quad \text{(PEST)}$$
$$\delta W_b = 7.42 \times 10^{-3} \quad \text{(PEST)}$$
$$Re(\delta W_K) = 1.89 \times 10^{-2} \quad \text{(MISK)}$$
$$Im(\delta W_K) = 8.41 \times 10^{-3} \quad \text{(MISK)}$$



The effect of rotation on stability is more complex with kinetic theory

$$\delta W_K \propto \int \left[\frac{\omega_{*N} + \left(\hat{\varepsilon} - \frac{3}{2}\right)\omega_{*T} + \omega_E - \omega - i\gamma}{\langle \omega_D \rangle + l\omega_b - i\nu_{\text{eff}} + \omega_E - \omega - i\gamma} \right] \hat{\varepsilon}^{\frac{5}{2}} e^{-\hat{\varepsilon}} d\hat{\varepsilon} \qquad \omega_E = \omega_\phi - \omega_{*N} - \omega_{*T}$$

Using self-similarly scaled rotation profiles, we can isolate and test the effect of rotation on stability.



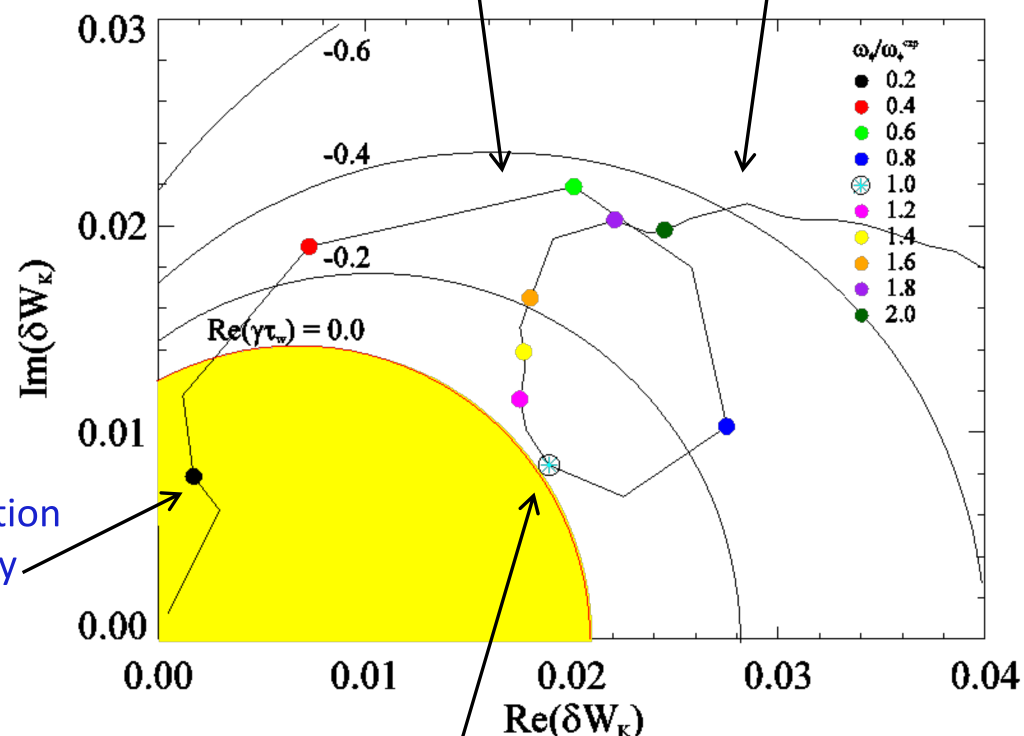
121083

moderate rotation stability

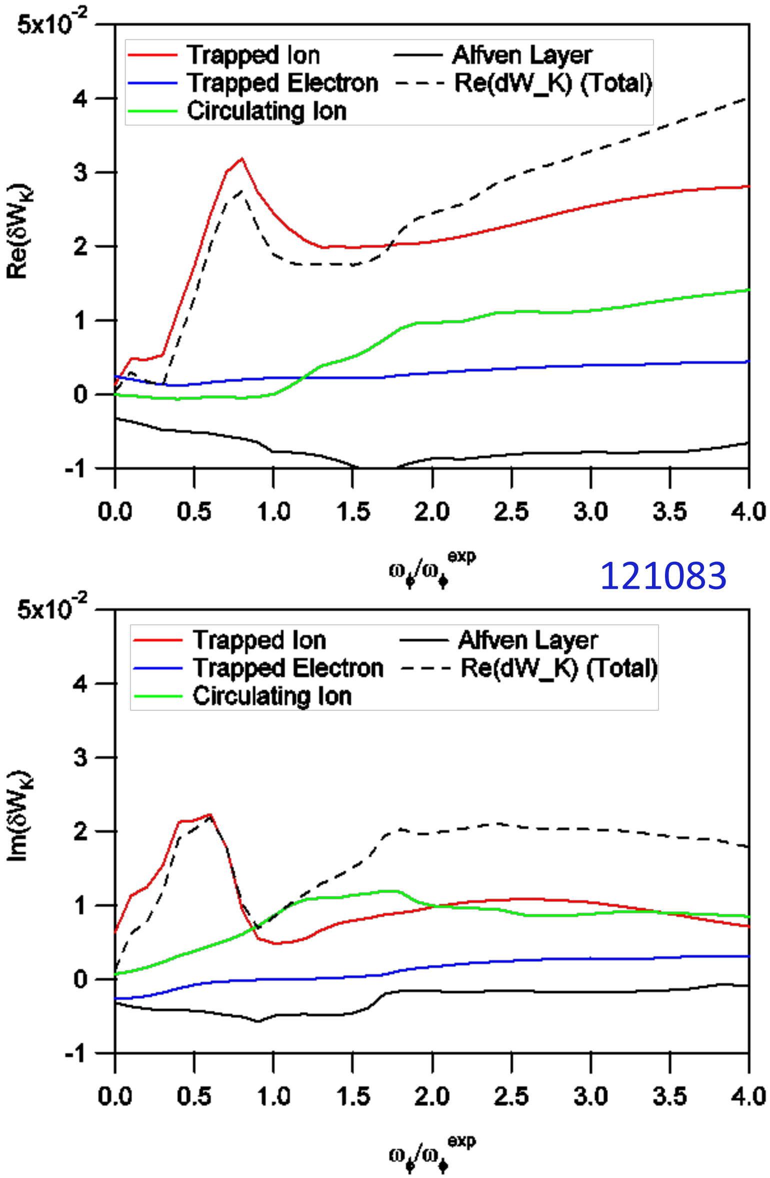
high rotation stability

low rotation instability

marginally stable

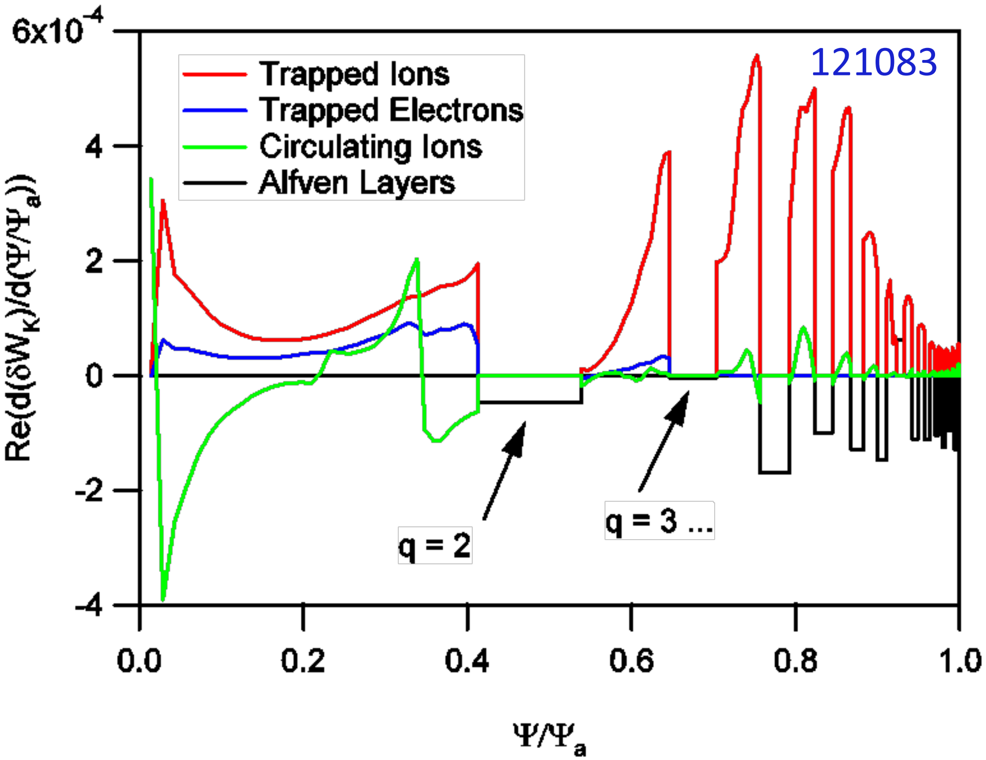


Resonances with multiple particle types contribute to the trends



- For $\omega_\phi/\omega_\phi^{exp}$ from 0 to 0.6 stability increases as the real and imaginary trapped ion components increase.
- From 0.6 to 0.8 the real part increases while the imaginary part decreases, leading to the turn back towards instability.
- For rotation levels above the experimental value, trapped ion and circulating ion components rise, leading to strong stability.

Strong trapped ion stabilization comes from the outer surfaces



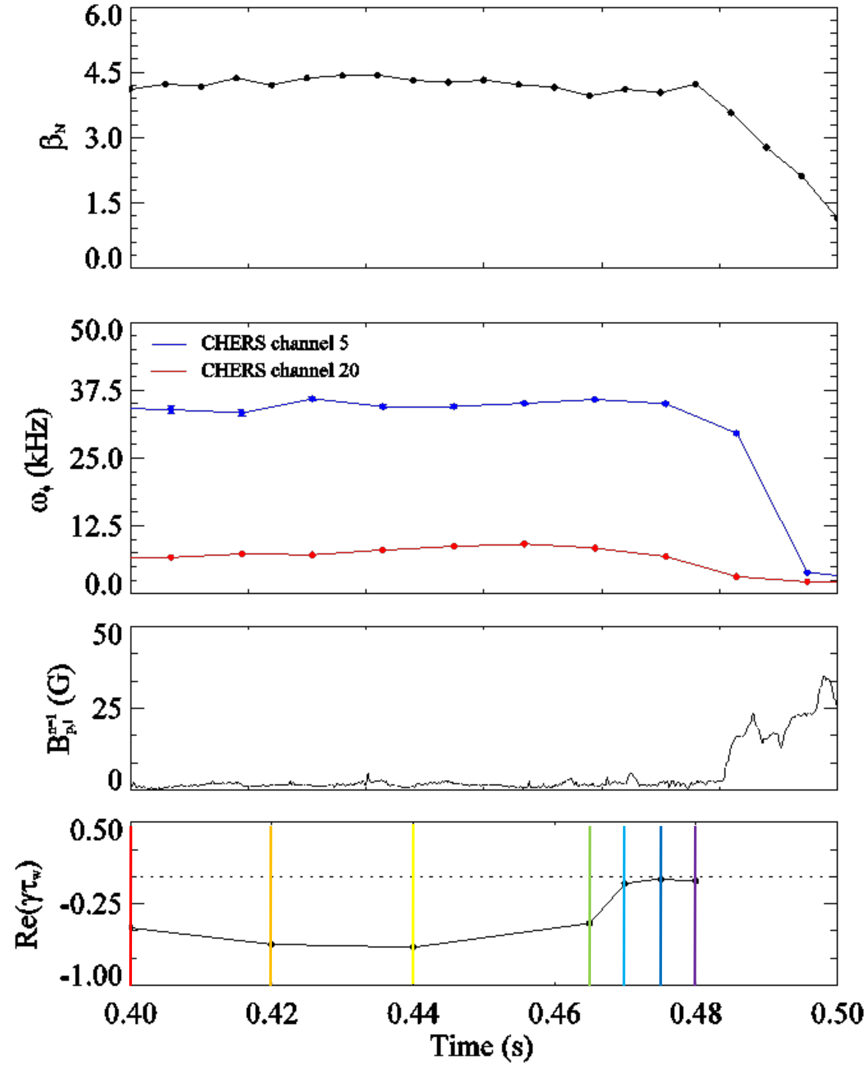
- Contributions to Re(δW_K)
  - A large portion of the kinetic stabilization comes from q>2, where ω_{*N} and $\omega_{*T} \geq \omega_E$.

The flat areas are rational surfaces (integer q ± 0.2) where the contributions have been zeroed out and dealt with analytically through calculation of inertial enhancement by shear Alfven damping.

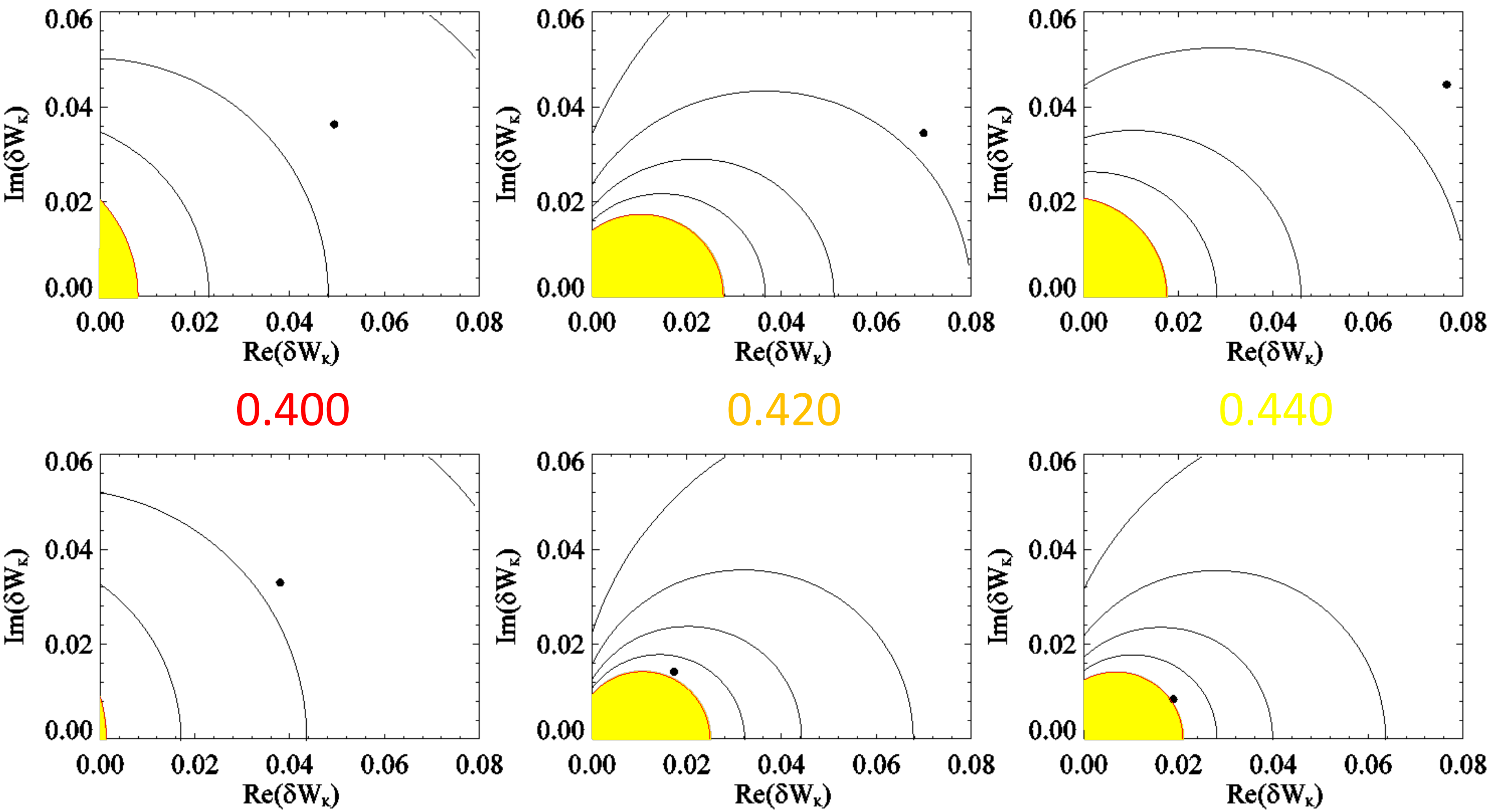
(Zheng et al., PRL, 2005)

Kinetic prediction of instability matches experimental result

- Examining the evolution of a shot
  - For NSTX shot 121083, β and ω_ϕ are relatively constant leading up the the RWM collapse.
  - Calculation of the RWM kinetic growth rate for multiple equilibria shows a turn towards instability just before the RWM.



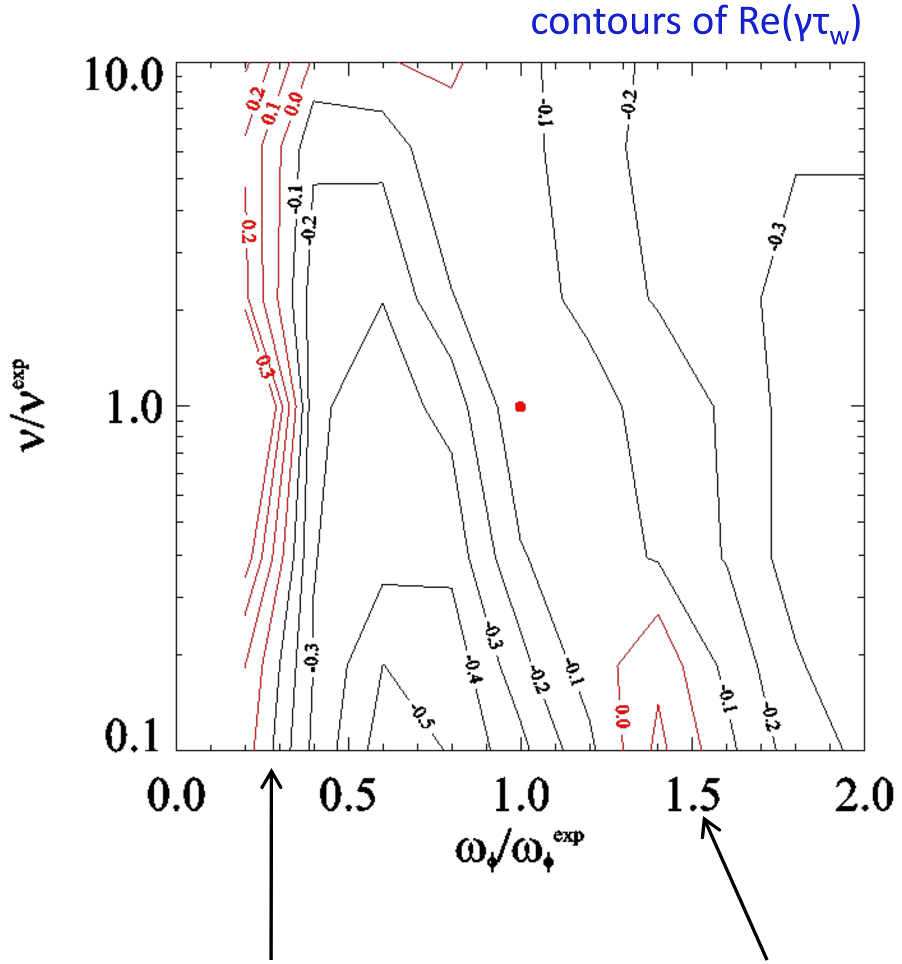
As time progresses the stabilizing δW_K decreases



0.400 0.420 0.440

0.465 0.470 0.475

Isolating and testing the effect of collisionality reveals band of marginal stability



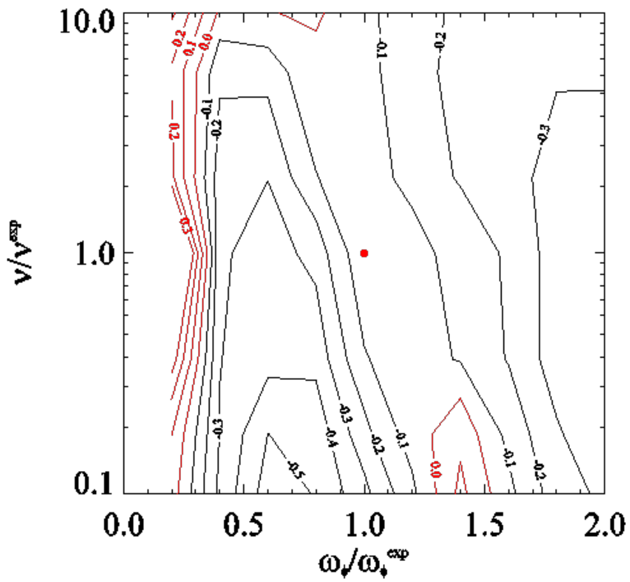
- Density and temperature profiles are self-similarly scaled while keeping β constant (ie, n*2 and T/2 or n/2 and T*2).
- We find a low rotation "critical" threshold.
- However, there is also a band of marginally stable moderate rotation, and it is here that the experiment goes unstable!

Similar to a "critical" rotation threshold.

Band of marginally stable moderate rotation.

Analysis of other NSTX shots shows the same characteristic behavior

121083



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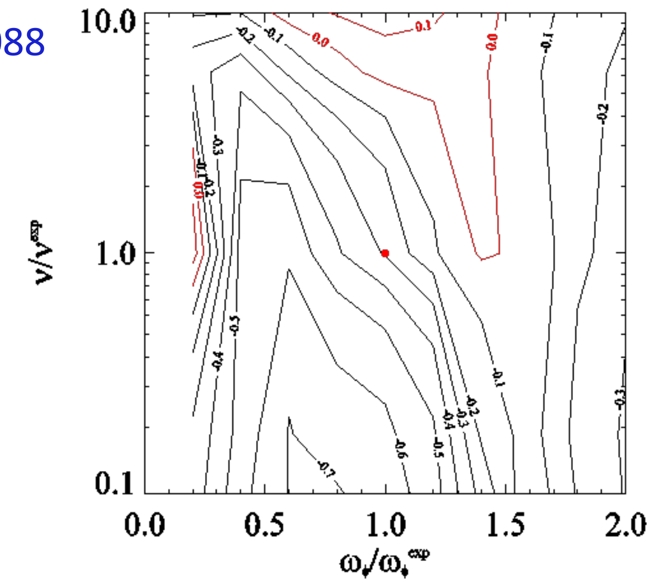

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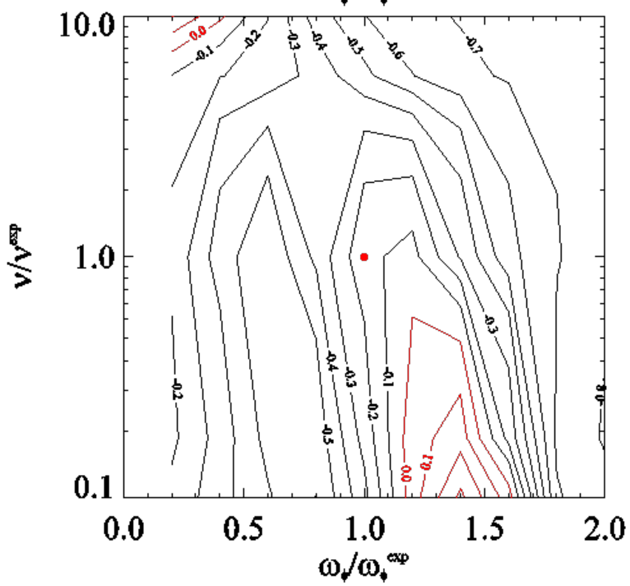

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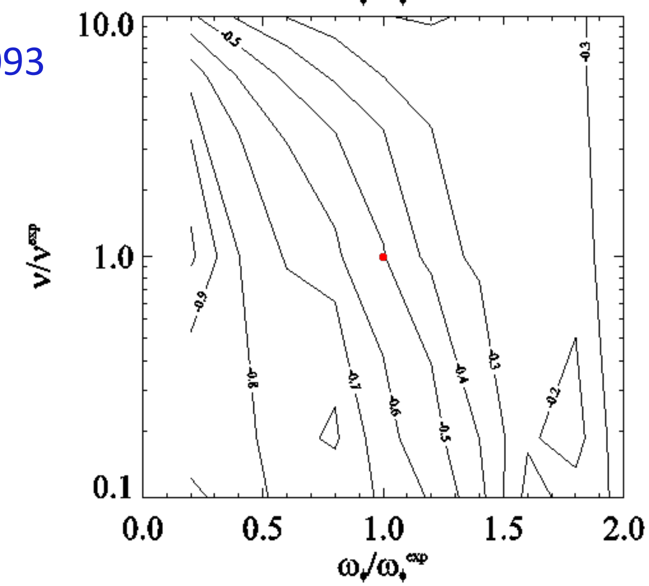
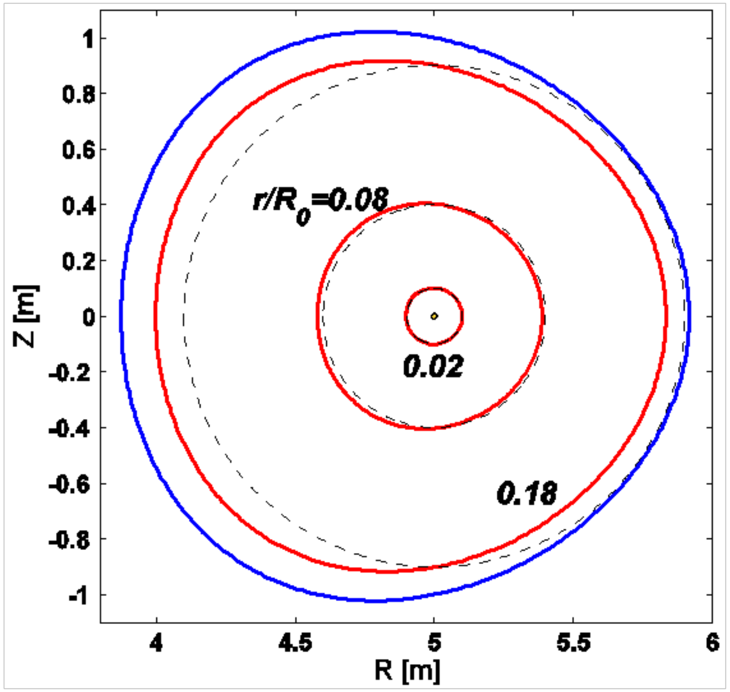


MISK and MARS-K were benchmarked using a Solov'ev equilibrium

$$\mu_0 P(\psi) = -\frac{1+\kappa^2}{\kappa R_0^3 q_0}\psi, \quad F(\psi) = 1$$

$$\psi = \frac{\kappa}{2R_0^3 q_0}\left[\frac{R^2 Z^2}{\kappa^2} + \frac{1}{4}\left(R^2 - R_0^2\right)^2 - a^2 R_0^2\right]$$



(Liu, ITPA MHD TG Meeting, Feb. 25-29, 2008)

– Simple, analytical solution to the Grad-Shafranov equation.

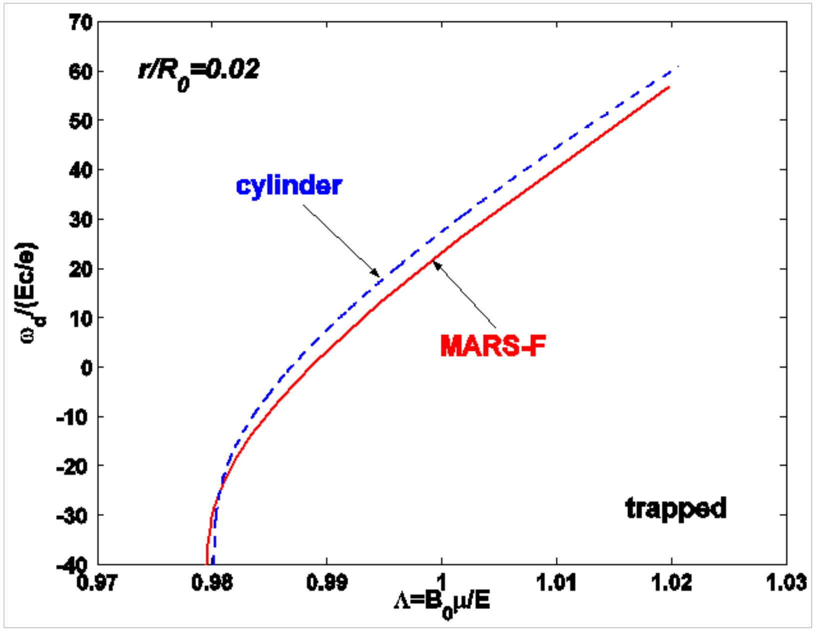
– Flat density profile means $\omega_{*N} = 0$.

– Also, ω, γ, and ν_{eff} are taken to be zero for this comparison, so the frequency resonance term is simply:

$$\delta W_K \propto \int \left[\frac{\left(\hat{\varepsilon} - \frac{3}{2}\right)\omega_{*T} + \omega_E}{\langle\omega_D\rangle + l\omega_b + \omega_E}\right]\hat{\varepsilon}^{\frac{5}{2}} e^{-\hat{\varepsilon}} d\hat{\varepsilon}$$

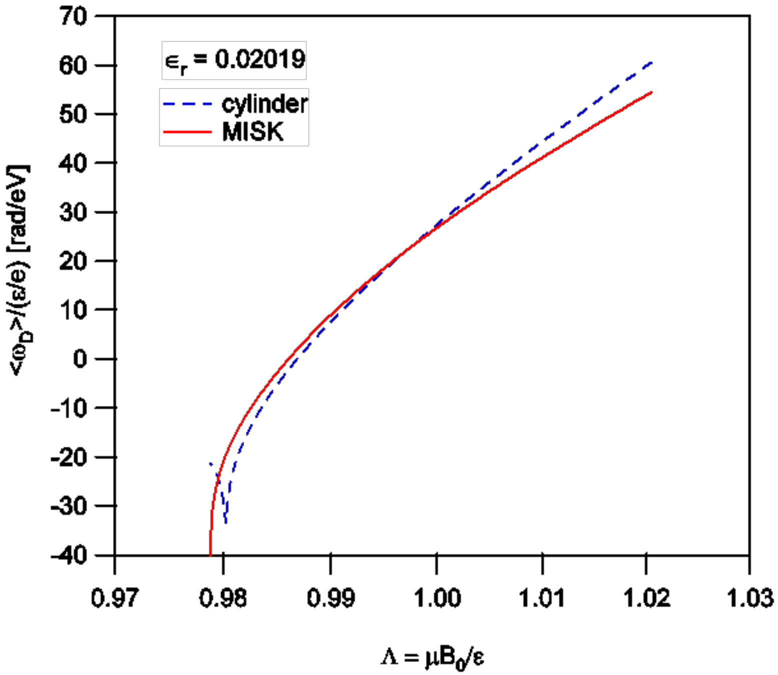
MARS-K: *(Liu, IAEA, 2008) and (Liu, B12.00003, Monday AM)*

Drift frequency calculations match for MISK and MARS-K



MARS

(Liu, ITPA MHD TG Meeting, Feb. 25-29, 2008)



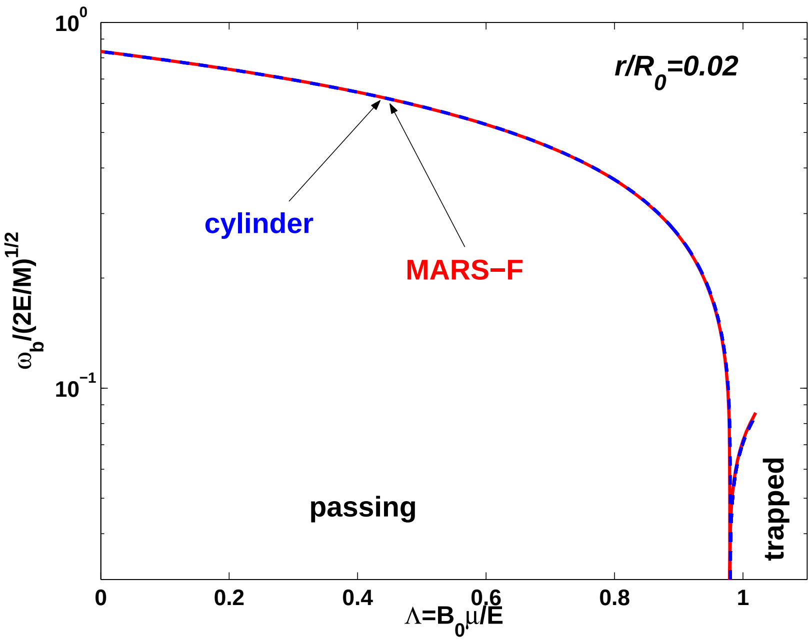
MISK

large aspect ratio approximation *(Jucker et al., PPCF, 2008)*

$$\frac{\langle\omega_D\rangle}{\varepsilon/e} = \frac{2q\Lambda}{R_0^2 B_0 \epsilon_r}\left[(2s+1)\frac{E\left(k^2\right)}{K\left(k^2\right)} + 2s\left(k^2-1\right) - \frac{1}{2}\right] \qquad k = \left[\frac{1-\Lambda+\epsilon_r\Lambda}{2\epsilon_r\Lambda}\right]^{\frac{1}{2}}$$

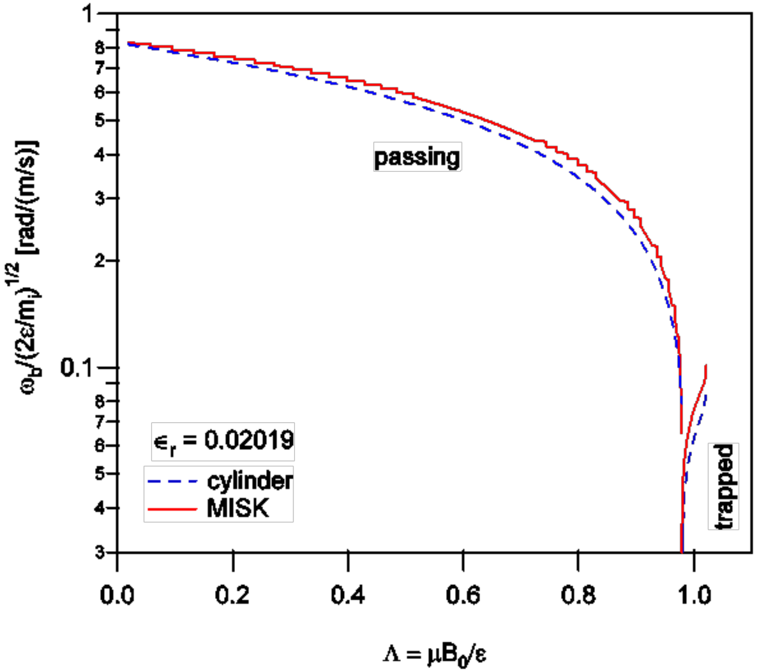
here, ϵ_r is the inverse aspect ratio, s is the magnetic shear, K and E are the complete elliptic integrals of the first and second kind, and $\Lambda = \mu B_0/\varepsilon$, where μ is the magnetic moment and ε is the kinetic energy.

Bounce frequency calculations match for MISK and MARS-K



MARS

(Liu, ITPA MHD TG Meeting, Feb. 25-29, 2008)



MISK

large aspect ratio approximation *(Bondeson and Chu, PoP, 1996)*

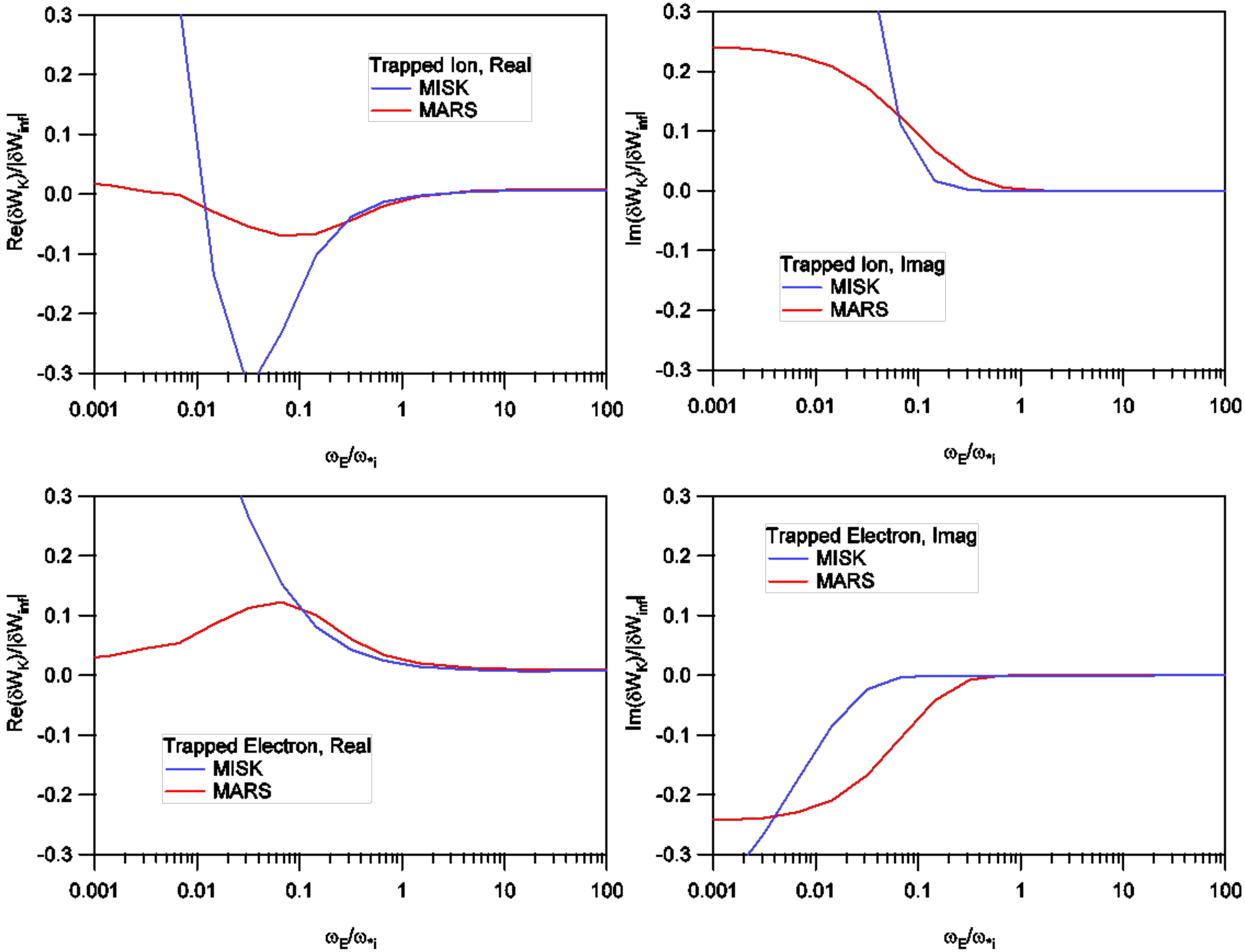
$$\frac{\omega_b}{\sqrt{2\varepsilon/m_i}} = \frac{\sqrt{2\epsilon_r\Lambda}}{4qR_0}\frac{\pi}{K(k)} \quad \text{(trapped)} \qquad \frac{\omega_b}{\sqrt{2\varepsilon/m_i}} = \frac{\sqrt{1-\Lambda+\epsilon_r\Lambda}}{2qR_0}\frac{\pi}{K(1/k)} \quad \text{(circulating)}$$

$$k = \left[\frac{1-\Lambda+\epsilon_r\Lambda}{2\epsilon_r\Lambda}\right]^{\frac{1}{2}}$$

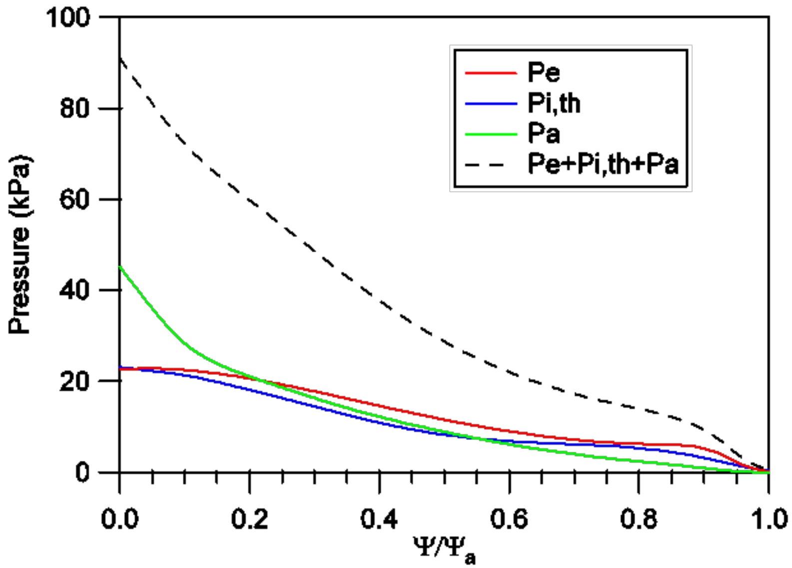
MISK and MARS-K match well at reasonable rotation

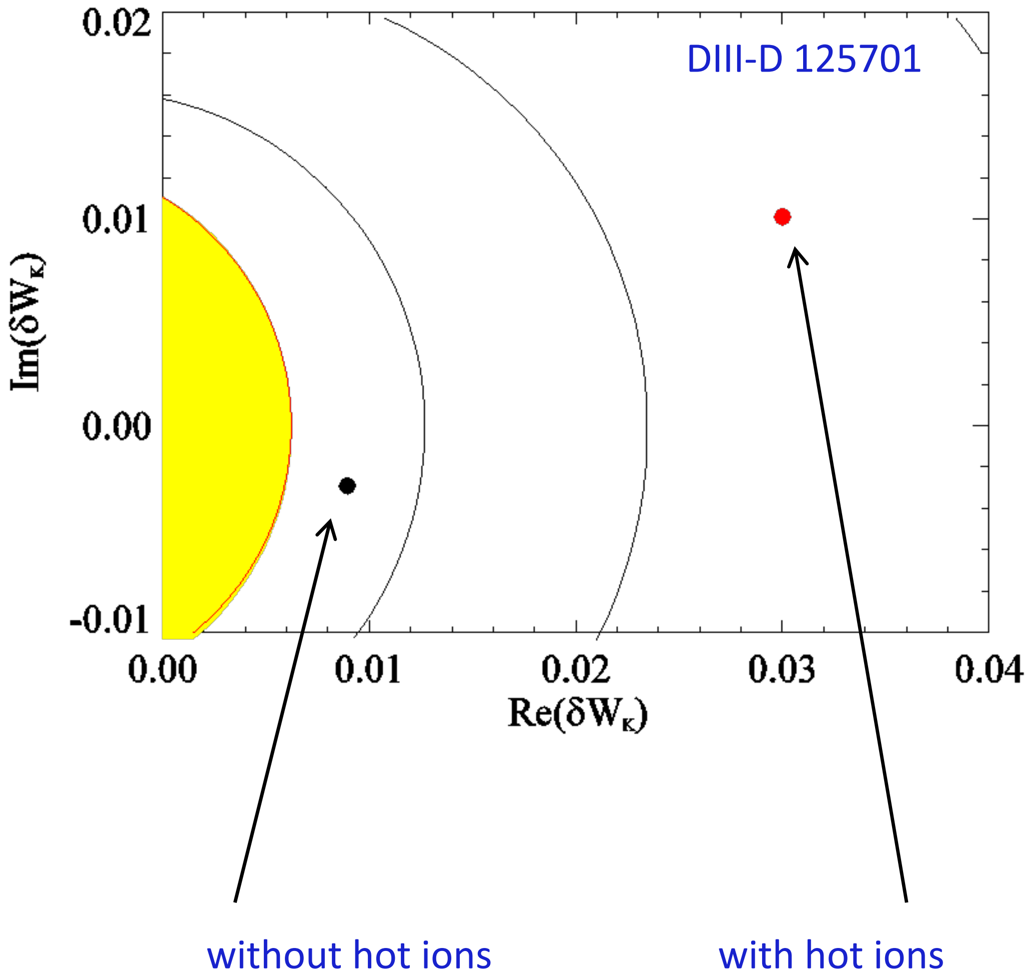
Good match for trapped ions and electrons at high rotation, but poor at low rotation.

The simple frequency resonance term denominator causes numerical integration problems with MISK that don't happen with realistic equilibria.



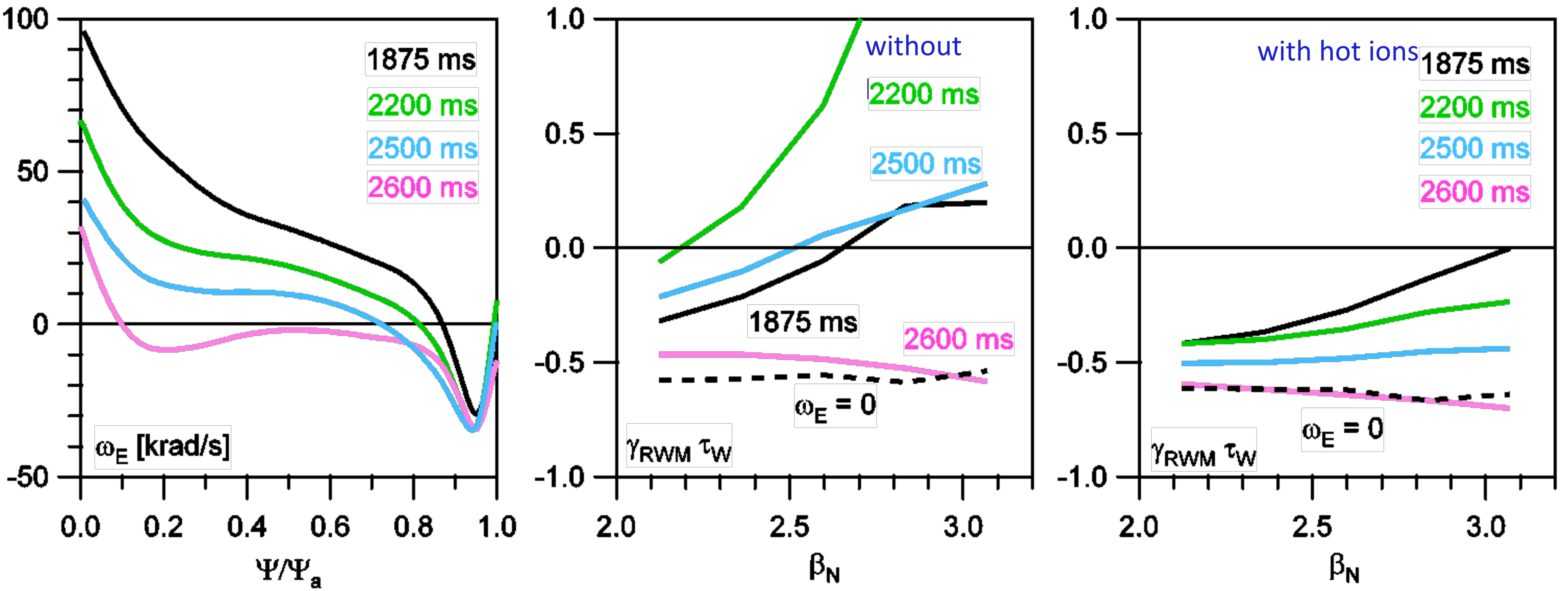
Hot ions contribute a large fraction of β to DIII-D, and have a strong stabilizing effect





– Hot ions not yet implemented for NSTX.

Hot ions have a strongly stabilizing effect for DIII-D

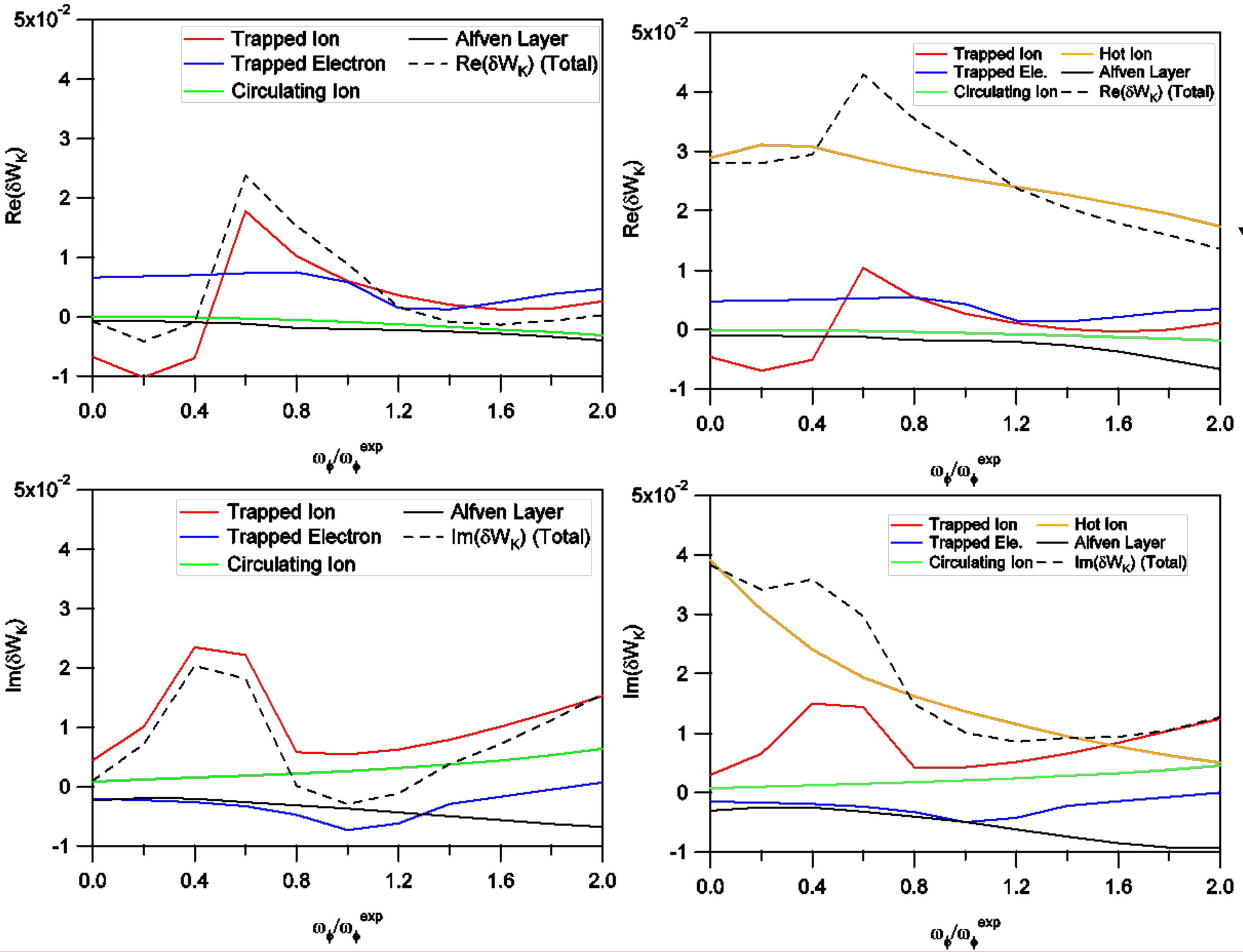


– Using the equilibrium from DIII-D shot 125701 @ 2500ms and rotation from 1875-2600ms, MISK predicts a band of instability at moderate rotation without hot ions, but complete stability with hot ions.

– This could help to explain why DIII-D is inherently more stable to the RWM than NSTX, and possibly why energetic particle modes can "trigger" the RWM.

(Matsunaga et al., IAEA, 2008)

Components of δW_K without and with hot ions



hot ion contribution is large.

Hot ions are included in MISK with a slowing-down distribution function

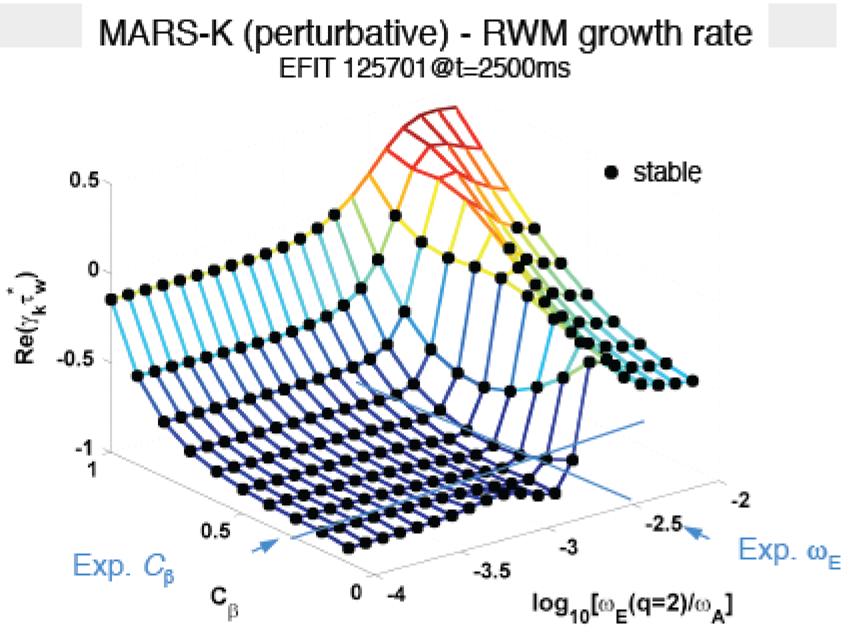
$$\delta W_K^a \propto \left(\int \frac{\hat{\varepsilon}_a^{\frac{1}{2}}}{\hat{\varepsilon}_a^{\frac{3}{2}} + \hat{\varepsilon}_c^{\frac{3}{2}}} d\hat{\varepsilon}_a\right)^{-1} \int \left[\frac{\omega_{*N}^a + \omega_f^a + \frac{1}{\hat{\varepsilon}_a^{\frac{3}{2}}+\hat{\varepsilon}_c^{\frac{3}{2}}} \frac{d}{d\Psi}\hat{\varepsilon}_c^{\frac{3}{2}} + \frac{\frac{3}{2}\hat{\varepsilon}_a^{\frac{1}{2}}}{\hat{\varepsilon}_a^{\frac{3}{2}}+\hat{\varepsilon}_c^{\frac{3}{2}}} Z_a\left(\omega_E^a - \omega - i\gamma\right)}{\langle\omega_D^a\rangle + l\omega_b^a - i\nu_{\mathrm{eff}}^a + Z_a\left(\omega_E^a - \omega - i\gamma\right)}\right] \frac{\hat{\varepsilon}_a^{\frac{5}{2}}}{\hat{\varepsilon}_a^{\frac{3}{2}} + \hat{\varepsilon}_c^{\frac{3}{2}}} d\hat{\varepsilon}_a$$

(Hu, Betti, and Manickam, PoP, 2006)

$$\hat{\varepsilon}_c = \left(\frac{3\sqrt{\pi}}{4}\right)^{\frac{2}{3}} \left(\frac{m_a}{m_i}\right) \left(\frac{m_i}{m_e}\right)^{\frac{1}{3}} \left(\frac{T_e}{\varepsilon_a}\right) \qquad \omega_f^a = \int \frac{\hat{\varepsilon}_a^{\frac{1}{2}}}{\hat{\varepsilon}_a^{\frac{3}{2}} + \hat{\varepsilon}_c^{\frac{3}{2}}} d\hat{\varepsilon}_a \frac{d\left(\left(\int \frac{\hat{\varepsilon}_a^{\frac{1}{2}}}{\hat{\varepsilon}_a^{\frac{3}{2}}+\hat{\varepsilon}_c^{\frac{3}{2}}} d\hat{\varepsilon}_a\right)^{-1}\right)}{d\Psi}$$

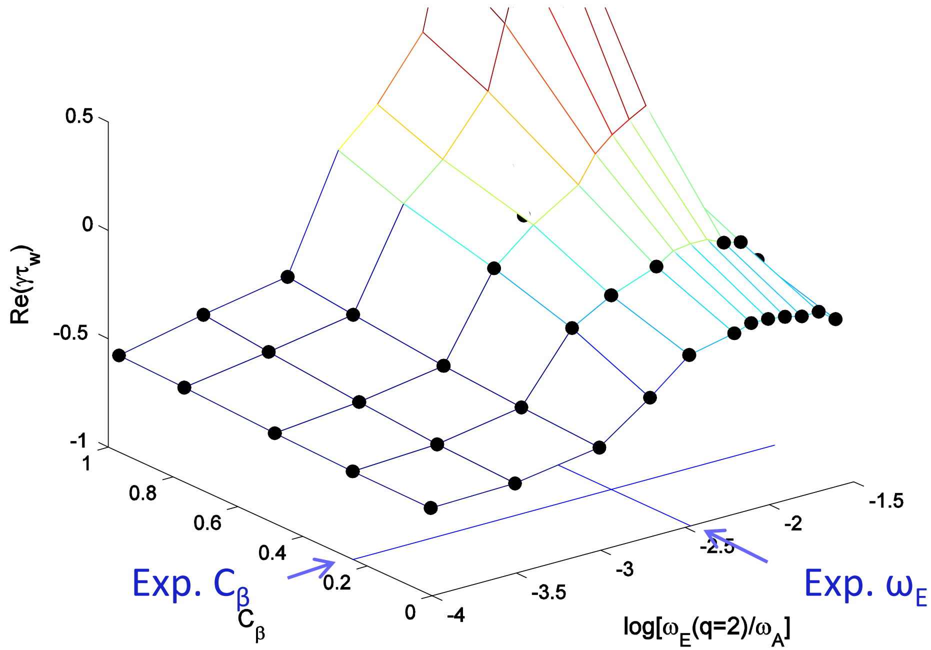
- Profiles of p_a and n_a, calculated by onetwo are used both directly and to find ε_a.
- The hot ion pressure is subtracted from the total pressure for the other parts of the calculation.

MARS-K (perturbative) and MISK produce similar results for the same DIII-D case



MARS

(Reimerdes, PO3.00011, Wed. PM)



MISK

- Results are qualitatively similar. Main differences are unstable magnitude and behavior at low ω_E.
- This case does not include hot ions.

Future work: inclusion of pitch angle dependent collisionality

$$\left[\frac{\partial}{\partial t} + \mathbf{v_g} \cdot \boldsymbol{\nabla}\right] \tilde{f} + \left[\frac{\tilde{\mathbf{F}}}{m}\right] \frac{\partial f}{\partial \mathbf{v}} = C(\tilde{f})$$

Collisionality enters through drift kinetic equation

$$C(\tilde{f}) = \nu_{\text{eff}} \tilde{f}$$

Effective collisionality can be included in various forms:

$$\nu_0 = 0 \text{ (collisionless)}$$

$$\nu_1(\Psi) = \frac{n_i e^4 \ln \Lambda_e}{12\pi^{\frac{3}{2}} \epsilon_0^2 m_i^{\frac{1}{2}} T_i^{\frac{3}{2}} \epsilon_r} \text{ (no energy dependence)}$$

(MARS)

$$\nu_2(\Psi, \varepsilon) = \nu_1 \hat{\varepsilon}^{-\frac{3}{2}} \text{ (simple energy dependence)}$$

(MISK)

$$\nu_3(\Psi, \varepsilon, \Lambda) = 2\nu_1 \left[Z_{\text{eff}} + \frac{1}{\sqrt{\pi \hat{\varepsilon}}} e^{-\hat{\varepsilon}} + \frac{1}{\sqrt{\pi}} \left(2 - \hat{\varepsilon}^{-1}\right) \int_0^{\sqrt{\hat{\varepsilon}}} e^{-t^2} dt \right] \times \frac{\partial}{\partial \Lambda} \left(\Lambda \sqrt{\frac{B_0}{B} - \Lambda} \right) \frac{\partial}{\partial \Lambda} \left(\sqrt{\frac{B_0}{B} - \Lambda} \right) \text{ (Lorentz operator, pitch angle dependence)}$$

(Future?)

(Fu et al., POFB, 1993)

Summary

- Kinetic effects contribute to stabilization of the RWM, and may explain the complex relationship between plasma rotation and stability in NSTX.
- The MISK code is used to calculate the RWM growth rate with kinetic effects, and it has been benchmarked against MARS.
- Results indicate that hot ions contribute greatly to the stability of DIII-D.

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