Comparison of a New Self-Adjoint Calculation of NSTX High Harmonic Current Drive with CQL3D

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GENRAY ADJ-QL / OUTLINE

- General linear RF current drive calculations implemented in GENRAY all frequencies rf ray tracing code w ADJ-QL
- Bounce averaged ADJ package is based on Karney et.al relativistic, Green function calculator (8th Top Conf RF Power in Plasmas, Irvine, 1989)
- We have added general non-circular equilibria and ray tracing-based calculation of RF fluxes, and interfaced to GENRAY
- The bounce-averaged relativistic momentum-space RF quasilinear flux is calculated at each point along rf ray trajectories, to be convolved w self-adjoint function.
- Comparisons are made between GENRAY-ADJ, CQL3D, and Ehst-Karney parameterization

Geometric optics equations

$$\frac{dr}{dt} = -\frac{c}{}\frac{\partial D/\partial N_r}{\partial D/\partial \omega} \qquad \frac{dN_r}{dt} = \frac{c}{}\frac{\partial D/\partial r}{\partial D/\partial \omega}$$

$$\frac{d\phi}{dt} = -\frac{c}{}\frac{\partial D/\partial M}{\partial D/\partial \omega} \qquad \frac{dM}{dt} = \frac{c}{}\frac{\partial D/\partial \phi}{\partial D/\partial \omega}$$

$$\frac{dz}{dt} = -\frac{c}{}\frac{\partial D/\partial N_z}{\partial D/\partial \omega} \qquad \frac{dN_z}{dt} = \frac{c}{}\frac{\partial D/\partial z}{\partial D/\partial \omega}$$

Dispersion Relation, general dielectric tensor **ε**

$$\begin{pmatrix} \varepsilon_{xx} - N_{||}^2 & \varepsilon_{xy} & \varepsilon_{xz} + N_{||} N \\ \varepsilon_{yx} & \varepsilon_{yy} - N^2 & \varepsilon_{yz} \\ \varepsilon_{zx} + N_{||} N & \varepsilon_{zy} & \varepsilon_{zz} N_{|}^2 \end{pmatrix} \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} = 0$$

$$D(\vec{R}, N_{||}, N, \omega) = \det D_{\alpha\beta}$$

ADJ-Module in GENRAY

- Small RF power $$f = f_m + \tilde{f}$$

- Linearized bounce averaged relativistic collision operator
$$<C(\tilde{f})> = C<(f_{e,m}, \tilde{f})> + C<(f_{e,m}, \tilde{f})> + <C(\tilde{f}, f_{m,i})>$$

- Self-adjoint operator
$$\int \psi <C(f_m \chi)> d^3u = \int \chi <C(f_m \psi)> d^3u$$

- The adjoint problem for passing particles (E-fld-like eqn)

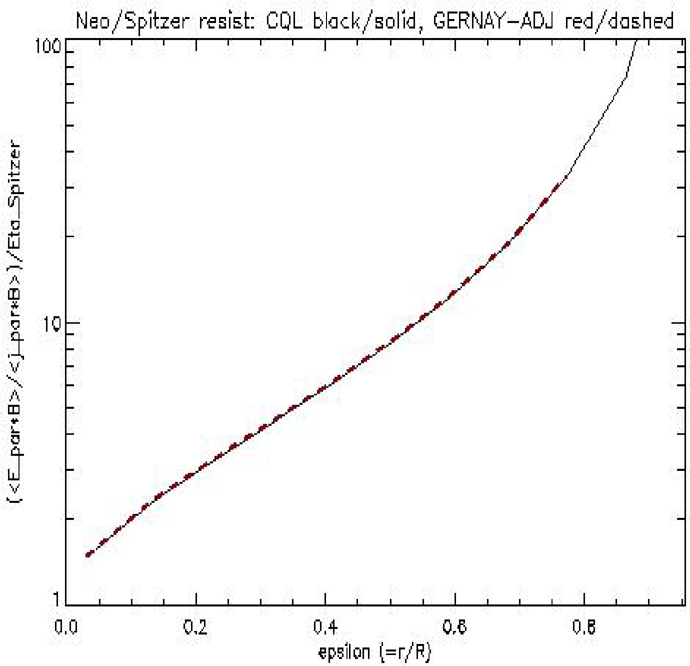
$$<C(f_m \chi)> f_m = -q \frac{v_0 \cos\theta_0}{\lambda}, \quad \lambda = \tau_b v_0 \cos\theta_0 / L$$

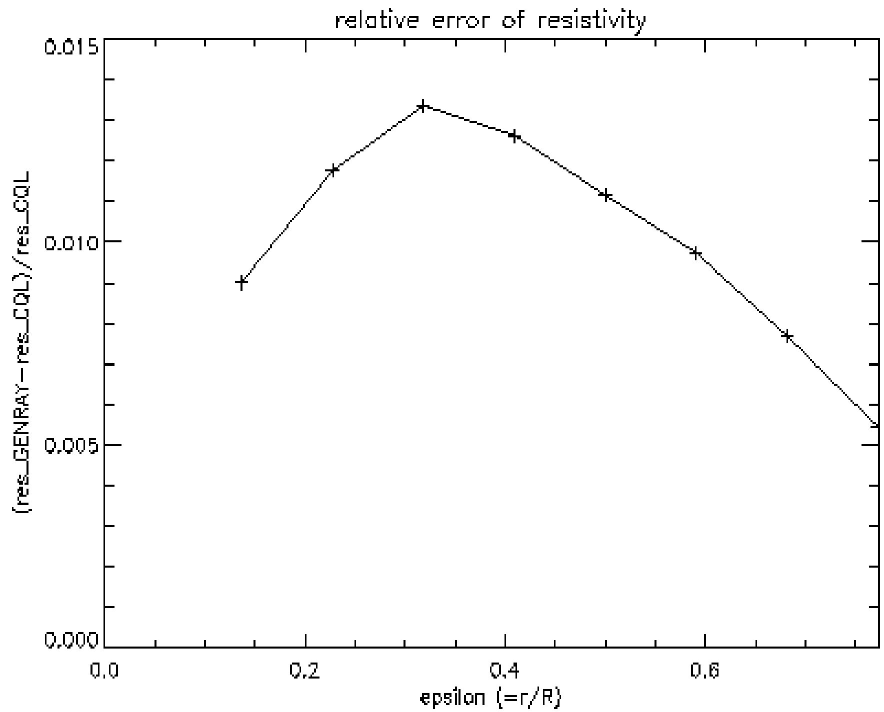
CQL3D-like collisional diffusion 2nd order partial diff operator

Neoclass. Elec. Conductivity: ADJ vs CQL3D

$$J_{\|0} = q\int \chi f_m v_{\|0} d^3u_0$$

Comparison of GENRAY-ADJ, CQL3D: DC current resistivity

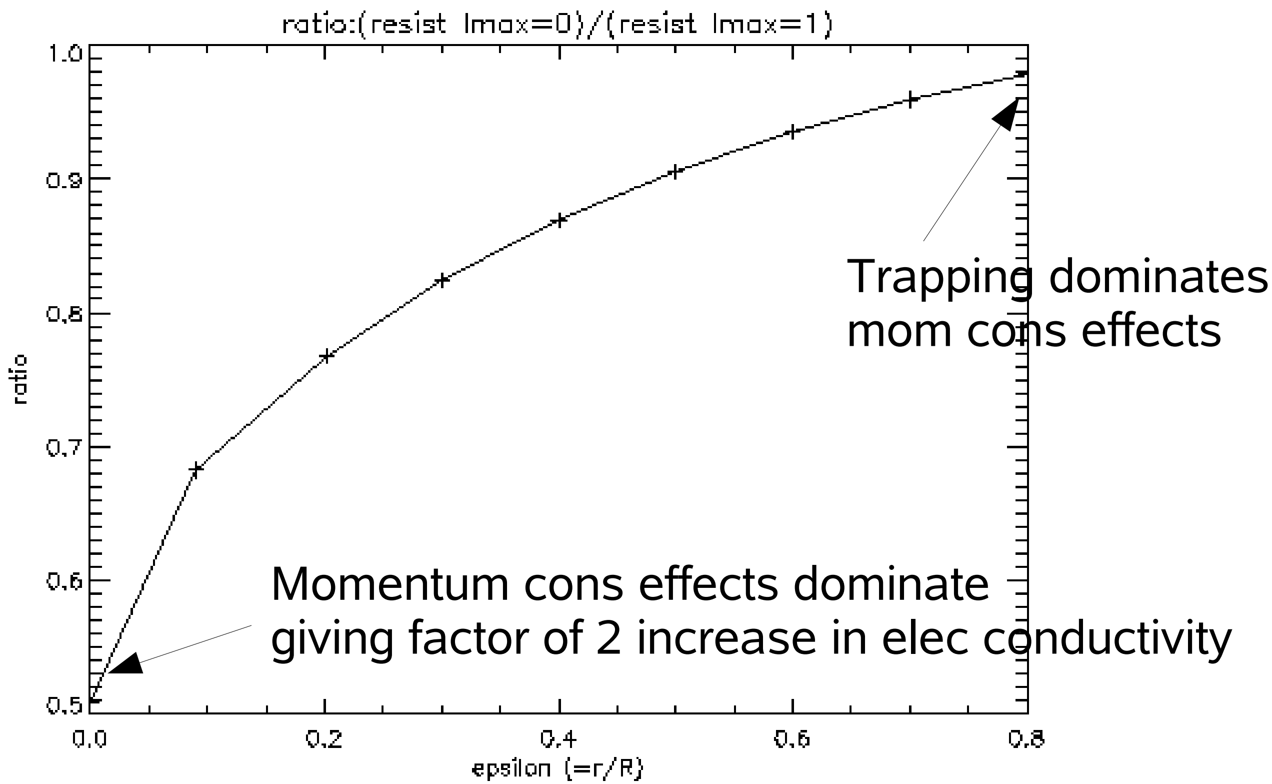




GENRAY-ADJ Electrical conductivity agrees to withing 1.4% over a full range of inverse aspect ratio (r/R), including strong toroidal trapping effect.

Effect of Momentum Conservation on ADJ DC conductivity vs (r/R)

- Lmax is the number of Legendre polynomials representing the momentum conservation part of the collision operator



Relativistic QL Flux

- Local QL Flux in midplane momentum space at Maxwellian distribution

$$S_{ql,u_0} = -D_{uu}\frac{df_m}{du_{0_{||}}}, \qquad S_{ql,\theta_0} = -D_{\theta u}\frac{df_m}{du_{0_{||}}}$$

- Diffusion coefficients

$$D_{uu} = \sum_n D_n \left[\frac{k_{||}v_{\perp}}{\omega\gamma}\cos\theta + \frac{n\omega_c}{\omega v}\sin\theta\right]^2$$

$$D_{\theta u} = \sum_n D_n \left[-0.5\left(\left(\frac{k_{||}v_{\perp}}{\omega\gamma}\right)^2 - \left(\frac{n\omega_c}{\omega\gamma}\right)^2\right)\sin 2\theta + \left(\frac{k_{||}v_{\perp}}{\omega\gamma}\right)\left(\frac{n\omega_c}{\omega v}\right)\cos 2\theta\right]$$

$$D_n = \frac{q^2}{2m}\pi\delta\left(\omega - k_{||}v_{||} - \frac{n\omega_c}{\gamma}\right)|\Theta_{n,k_{\perp}}|^2$$

- The relativistic resonance condition determines the argument of delta function.

$$\Theta_{n,k_{¿}} = \frac{E_x - iE_y}{2} J_{n+1}\left(\frac{k_{¿}u_{¿}}{\omega_c}\right) + \frac{E_x + iE_y}{2} J_{n-1}\left(\frac{k_{¿}u_{¿}}{\omega_c}\right)$$

$$+ E_z \frac{u_{||}}{u_{¿}} J_n\left(\frac{k_{¿}u_{¿}}{\omega_c}\right)$$

ADJ-QL RF Current Drive Efficiency

- Parallel current drive density

$$J_{\|0} = \int \lambda < \mathbf{S}_{ql,0} > \cdot \frac{\partial \chi}{\partial \mathbf{u}_0} d^3 u_0$$

RF QL Flux   Vel grad ADJ Function

- Power density

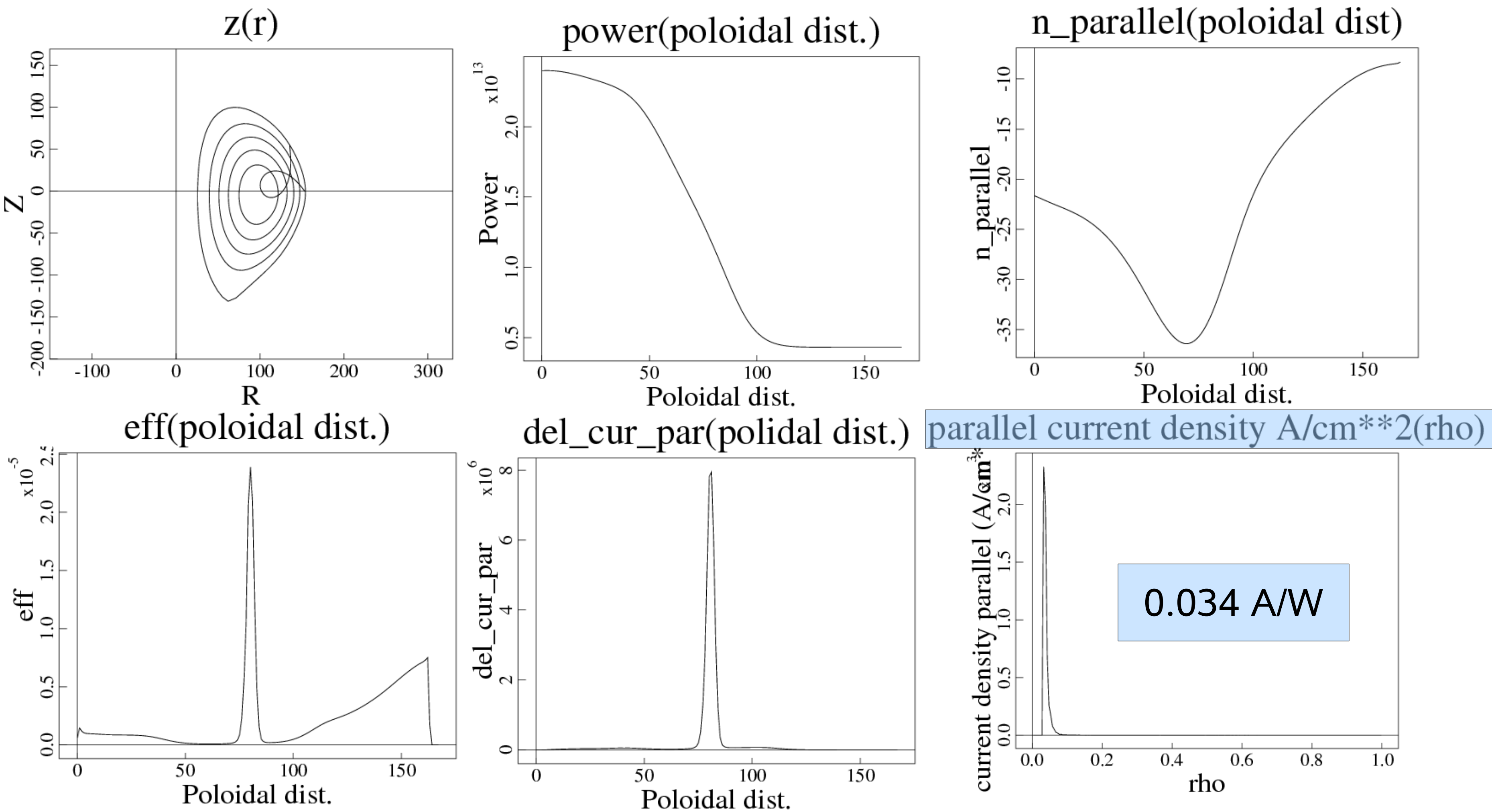
$$P_0 = m \int \lambda < \mathbf{S}_{ql,0} > \cdot \mathbf{v}_0 d^3 u_0$$

- CD efficiency

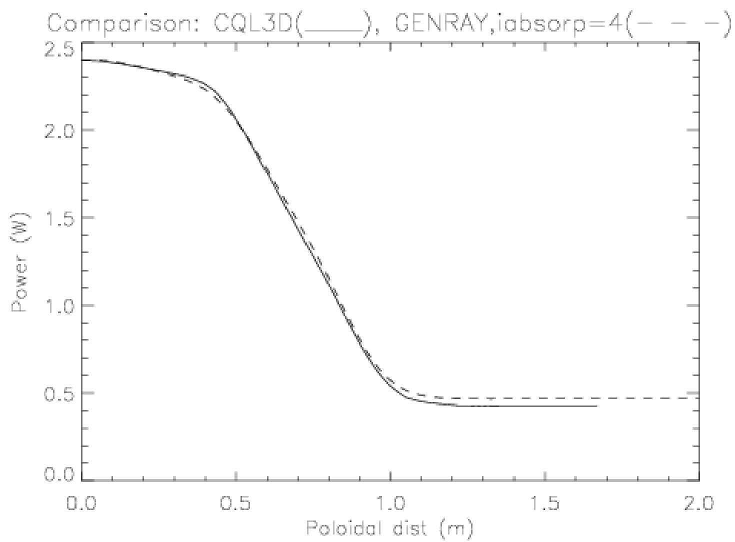
$$\eta = \frac{J_{\|0}}{P_0}$$

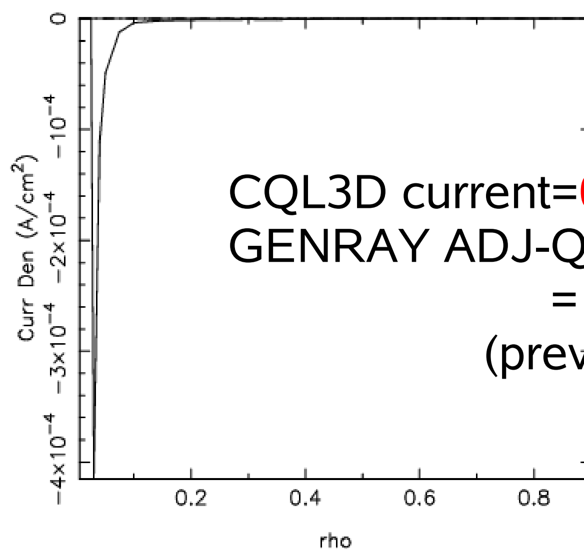
- Calc CD effic at each point along the ray, then multiply by absorbed power => CD from ray element

HHFW Rays and for NSTX Test Data
(30 MHz, n_par=-21.)



Comparison of ADJ-QL with CQL3D for NSTX Test Case



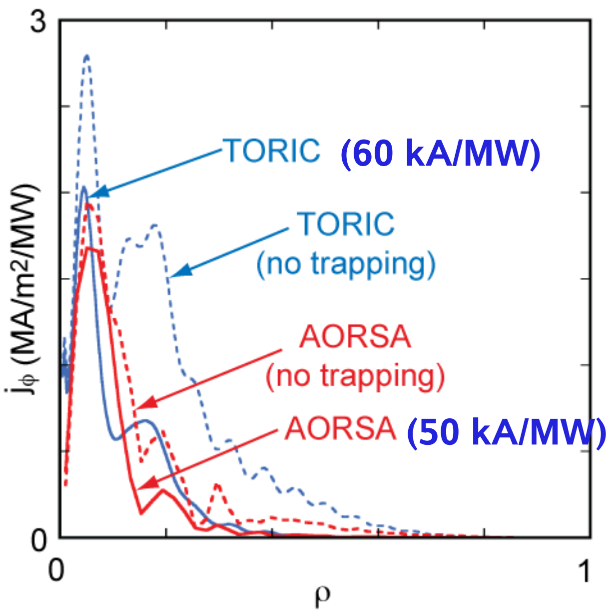


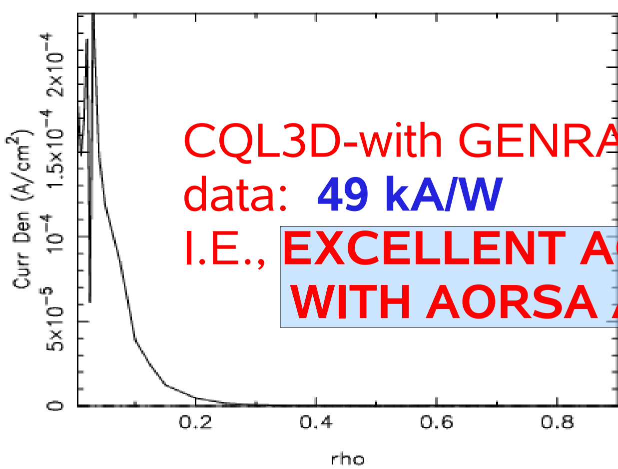
CQL3D current=0.010A/W
GENRAY ADJ-QL current
= 0.034A/W
(previous page)

Above excellent agreement between GENRAY and CQL3D absorption supports that polarizations have been correctly calculated in GENRAY, since CQL3D power absorption is calcuated from vel integral of energy*(df/dt)|QL.

Both CQL3D and GENRAY ADJ results appear well converged. THEREFORE: Presently have a bug (in GENRAY, I believe) for 0^{th} harmonic CD. The integration is different from cyclotron cases, which work.

GENRAY/CQL3D CD Calc Vs. AORSA-TORIC/Ehst-Karney for NXTX shot 123435

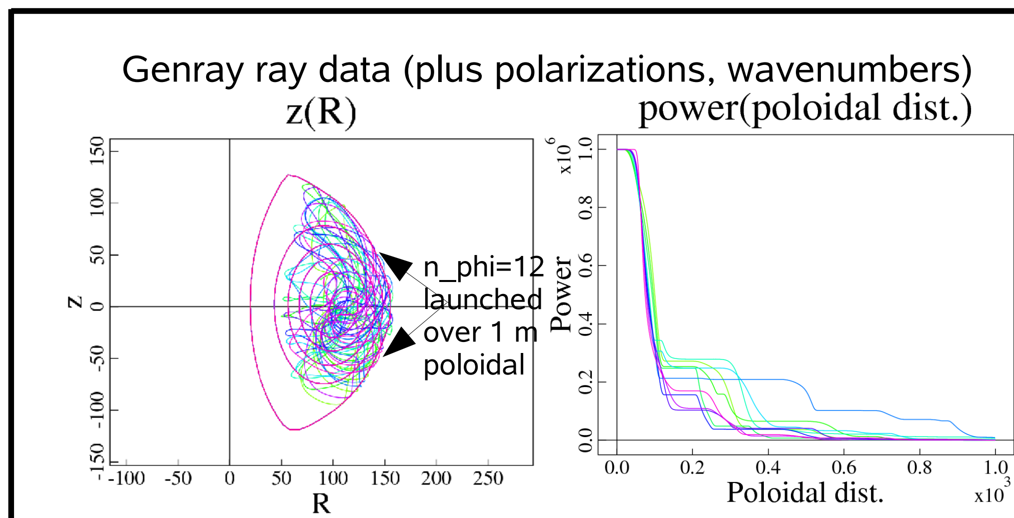




CQL3D-with GENRAY ray data: **49 kA/W**

I.E., **EXCELLENT AGREEMENT WITH AORSA AND TORIC**

AORSA and TORIC Simulations with and without trapping, for N_phi=12. Ehst-Karney CD effic. Phillips et al., IAEA 2008



CONCLUSIONS

- Bounce averaged relativistic ADJ QL package implemented in GENRAY.
- This approach includes: momentum conservation, wave polarization effects, and the influence of trapped electrons.
- The bounds of assumptions: the electron distribution function remains close to a relativistic Maxwellian, small banana width, toroidal geometry and low collisionality.
- The method provides a general, linear CD calculation, and is directly comparable to CQL3D in the low power (linear) regime. Comparisons with CQL3D for electron cyclotron harmonic current drive agree well. **Moreover, calculation of HHFW CD in an recent NSTX shot with GENRAY ray data plus CQL3D current calculation agree well with AORSA/Ehst-Karney CD calculation (49 kA/MW vs 50 kA/MW).**
- However, at the moment, at zeroth harmonic HHFW ADJ-QL current calculation appears to overestimate CD efficiency by 3X, in a test case. This is being worked on, and hopefully, the bug will be eliminated shortly.