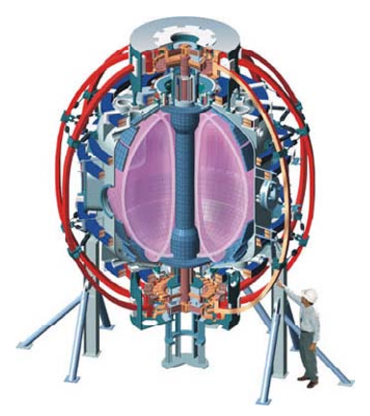

Model-Based Shape Control Design for the National Spherical Torus Experiment (NSTX)

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Introduction

- Plasma shape and position control present difficulties associated with real-time shape identification, plasma parameters measurement, and control method selection.
- The recent implementation of the real-time equilibrium reconstruction code rtEFIT on NSTX allows plasma shaping by controlling the magnetic flux at the plasma boundary. A non-model-based shape controller that exploits this capability has been recently proposed [1].
- We describe current efforts to develop a model-based multi-input-multi-output (MIMO) H_∞ controller to provide real-time shaping, current and position control.
- The control design is based on linear plasma response models derived from fundamental physics assumptions.
- Computer simulations illustrate preliminary results.

[1] D.A. Gates, et al., Nucl. Fusion 46 (2006) 17–23.

National Spherical Torus Experiment (NSTX)

Advantages of NSTX:

1. Encompasses a low aspect ratio in comparison to traditional tokamaks.
2. NSTX is predicted to exhibit more efficient magnetic utilization field than conventional tokamaks.
3. Has the ability to operate with relatively high ratio (β) of plasma pressure to magnetic field pressure.
4. Power density is proportional to β^2, so a considerable increase in this factor could significantly improve the chances for realization of practical fusion energy.
5. The naturally elongated plasma shape of the ST configuration leads to a reduction in power density deposited on plasma facing surfaces.

Plasma Response Model

Summary of the linearized plasma model equations [2]:

1. Poloidal field (PF) coil current dynamics

$$M_{cc}^* \dot{I}_c + R_c I_c + M_{cv}^* \dot{I}_v + M_{cp}^* \dot{I}_p = V_c$$

2. Vessel (passive conductor) current dynamics

$$M_{vv}^* \dot{I}_v + R_v I_v + M_{vc}^* \dot{I}_c + M_{vp}^* \dot{I}_p = 0$$

3. Plasma current equation

$$L_p^* \dot{I}_p + R_p I_p + M_{pc}^* \dot{I}_c + M_{pv}^* \dot{I}_v = V_{non-ohmic}$$

Define $x = [I_c^T \;\; I_v^T \;\; I_p]^T, u = [V_c^T \;\; 0 \;\; 0]^T$ to obtain state-space representation

State equation:

$$\dot{x} = Ax + Bu; \qquad x \in R^{45x1}, \;\; u \in R^{9x1}$$

Output equation:

$$y = Cx; \qquad y \in R^{160x1}$$

I_c, R_c	PF coils currents and resistances
I_v, R_v	Vessel currents and resistances
I_p, R_p	Plasma coils currents and resistances
V_c	Vector of voltages applied to PF coils
$V_{non\text{-}ohm}$	Effective voltage applied to drive plasma current by noninductive sources
	$M^*_{ab} = M_{ab} + X_{ab}(J, B_{vac})$
	$a,b \in \{c,v,p\}$ denote coil, vessel, plasma
M_{ab}	Conductor-to-conductor mutual inductance
X_{ab}	Linearized plasma motion-motivated inductance
J	Plasma toroidal current distribution
B_{vac}	Vacuum magnetic field

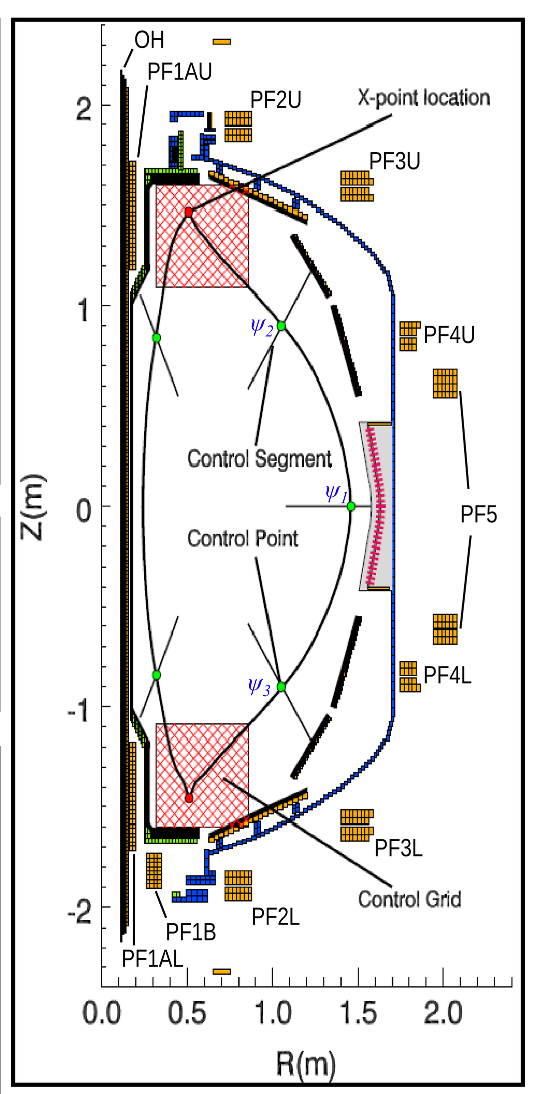
The system has 45 states:

- State 1: OH current
- States 2-9: PF currents (PF1AU-PF1B)
- States 10-44: Vessel currents
- States 45: Plasma current

System Inputs (9)	
1	V_oh1
2	V_pf1au
3	V_pf2u1
4	V_pf3u1
5	V_pf5u2
6	V_pf3l1
7	V_pf2l2
8	V_pf1al
9	V_pf1b

(Original) System Outputs (160)

- Outputs 1-26:
  - 1 Ohmic coil current (I_{oh})
  - 21 Poloidal field coils current
  - 4 Toroidal field coils current
- Outputs 27-87:
  - 61 Flux loops
- Outputs 88-157:
  - 70 Magnetic probes
- Output 158: Radial Position (R_p)
- Output 159: Vertical Position (Z_p)
- Output 160: Plasma Current (I_p)



[2] M. Walker, D. Humphreys, Fusion Science and Technology, v 50, n 4, November, 2006, p 473-489

Control System Architecture

Only 6 outputs are used for control:

$$y = C_r x \qquad y \in R^{6\times1}$$

where C_r is the reduced C matrix.

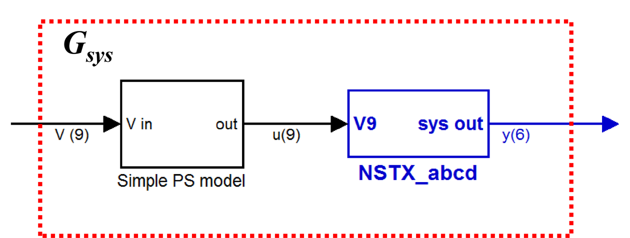
- The fluxes ψ_1, ψ_2, ψ_3 at the control points
- The plasma radial position R_p
- The plasma vertical position Z_p
- The plasma current I_p

The fluxes at the control points (ψ_1, ψ_2, ψ_3) are calculated as

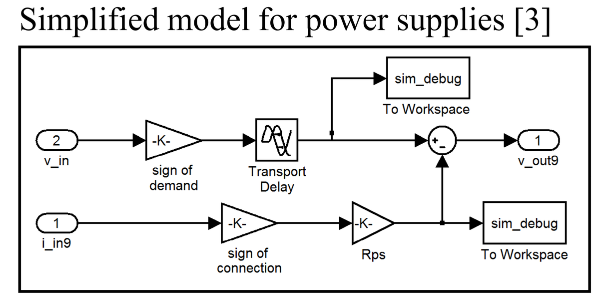
$$\begin{bmatrix}\psi_1\\ \psi_2\\ \psi_3\end{bmatrix} = M_{pc} I_c + M_{pv} I_v + \frac{\psi_p^{eq}}{I_p^{eq}} I_p$$

where M_{pc} and M_{pv} are the plasma-coil and plasma-vessel mutual inductance matrices, and ψ_p^{eq} is the flux distribution due to the plasma current at equilibrium.

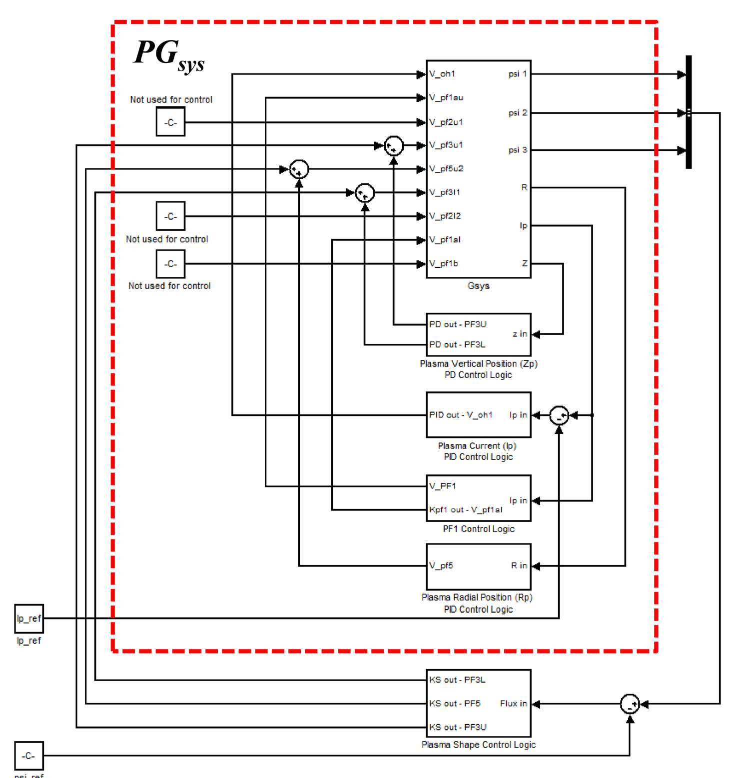
Reduced system with 6 outputs (G_{sys}):



Simplified model for power supplies [3]



[3] J. Leuer, M. Walker, General Atomics Engineering Physics Memo, EPMja1031212.



The feedback control system uses isoflux control to shape the plasma boundary. The goal of the feedback controller is to control:

- Plasma position
- Plasma current
- Plasma shape

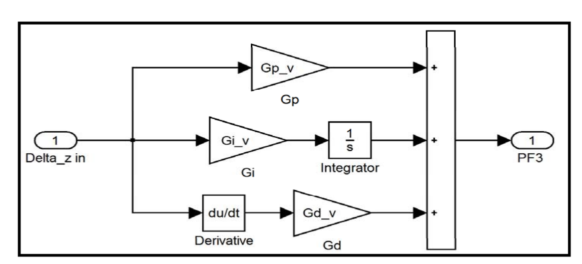
The plasma position and current feedback control consists of three Proportional-Integral-Derivative (PID) controllers and one proportional controller.

Position and Current Controllers

1. Vertical Position Controller

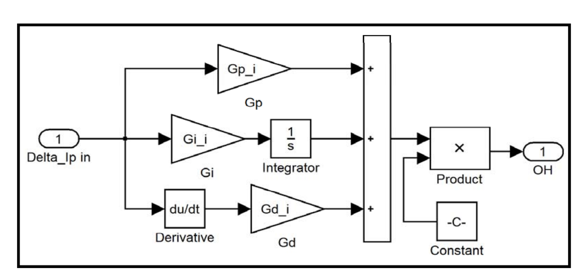
$$u_v = \pm\Delta V_{PF3} = (-1)^j \left(G_P^v Z_p + G_I^v \int_0^t Z_p dt + G_D^v \frac{dZ_p}{dt} \right)$$

$j = 0,1$ refer to [upper, lower] PF3 coils.



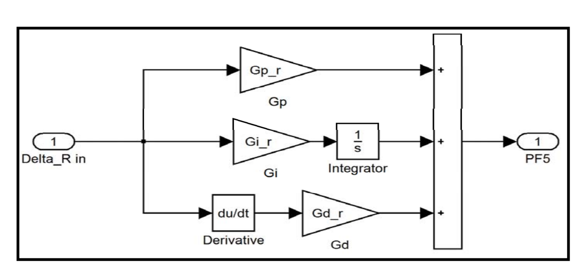
2. Plasma Current Controller

$$u_{Ip} = V_{OH1} = G_p^{Ip} I_p + G_I^{Ip} \int_0^t I_p dt + G_d^{Ip} \frac{dI_P}{dt}$$



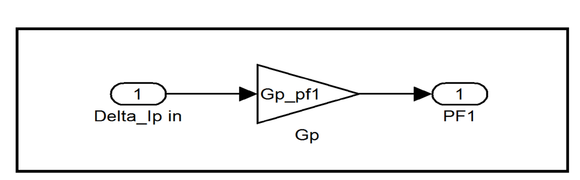
3. Plasma Radial Position Controller

$$u_r = V_{PF5} = G_p^r R_p + G_I^r \int_0^t R_p dt + G_d^r \frac{dR_p}{dt}$$



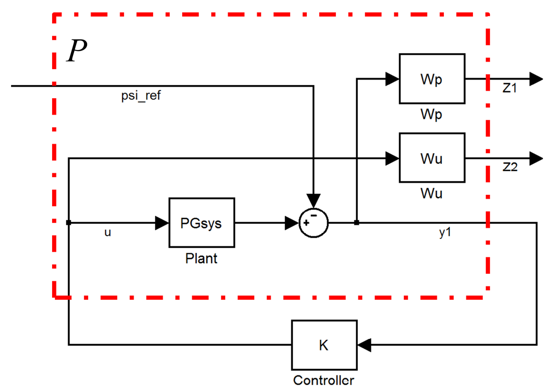
4. PF1 Controller

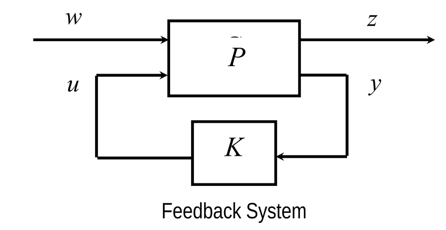
$$u_{PF1} = V_{PF1} = G_p^{pf1} I_p$$



H_∞ Multi-Input Multi-Output (MIMO) Shape Controller Design

General Control Problem Formulation (Frequency Domain):





u: Control input
y: Measured output for feedback control
w: External input signal (reference ψ^{ref})
z: External performance signal

Using Laplace transform to obtain a frequency-domain representation

$$\begin{bmatrix} z \\ y \end{bmatrix} = \begin{bmatrix} z_1 \\ z_2 \\ y_1 \end{bmatrix} = P(s)\begin{bmatrix} w \\ u \end{bmatrix} = \begin{bmatrix} P_{11}(s) & P_{12}(s) \\ P_{21}(s) & P_{22}(s) \end{bmatrix}\begin{bmatrix} w \\ u \end{bmatrix}$$

$$u = K(s)y$$

The controller K stabilizes the system and minimizes the H_∞ norm of the transfer function between w and z, i.e., $T_{zw}(P,K)$ is

$$\min_{K(s)} \|T_{zw}(P,K)\|_\infty = \min_{K(s)} \left(\sup_w \bar{\sigma}[T_{zw}(P,K)(jw)] \right) \qquad \bar{\sigma} - \text{maximum singular value}$$

$$z = T_{zw}(P,K)w = F_l(P,K)w, \quad \text{where } F_l(P,K) = N = P_{11} + P_{12}K(I \quad P_{22}K)^{1} P_{21} = \begin{bmatrix} W_p S \\ W_u KS \end{bmatrix}$$

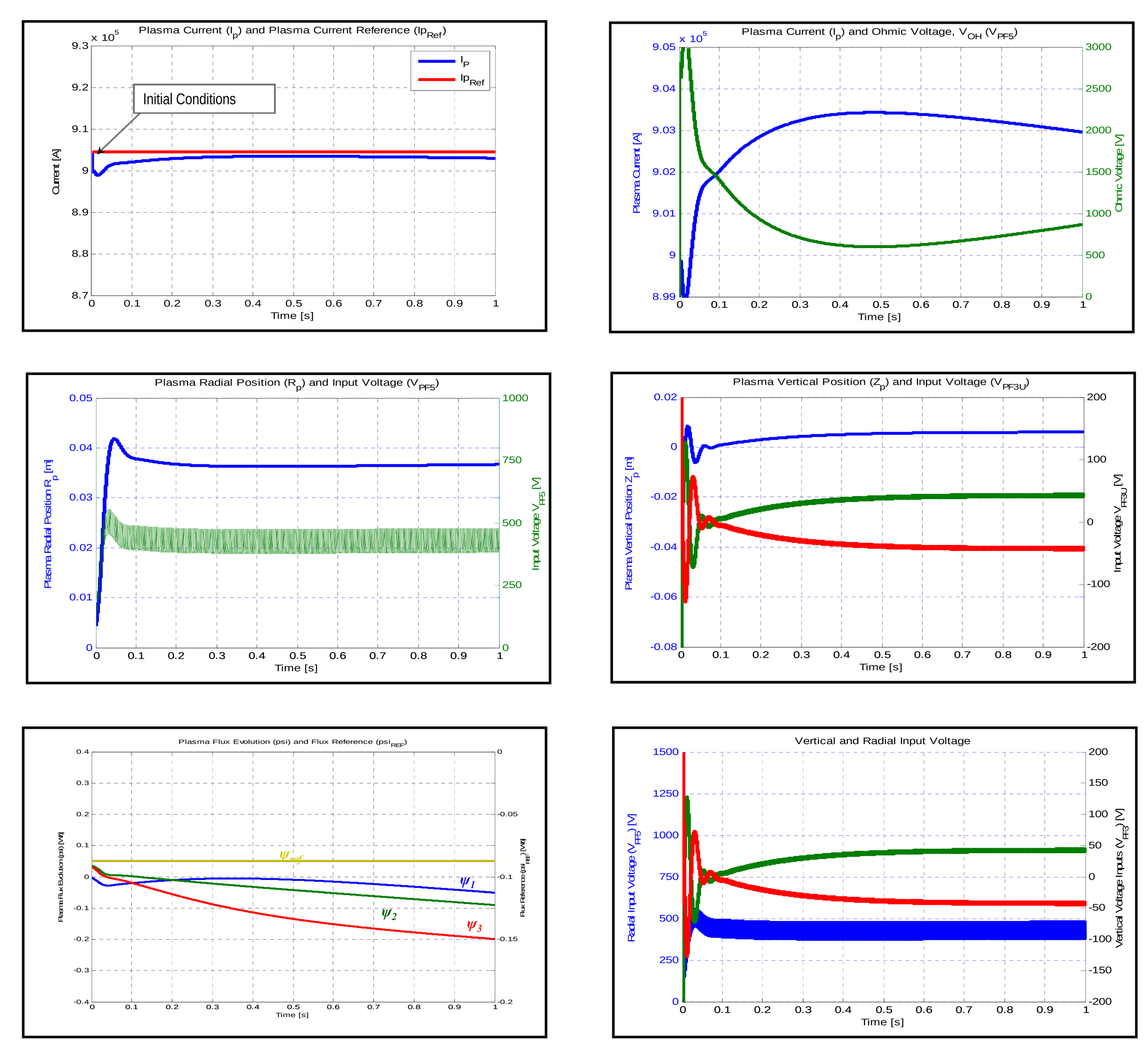
The controller K is designed using *Mixed Sensitivity H_∞ Control,* where the goal is to minimize the error tracking ($W_p S$) and the control effort ($W_u KS$) at the same time. That is, we seek a controller that minimizes the norm.

$$\left\| \begin{bmatrix} W_p S \\ W_u KS \end{bmatrix} \right\|_\infty$$

where $S = (I + PK)^{-1}$ denotes the sensitivity function of the closed-loop system.

Simulations Results: Without Shape Controller

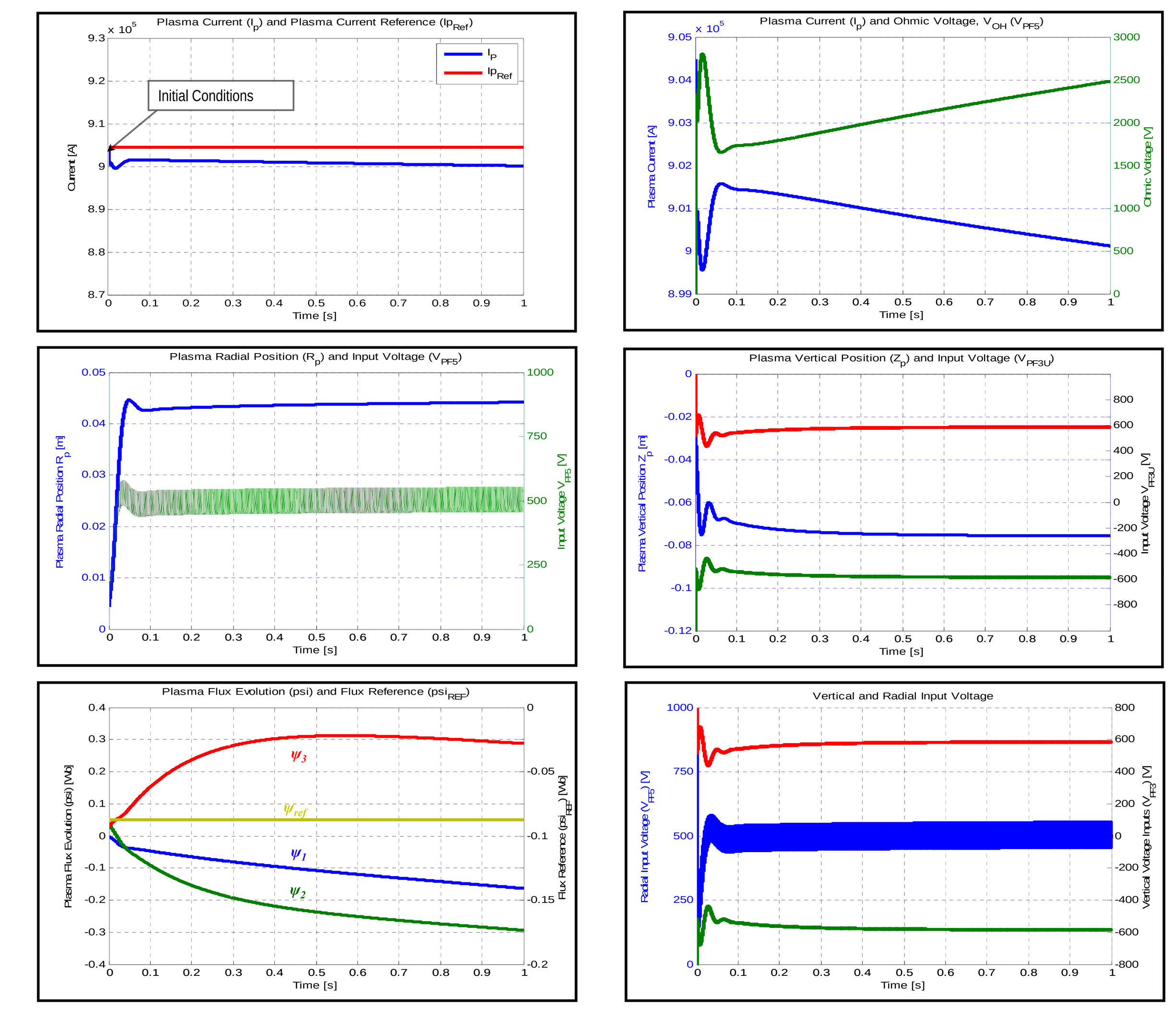
Gains			
Controller	G_p	G_i	G_d
Vertical Position	7,500	0	50
Plasma Current	0.5	0	0
PF1	0.015	0	0
Radial Position	10,000	0	100



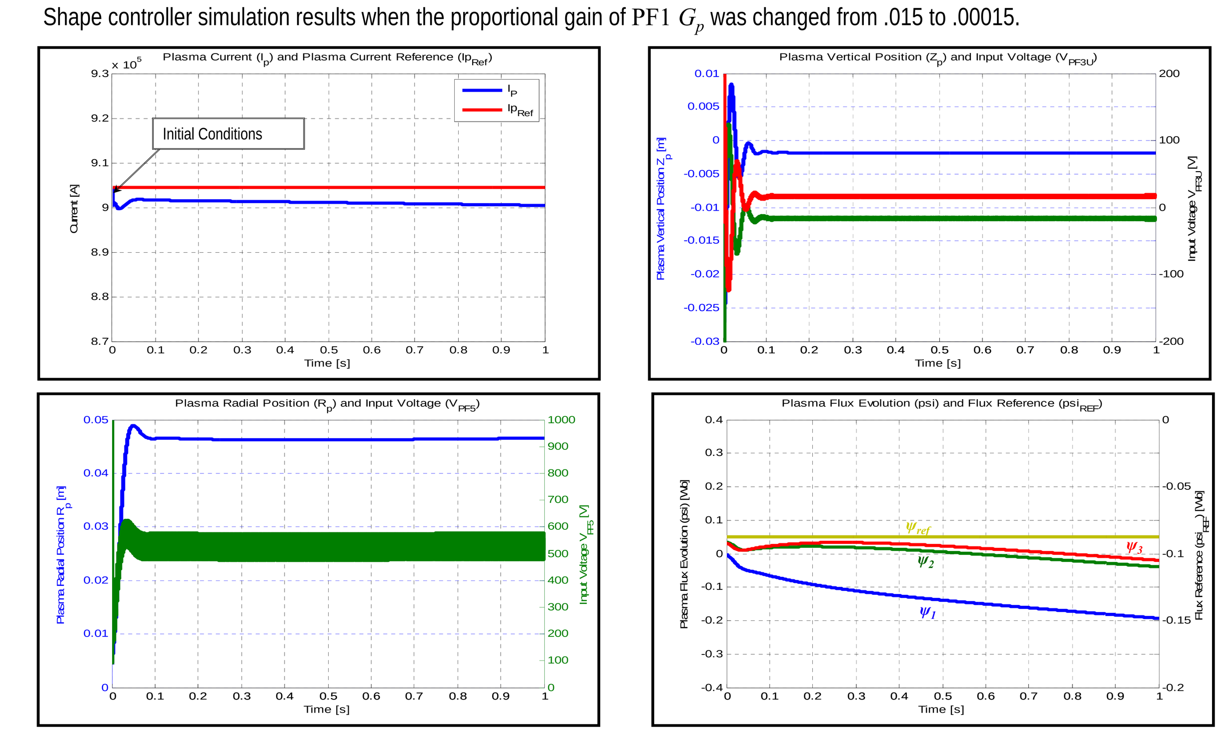
Simulation Results: With Shape Controller

$$W_p(s) = \frac{s + \omega_{b2} A_2}{s/M_2 + \omega_{b2}} I^{3x3} \qquad W_u(s) = \frac{s/M_1 + \omega_{b1}}{s + \omega_{b1} A_1} I^{3x3}$$

Weights Parameters					
W_p			W_u		
M_1	ω_{b1}	A_1	M_2	ω_{b2}	A_2
2	0.5	0.001	3	50	0.01



Shape controller simulation results when the proportional gain of PF1 G_p was changed from .015 to .00015.



NOTE: These are preliminary simulation results where the references for the fluxes at the control points are arbitrary and probably unrealistic. The use of more realistic control point flux references in the future, after incorporating real time EFIT, may reduce the competition among different controllers.

Conclusions and Future Work

- A model-based MIMO shape controller has been designed using H_∞ techniques.
- The model-based shape controller is part of a control architecture where the plasma position and current are regulated via SISO PID loops.
- Preliminary simulation results are presented using unrealistic references for the flux at the control points.
- Real-time EFIT will be incorporated into the SIMULINK-based simulation toolbox in order to be able to assess the performance of the controller using more realistic references for the control point fluxes.
- After rtEFIT is incorporated, further refinement of the controllers will be carried out based on the frequency sharing principle.
- The study of nonlinear phenomena such as actuator saturations and change of the plasma equilibrium is part of our future work.