Abstract

In order to better understand the effects of the time-varying error field[1] in NSTX on rotation braking, which impedes RWM stabilization, we model the plasma response to an applied low-*n* external field perturbation using the resistive MHD model in the M3D code[2]. As an initial benchmark, we apply an m=2, n=1 perturbation to the flux at the boundary of a non-rotating model equilibrium and compare the resulting steady-state island sizes with those predicted by the ideal linear code IPEC[3]. For sufficiently small perturbations, the codes agree; for larger perturbations, the nonlinear correction yields an upper limit on the island width beyond which stochasticity sets in. The ideal MHD current sheet at the resonant surface and its shielding effect on the plasma interior to this surface, which form the basis of the IPEC model, are difficult to reproduce in toroidal calculations though they readily appear in a straight cylinder. The M3D-C[1] code[4] is also being evaluated as a tool for this analysis.

[1]J.E. Menard, *et al*., *Nucl. Fus*. **47**, S645 (2007).
[2]W. Park, *et al*., *Phys. Plasmas* **6**, 1796 (1999).
[3]J.K. Park, et al., *Phys. Plasmas* **14**, 052110 (2007).
[4]S.C. Jardin, *et al*., *J. Comp. Phys*. **226**, 2146 (2007).

Motivation

- Left uncorrected, the NSTX error field produces magnetic islands that can mode lock, braking plasma rotation and destabilizing RWMs.
- Analysis with IPEC has helped to predict these effects and design effective mitigation strategies.
- Analysis with M3D can extend these results to the nonlinear, resistive, rotating plasma regime inaccessible to the ideal linear code.
- M3D analysis should be extensible to other RMP effects, such as potential ELM mitigation or destabilization.

The M3D Code

M3D is a parallel 3D multilevel physics code in toroidal geometry maintained by a multi-institutional collaboration, centered at PPPL, and supported by SciDAC.

- Contains MHD, extended MHD, and fluid-kinetic hybrid models.
- Uses linear, 2^{nd}, or 3^{rd}-order finite elements in-plane on an unstructured triangular mesh.
- Uses 4^{th}-order or pseudo-spectral finite differences between planes.
- Partially implicit treatment allows efficient time advance but requires small time steps.
- <u>Linear</u> operation: full nonlinear + filtering, active equilibrium maintenance.
- <u>Nonlinear</u> operation: all components of all quantities evolve nonlinearly.

The M3D Resistive MHD Model

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}_i) = 0$$

$$\rho\left[\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v}\cdot\nabla\mathbf{v}\right] = -\nabla \mathbf{p} + \mathbf{J}\times\mathbf{B} + \mu\nabla^2\mathbf{v}$$

$$\mathbf{E} + \mathbf{v}\times\mathbf{B} = \eta\mathbf{J}$$

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla\times\mathbf{E}$$

$$\mathbf{J} = \nabla\times\mathbf{B}$$

$$\frac{\partial p}{\partial t} + \mathbf{v}\cdot\nabla p = -\gamma p \nabla\cdot\mathbf{v} + \nabla\cdot n\chi_\perp \nabla\left(\frac{p}{\rho}\right)$$

Artificial sound wave model for $\chi_\parallel$:

$$\frac{\partial T}{\partial t} = s\frac{\mathbf{B}\cdot\nabla u}{\rho}$$

$$\frac{\partial u}{\partial t} = s\mathbf{B}\cdot\nabla T + \nu\nabla^2 u$$

M3D representation:

$$\mathbf{B} = \nabla\psi\times\nabla\phi + \frac{1}{R}\nabla_\perp F + \left(R_0 + \tilde{I}\right)\nabla\phi$$

$$\mathbf{V} = \frac{R^2}{R_0}\nabla U\times\nabla\phi + \nabla_\perp\chi + V_\phi\hat{\phi}$$

Calibration with IPEC

- In order to establish a baseline for comparison, we first compared the steady-state predictions of island widths in response to boundary perturbations between codes.
    - Add various low-*m*, *n*=1 perturbations of specified amplitude to initial poloidal flux on plasma boundary:

$$\tilde{\psi}_{boundary}(\theta,\varphi)=\tilde{\psi}_0\cos(\varphi-m\theta)$$

    - Measure plasma displacements, singular currents with IPEC; infer island widths.
    - Solve for instantaneous equilibrium+vacuum field (or evolve M3D nonlinearly until saturation of *n*=1 islands to include plasma response), measure island widths directly, compare to linear results.

1st Test: DIII-D Equilibrium, q_0=1.07

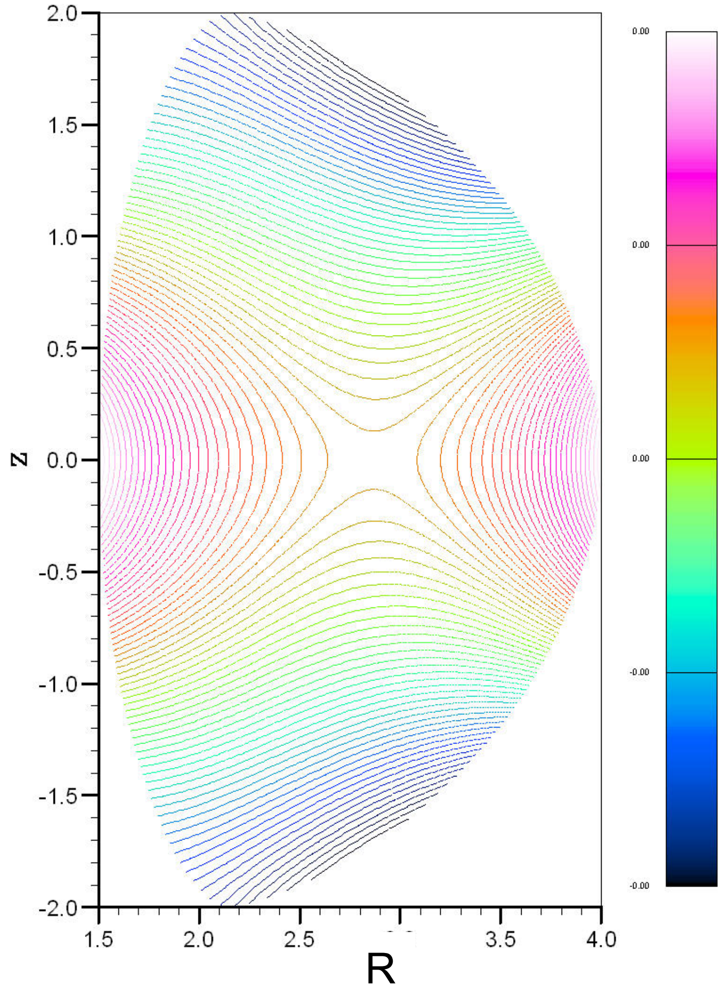
- Begin by solving the Poisson equation

$$\frac{\partial^2\psi}{\partial R^2}-\frac{1}{R}\frac{\partial\psi}{\partial R}+\frac{\partial^2\psi}{\partial z^2}=-RJ_\phi$$

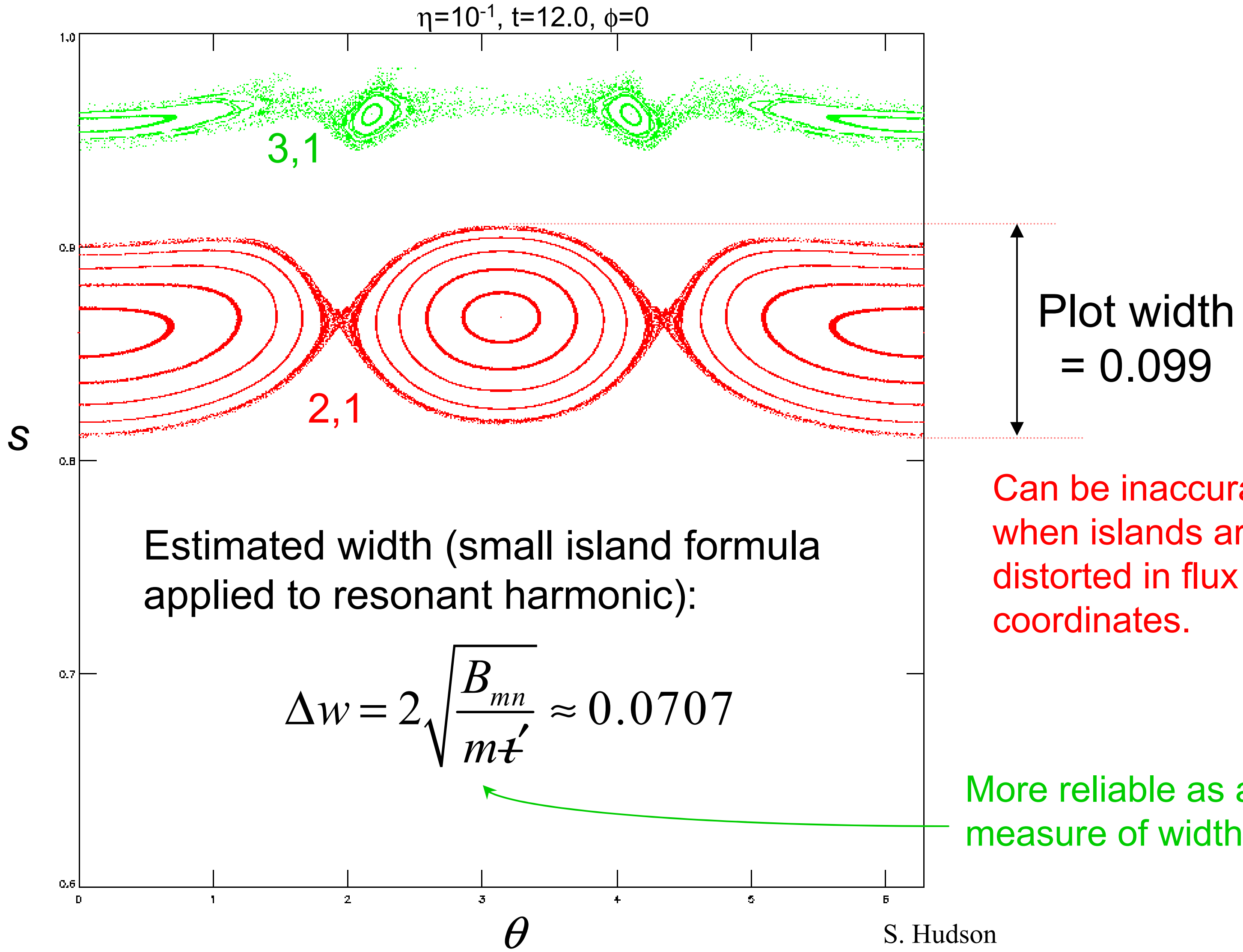
for ψ, subject to the perturbed boundary condition, where J_ϕ is the unperturbed equilibrium toroidal current density.

- Time-evolving from this state with various choices of resistivity, viscosity, etc. will show the effect of the plasma response on the islands.

Instantaneous Perturbed Flux



Island Widths are Characterized using Field-line-following diagnostic



Estimated width (small island formula applied to resonant harmonic):

$$\Delta w = 2\sqrt{\frac{B_{mn}}{m\,\iota'}} \approx 0.0707$$

Can be inaccurate when islands are distorted in flux coordinates.

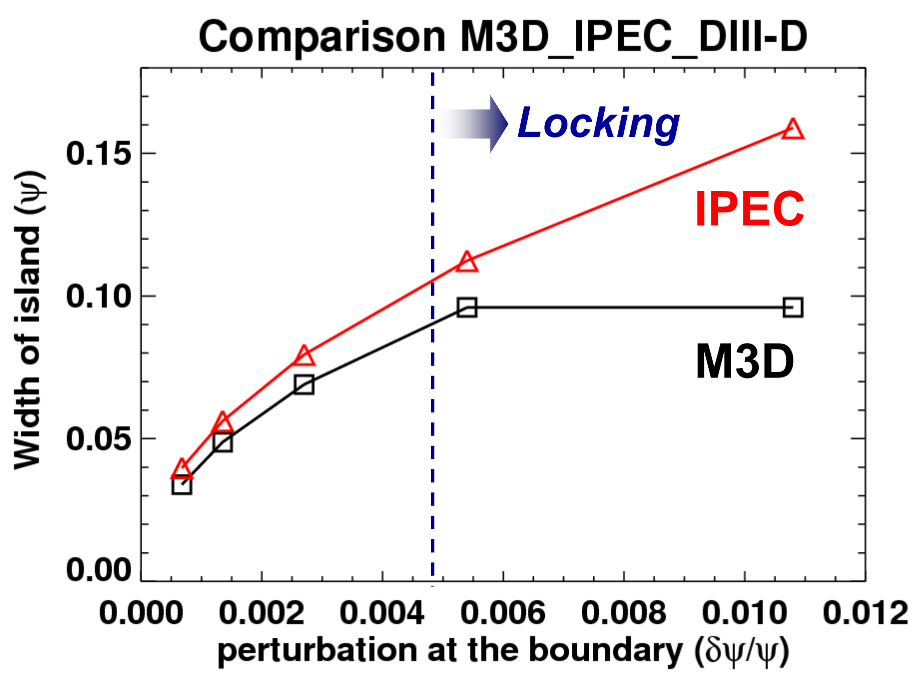
More reliable as a measure of width.

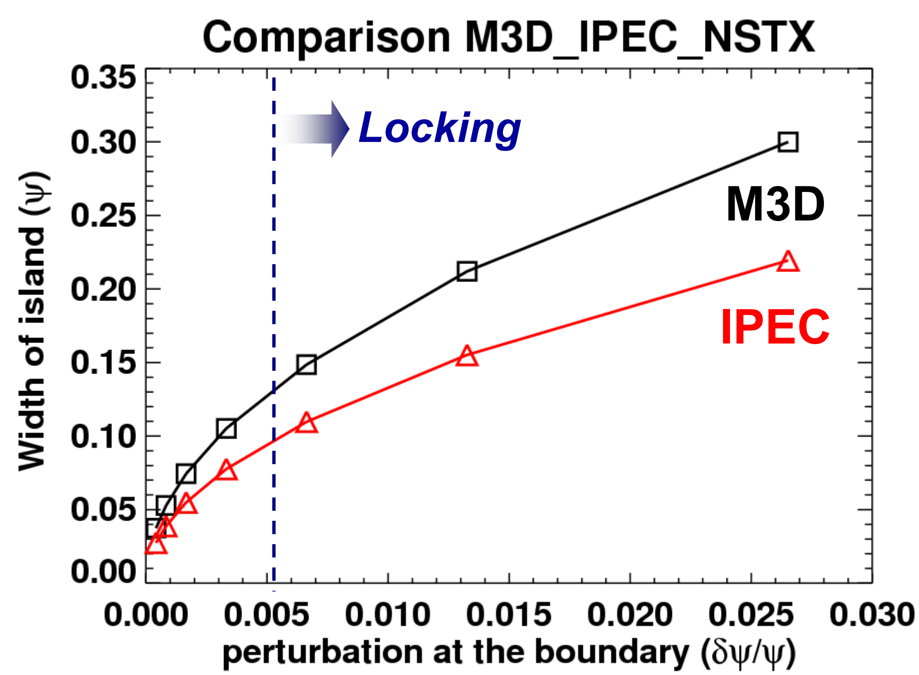
S. Hudson

2,1 Island Widths agree well with Inferred IPEC Widths in Linear Regime

IPEC prediction is inferred from formula based on singular current sheet at rational surface in ideal model, shielding interior.

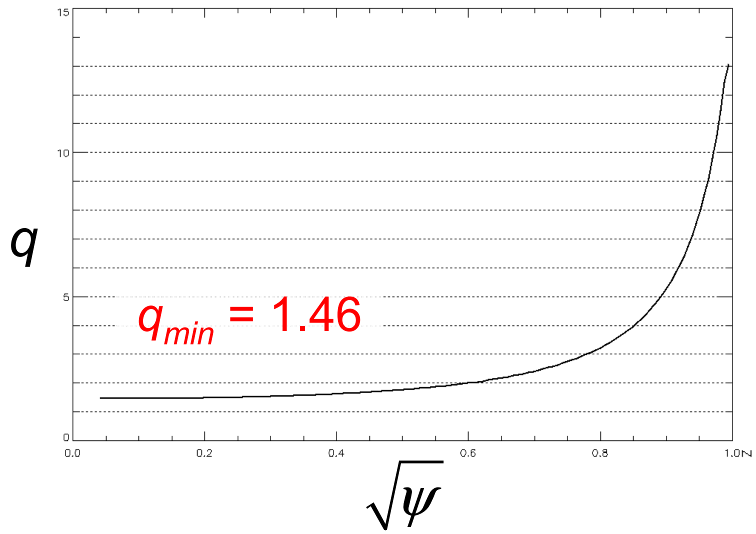
m=2, *n*=1 perturbation applied at boundary

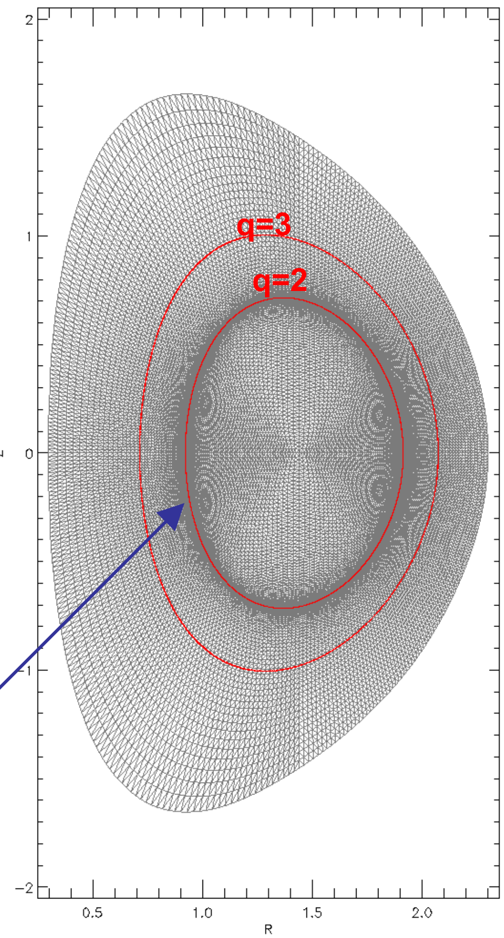


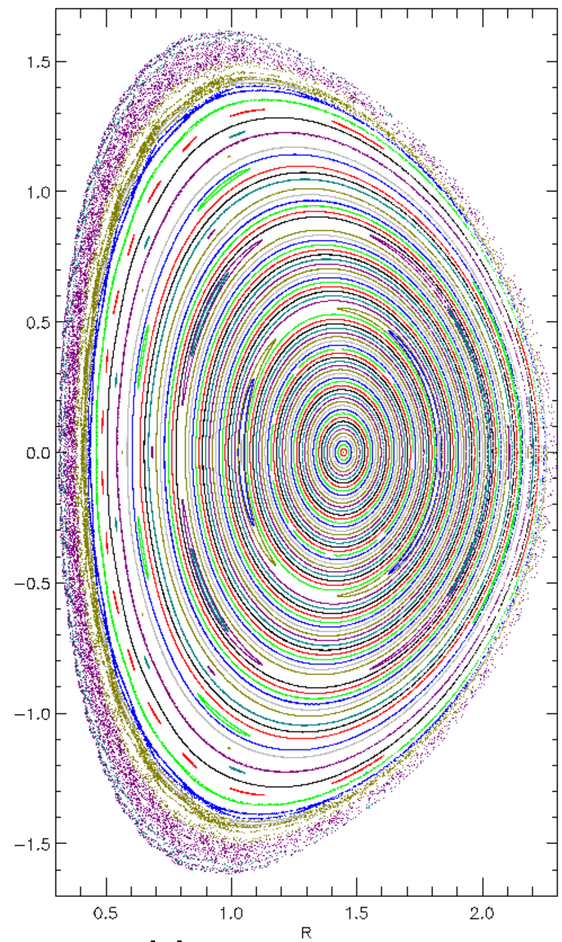


J.K. Park

Nonlinear Studies Based on EFIT Reconstruction of NSTX shot 122444



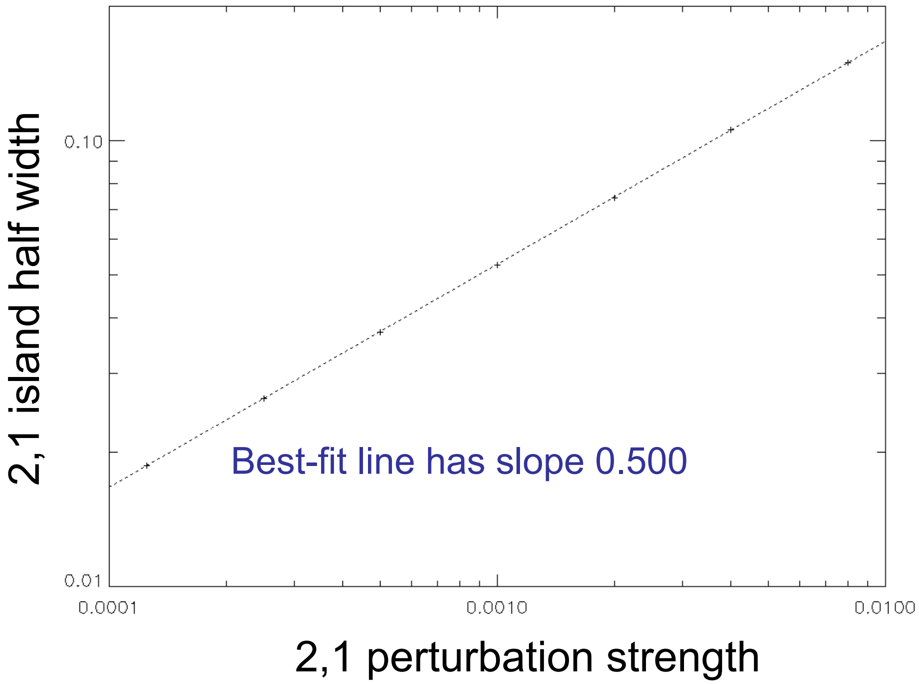




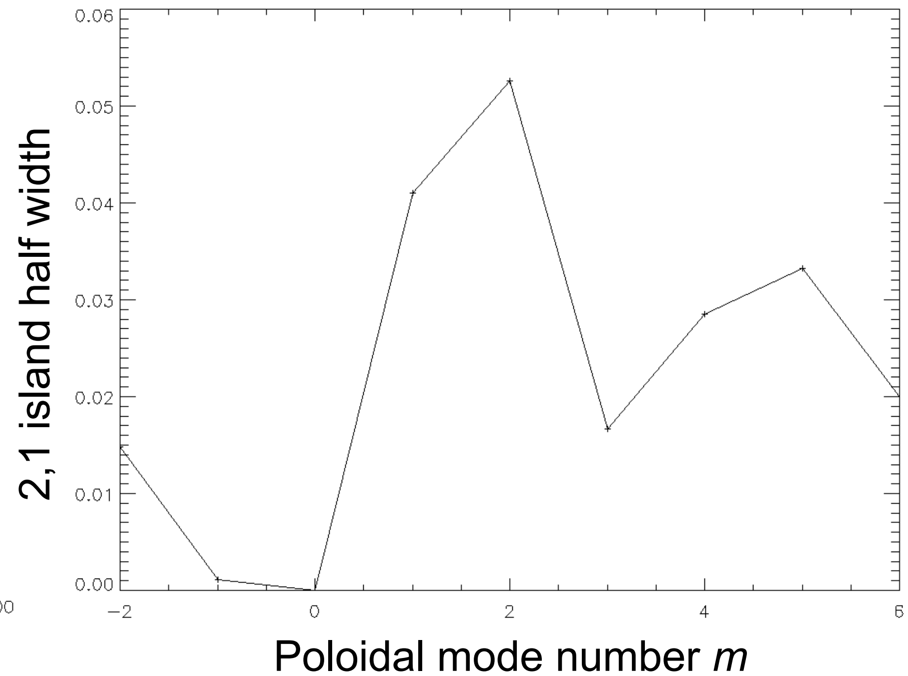
- ❑ Radial zones are packed at q=2 surface to help resolve small islands.
- ❑ Large perturbation is required at edge to produce a measurable island at q=2 (s=0.6).

Steady State Response

NSTX equilibrium, q_0 = 1.46

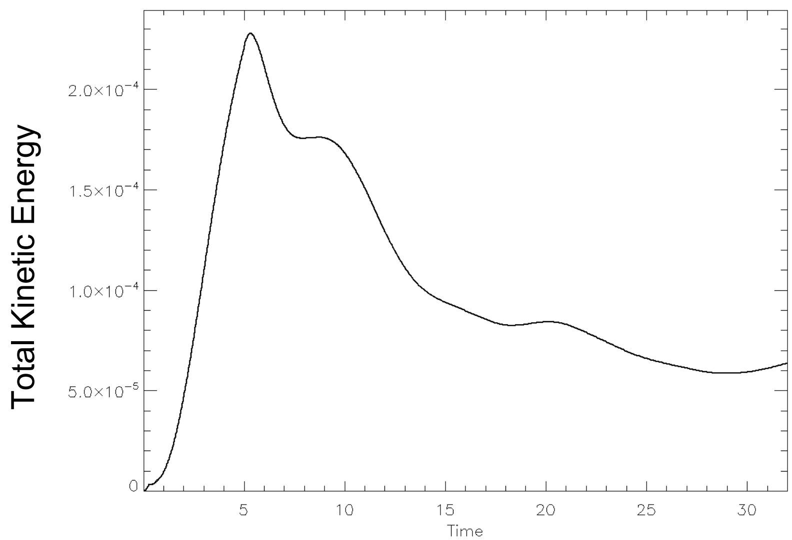


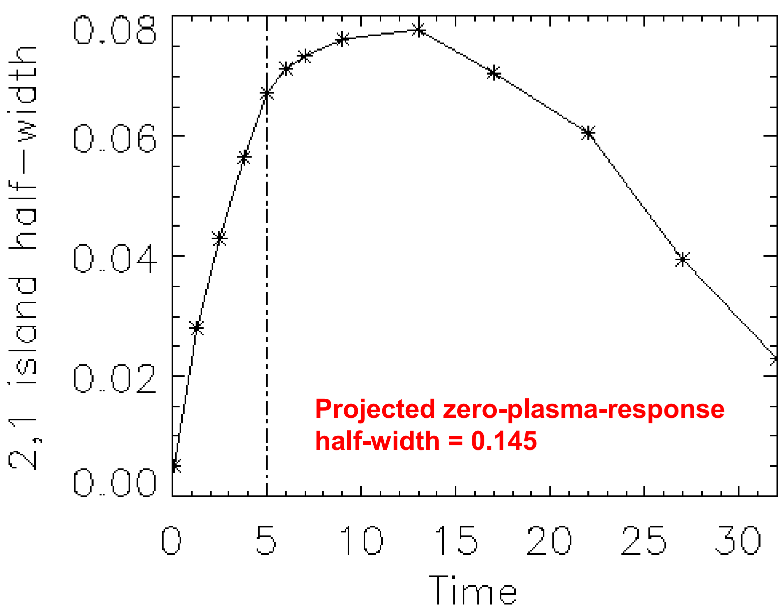
NSTX equilibrium, q_0 = 1.46
n=1 perturbations, pmag=10^{-3}



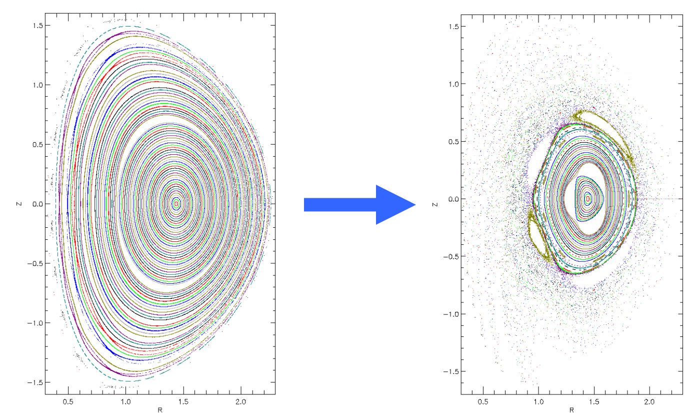
- ❑ Island width has expected scaling with perturbation amplitude.
- ❑ Peak response is at m=2.

Time-Dependent Response

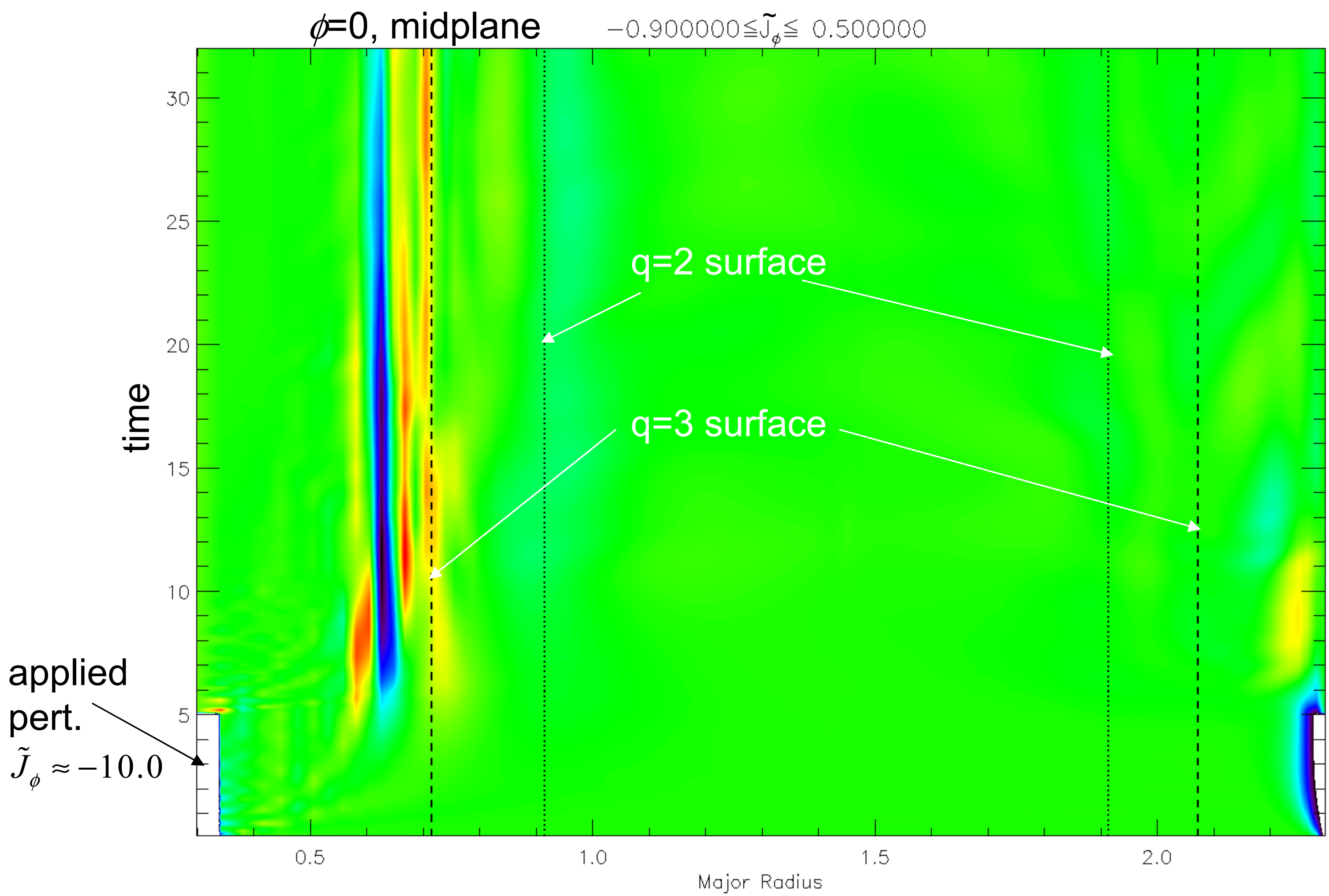




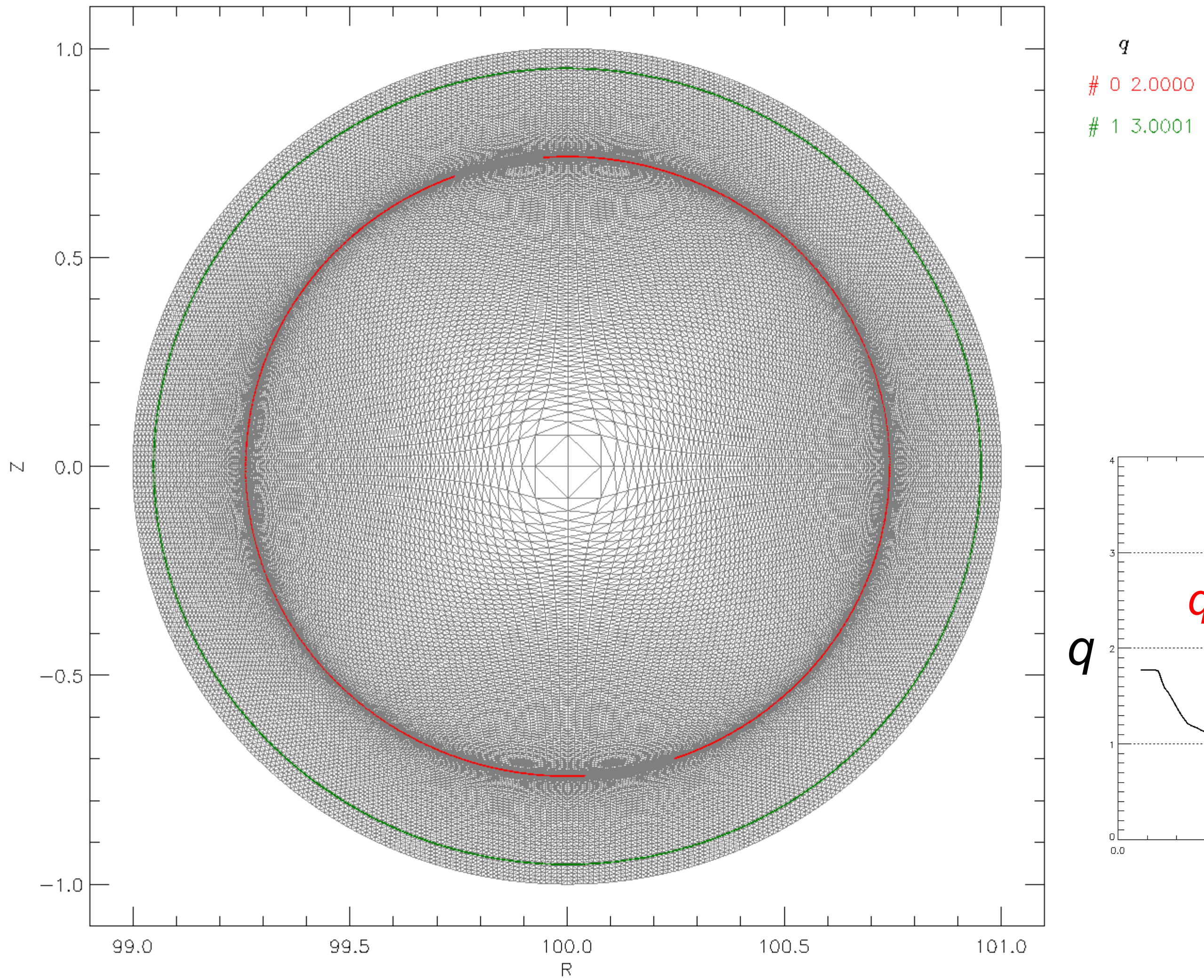
- ❑ Start with zero perturbation, ramp up linearly to full size in five Alfvén times to produce current sheets.
- ❑ Magnetic Reynolds number S = 2000; Prandtl number Pr =0.02; perturbation amplitude = 7.5×10^{-3}.
- ❑ Island size lags perturbation slightly, becomes stochastic on longer timescale.



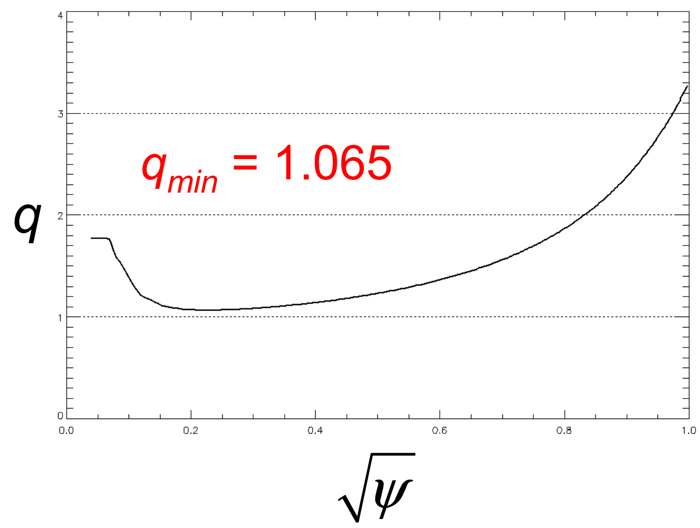
Sharp current sheets form away from the q=2 surface



Circular Cross-Section at High Aspect Ratio



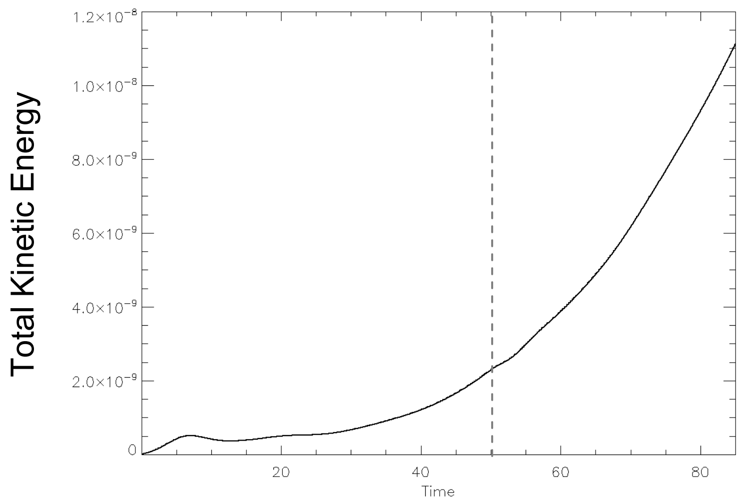
- ❑ Eliminate stochasticity complication by placing q=2 closer to plasma boundary.
- ❑ Assess effect of ramp-up speed on current sheet formation without shaping effects.

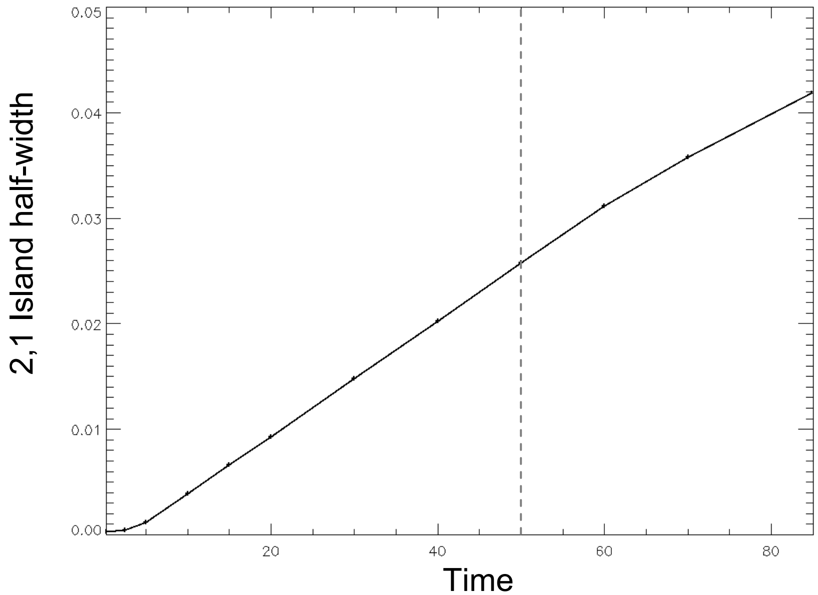


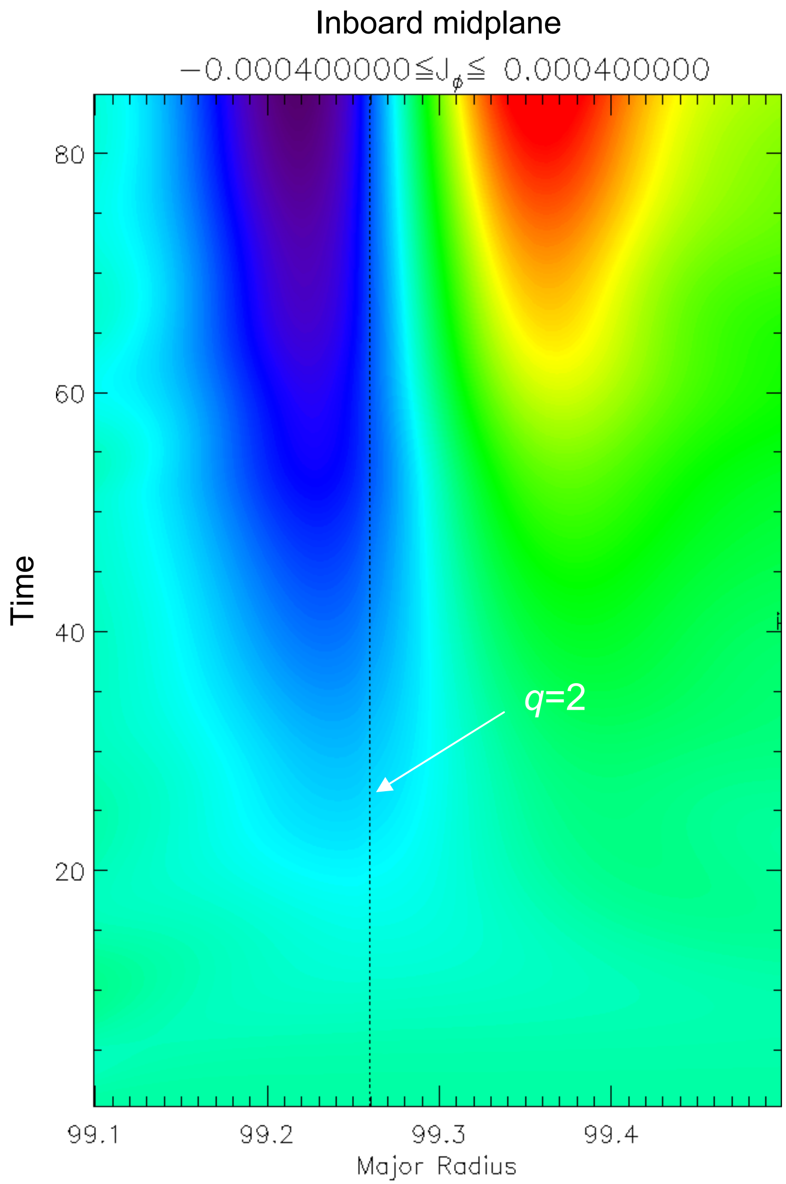
- ❑ Radial zones are packed at q=2 surface to help resolve small islands.

Ramp up pert. over 50 τ_A

- S = 2000, Pr = 0.1, pmag=5.0×10^{-4}

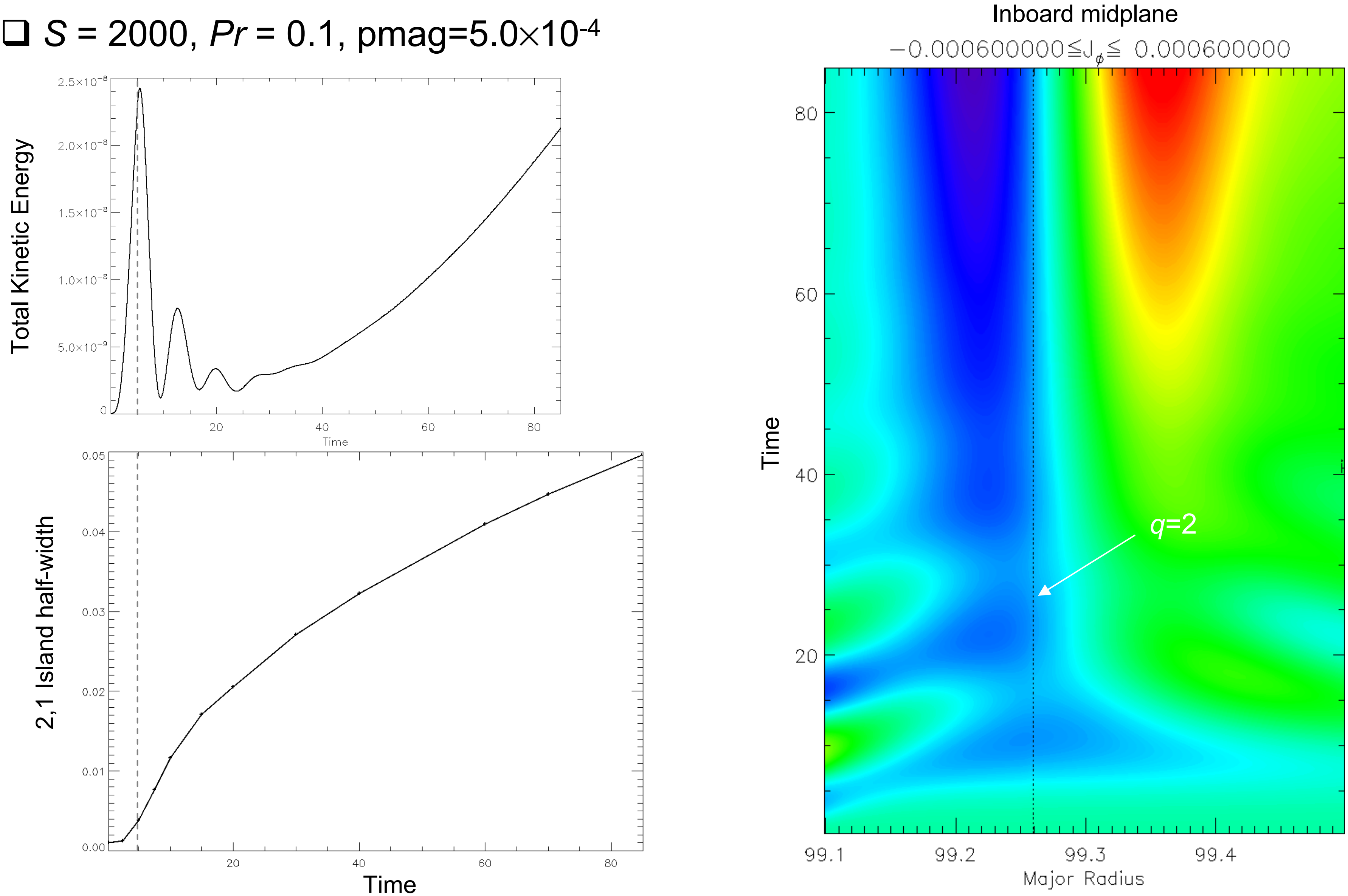






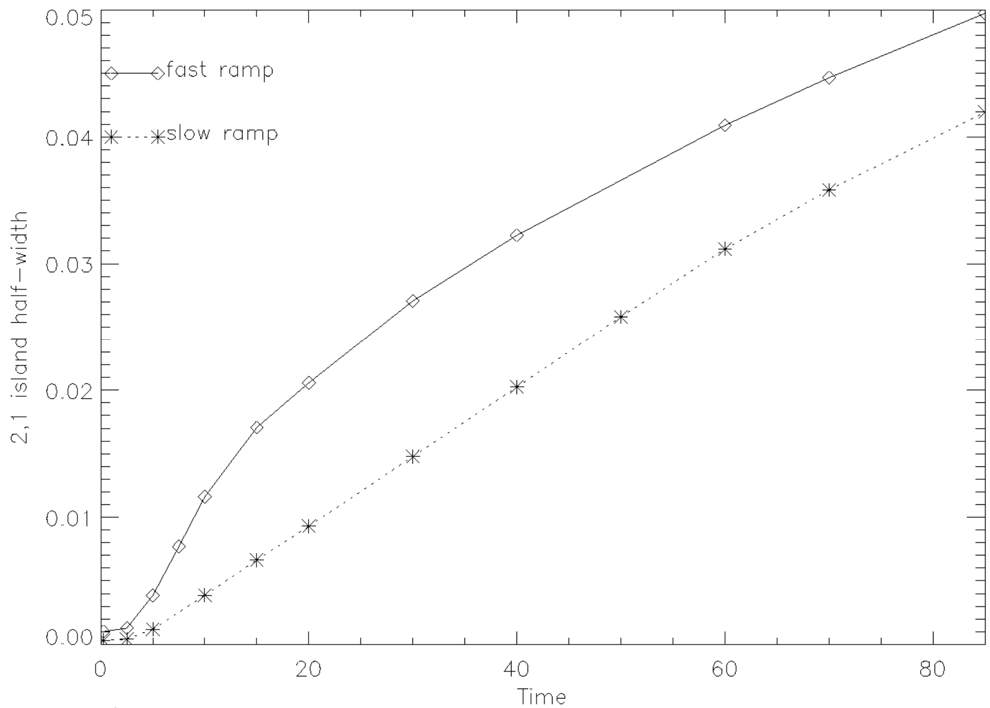
Ramp up pert. over 5 τ_A

- S = 2000, Pr = 0.1, pmag=5.0×10^{-4}

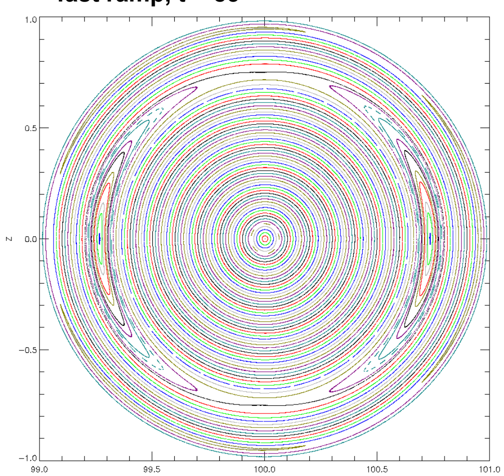


Island Width Comparison

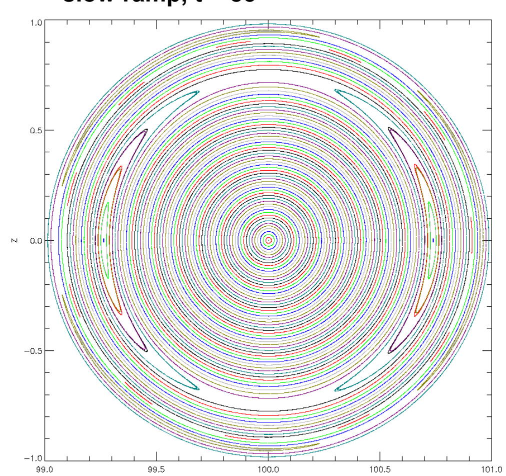
Circular cross-section, $\eta = 5\times10^{-4}$, $\nu = 5\times10^{-5}$, pmag = 5×10^{-4}



fast ramp, *t* = 85



slow ramp, *t* = 85

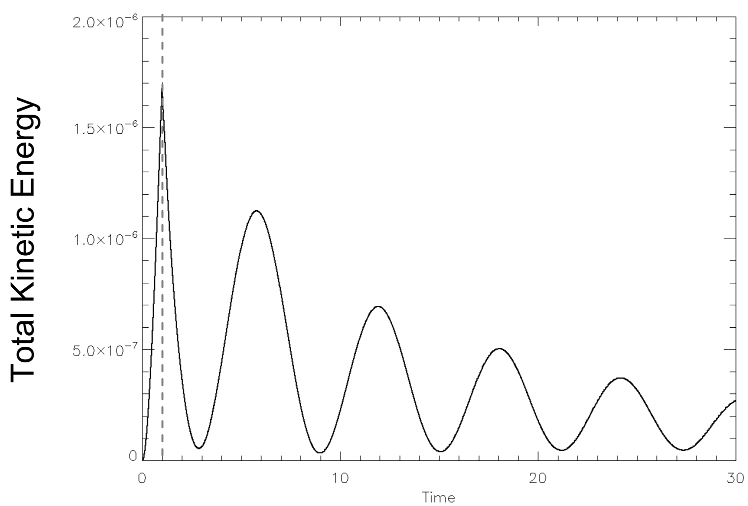


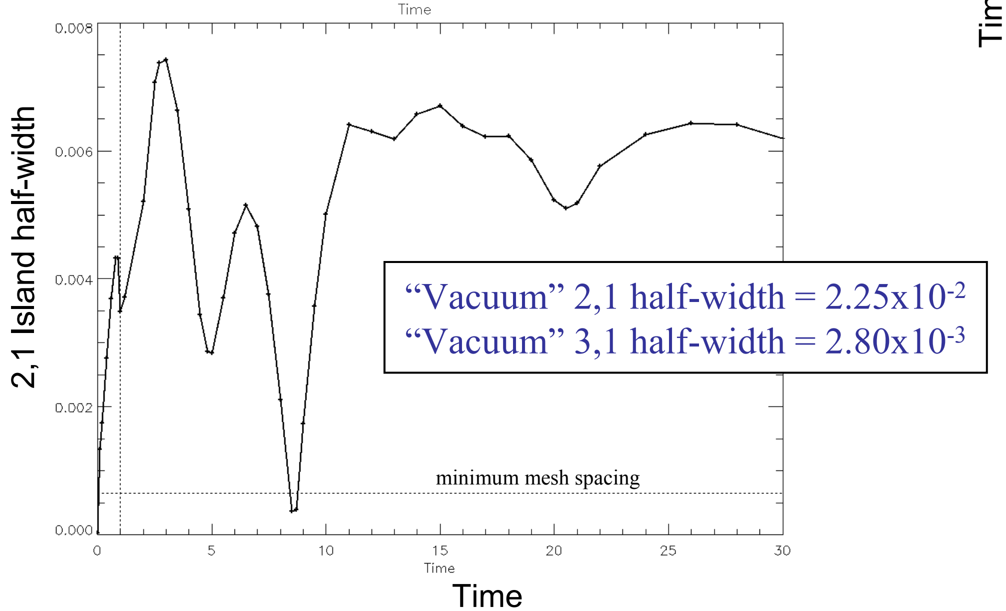
- Although the plasma response lags the perturbation, the final state should be independent of the ramp-up rate.

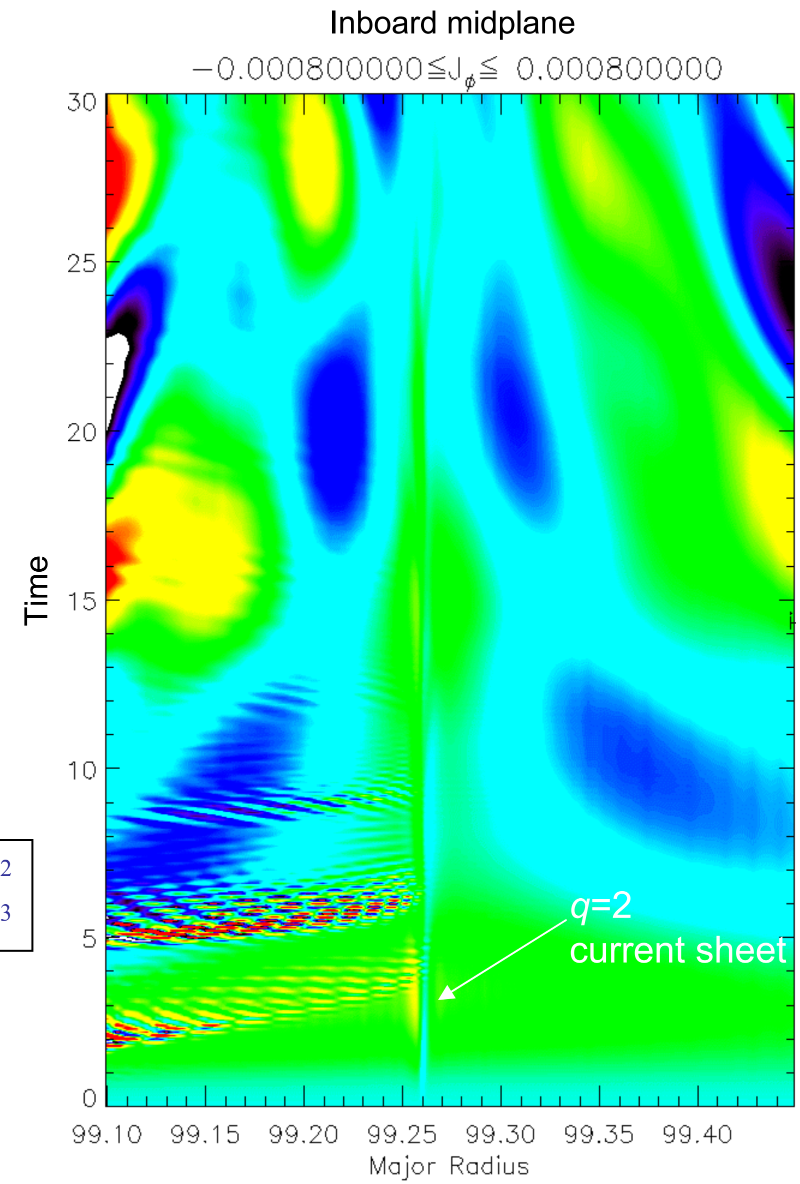
- Current sheet formation and shielding of interior plasma are not observed. To find them, use larger S, faster ramp...

Ramp up pert. over 1 τ_A

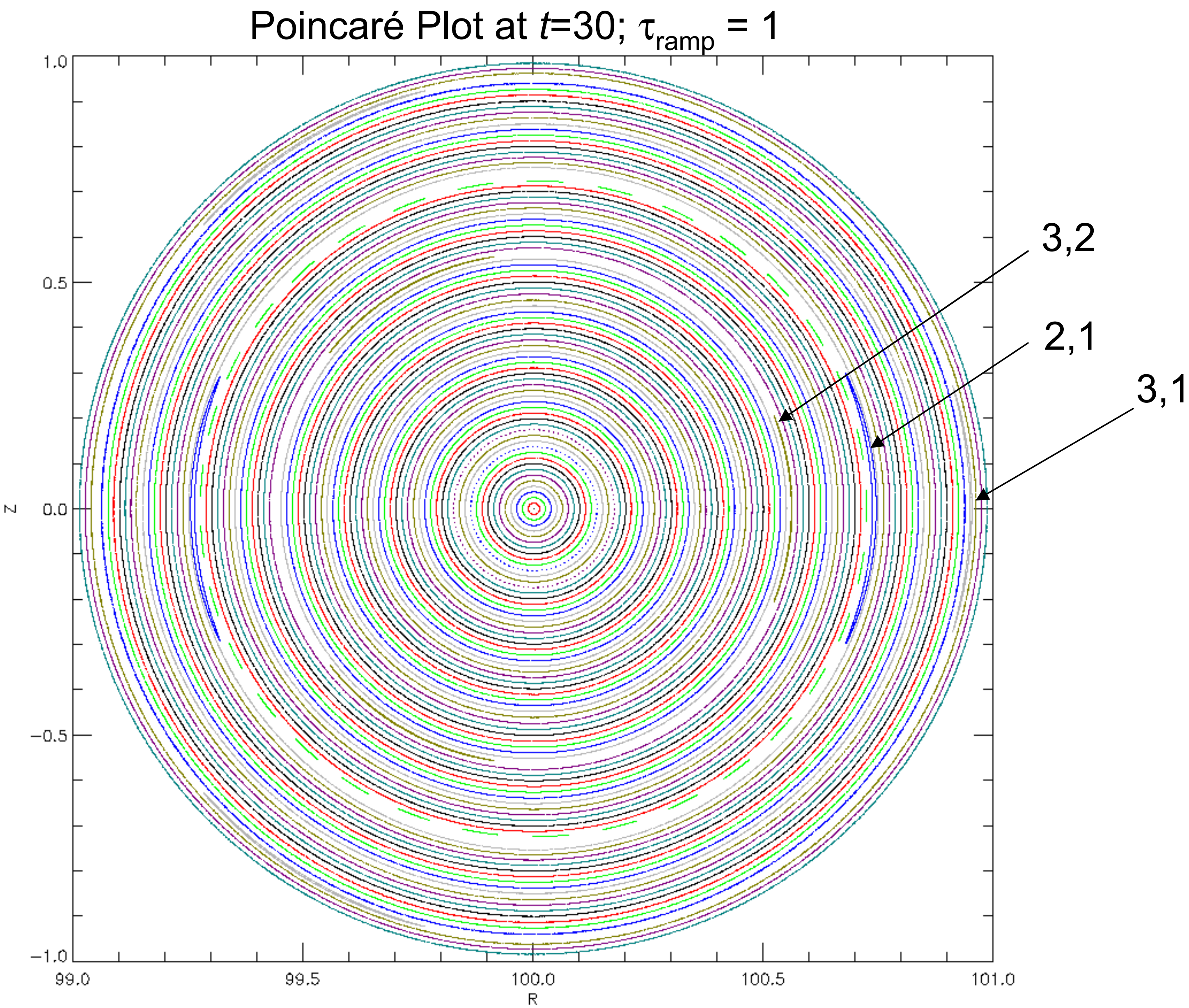
- $S = 10^6$, $Pr = 25$, pmag=2.5×10^{-4}



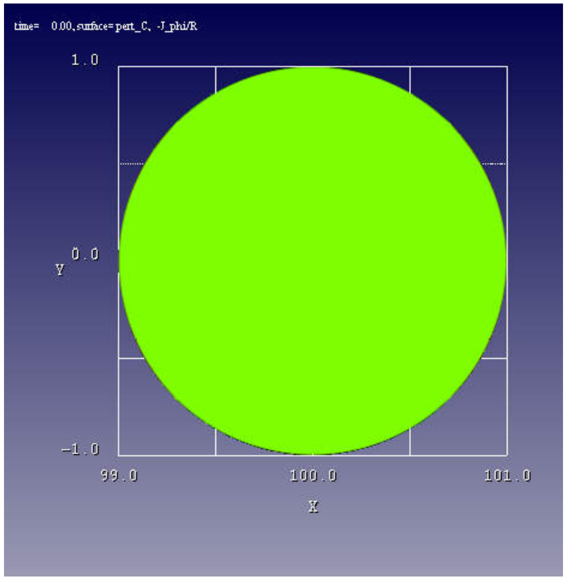




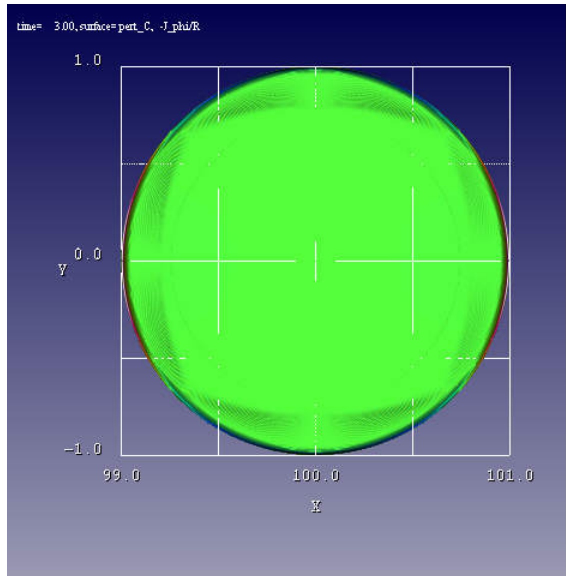
Small Islands Form as Current Sheet Decays



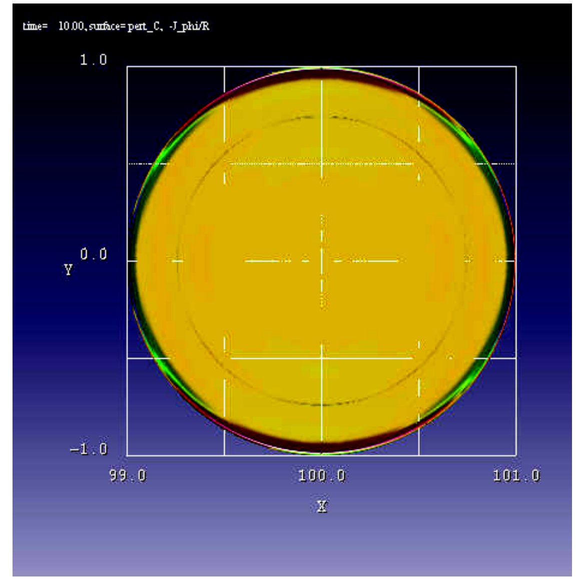
Penetration of Perturbed Current



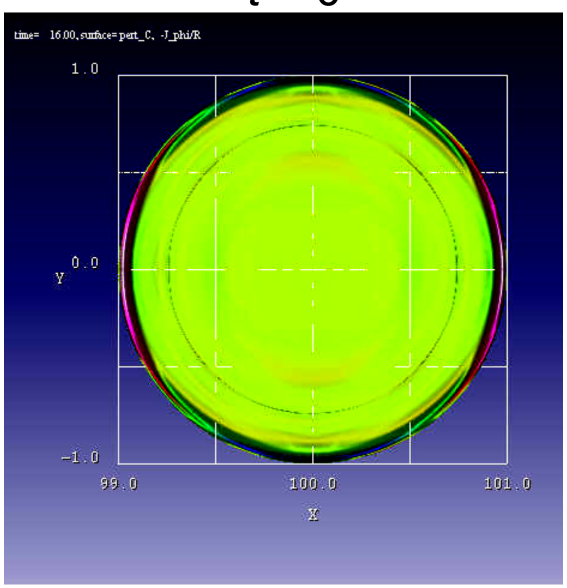
t = 0



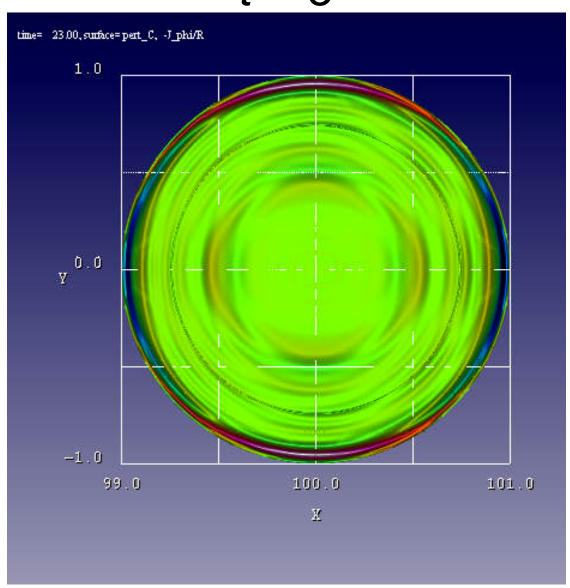
t = 3



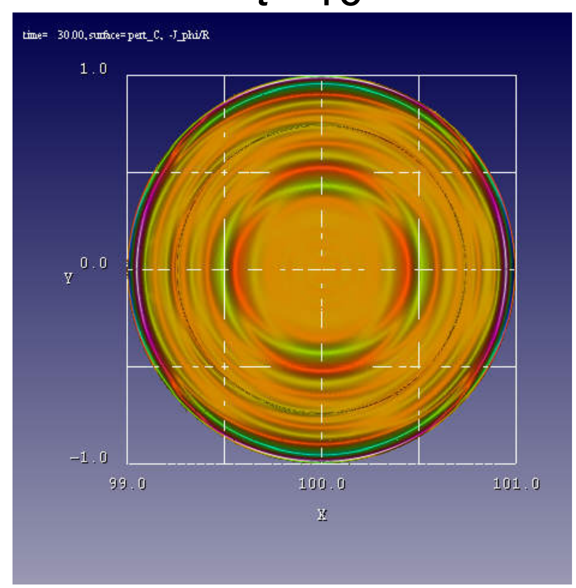
t =10



t = 16



t = 23



t = 30

Summary & Future Plans

- M3D agrees reasonably well with IPEC ideal predictions of steady-state island width responses to model perturbations in linear regime.

- Current sheet formation is only detectable with large *S*, large perturbations, and rapid ramp-ups.

- When current sheet appears, interior is shielded for several ramp-up times even as 2,1 island grows rapidly.

- Scans to follow will also include rotation effects, and may make use of the new linear M3D-C^1 code for greater computational efficiency.

- More accurate models of the NSTX error (or applied RMP) fields will give better predictive capability.