Modeling and Control of Plasma Rotation for NSTX Using Neoclassical Toroidal Viscosity

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Abstract

A one-dimensional plasma *toroidal rotation model* and its *controller* for a magnetically confined fusion device are developed in an effort to assist the continuous extraction of fusion energy.

This study is based on the experimental measurements from the National Spherical Torus Experiment (NSTX) and is aimed to capture and sustain the rotation (toroidal) momentum transport in a stable fashion and to achieve desirable plasma geometry inside the tokamak .

The neutral beam injection (NBI) being fixed, the neoclassical toroidal viscosity (NTV) will be considered in our model as the *actuator* for the controller design.

Motivation and Outline

- Plasma rotation profile can favorably affect
  - Macroscopic stability (kinks ,RWMs, ELMs, tearing modes) for disruption avoidance
  - Stability of turbulent fluctuations (drift waves from ITG and ETG)
  - Thermal diffusivity through drift wave suppression

- Rotation profile control is an important operational goal for NSTX-U to examine such physics

Outline

- Define a model for toroidal rotation and its actuators
- Build a simplified reduced order model and validate it
- Design a controller to track a desired profile

One dimensional toroidal momentum equation

$$\sum_i n_i m_i \left\langle R^2 \right\rangle \frac{\partial \omega}{\partial t} + \omega \left\langle R^2 \right\rangle \sum_i m_i \frac{\partial n_i}{\partial t}$$

Temporal change in toroidal momentum

$$+ \sum_i n_i m_i \omega \frac{\partial \left\langle R^2 \right\rangle}{\partial t} + \sum_i n_i m_i \left\langle R^2 \right\rangle \omega \left(\frac{\partial V}{\partial \rho} \right)^{-1} \frac{\partial}{\partial t} \frac{\partial V}{\partial \rho}$$

Viscous dissipation term

$$= \left(\frac{\partial V}{\partial \rho} \right)^{-1} \frac{\partial}{\partial \rho} \left[\frac{\partial V}{\partial \rho} \sum_i n_i m_i \chi_\phi \left\langle R^2 (\nabla \rho)^2 \right\rangle \frac{\partial \omega}{\partial \rho} \right]$$

Momentum loss due to the pinch

$$- \left(\frac{\partial V}{\partial \rho} \right)^{-1} \frac{\partial}{\partial \rho} \left[\frac{\partial V}{\partial \rho} \sum_i n_i m_i \omega \left\langle R^2 (\nabla \rho)^2 \right\rangle \frac{v_\rho}{|\nabla \rho|} \right]$$

$$- \sum_i n_i m_i \left\langle R^2 \right\rangle \omega \left(\frac{1}{\tau_{\phi cx}} + \frac{1}{\tau_{c\delta}} \right) + \sum_j T_j$$

Momentum loss due to charge exchange and field ripple

Torques

Toroidal momentum equation simplified into diffusion equation with torques added

$$\sum_i n_i m_i \left\langle R^2 \right\rangle \frac{\partial \omega}{\partial t} + \cancel{\omega \left\langle R^2 \right\rangle \sum_i m_i \frac{\partial n_i}{\partial t}}$$

Change in density assumed small

$$+ \cancel{\sum_i n_i m_i \omega \frac{\partial \left\langle R^2 \right\rangle}{\partial t}} + \cancel{\sum_i n_i m_i \left\langle R^2 \right\rangle \omega \left(\frac{\partial V}{\partial \rho} \right)^{-1} \frac{\partial}{\partial t} \frac{\partial V}{\partial \rho}}$$

Changes in geometry assumed small

$$= \left(\frac{\partial V}{\partial \rho} \right)^{-1} \frac{\partial}{\partial \rho} \left[\frac{\partial V}{\partial \rho} \sum_i n_i m_i \chi_\phi \left\langle R^2 (\nabla \rho)^2 \right\rangle \frac{\partial \omega}{\partial \rho} \right]$$

$$- \left(\frac{\partial V}{\partial \rho} \right)^{-1} \frac{\partial}{\partial \rho} \left[\frac{\partial V}{\partial \rho} \sum_i \cancel{n_i m_i \omega \left\langle R^2 (\nabla \rho)^2 \right\rangle \frac{v_\rho}{|\nabla \rho|}} \right]$$

Pinch ignored

$$- \sum_i n_i m_i \left\langle R^2 \right\rangle \omega \left(\cancel{\frac{1}{\tau_{\phi cx}}} + \frac{1}{\tau_{c\delta}} \right) + \sum_j T_j$$

Important only at the edges

One dimensional *Simplified* toroidal momentum equation

$$nm\left\langle R^2\right\rangle\frac{\partial\omega}{\partial t}=\left(\frac{\partial V}{\partial\rho}\right)^{-1}\frac{\partial}{\partial\rho}\left[\frac{\partial V}{\partial\rho}nm\chi_\phi\left\langle R^2(\nabla\rho)^2\right\rangle\frac{\partial\omega}{\partial\rho}\right]+T_{\mathrm{NBI}}+T_{\mathrm{NTV}}$$

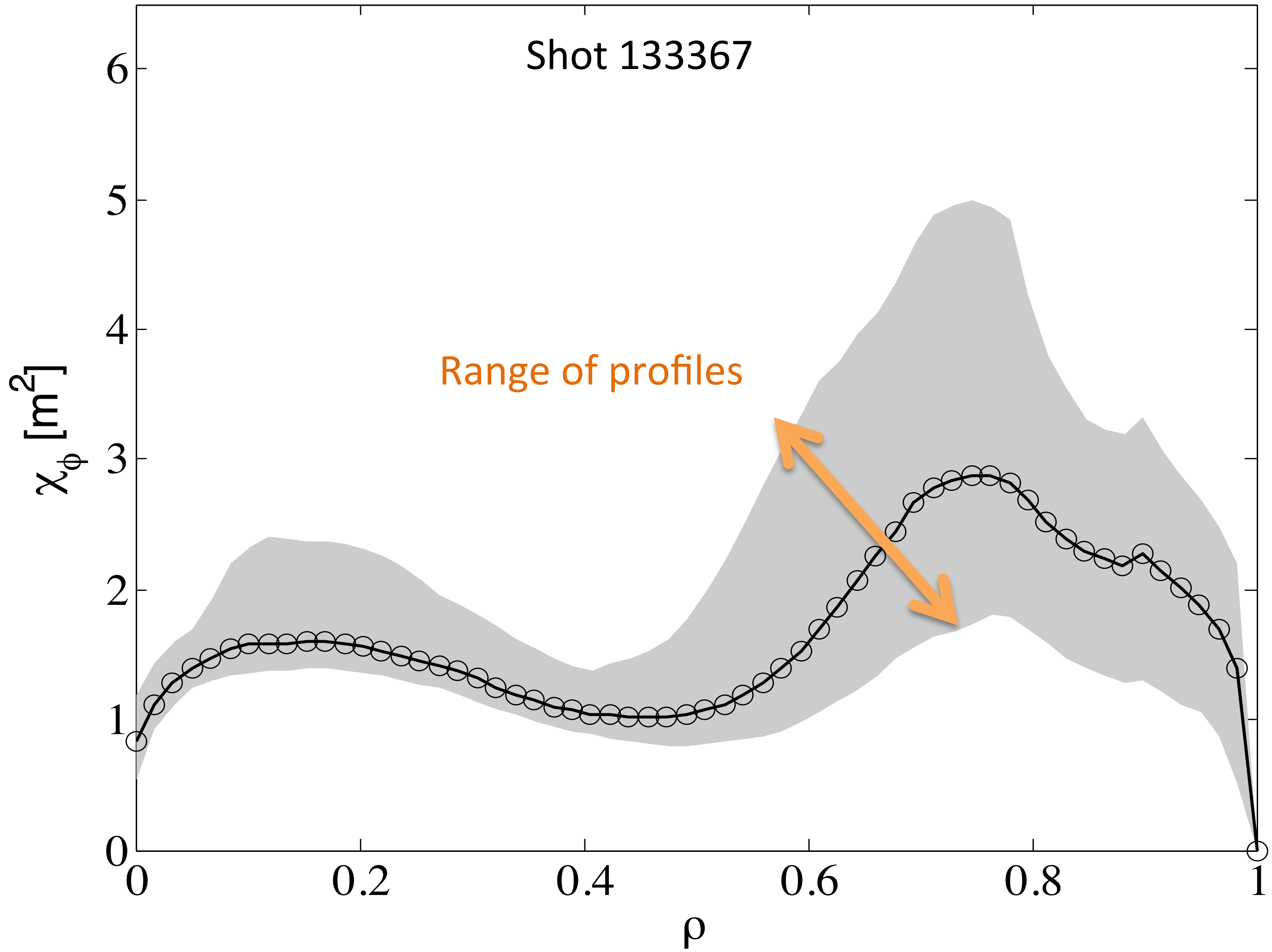
Component 1 Component 2 Component 3

Boundary conditions: Symmetry at plasma center and Dirichlet condition at the edge:

$$\left.\frac{\partial\omega}{\partial\rho}\right|_{\rho=0}=0\quad\text{and}\quad\left.\omega\right|_{\rho=1}=0$$

T_{NBI} And T_{NTV} Will be our actuators for control purposes

Component 1: The momentum diffusivity χ_ϕ



χ_ϕ is calculated from TRANSP (Analysis)

The time-average values and their curve-fits are shown by the circles and the solid lines respectively.

Component 2: Neutral Beam Injection Torque T_{NBI}

$$T_{\mathrm{NBI}}(t,\rho) = \overline{T}_{\mathrm{NBI}}(t) F_{\mathrm{NBI}}(\rho)$$

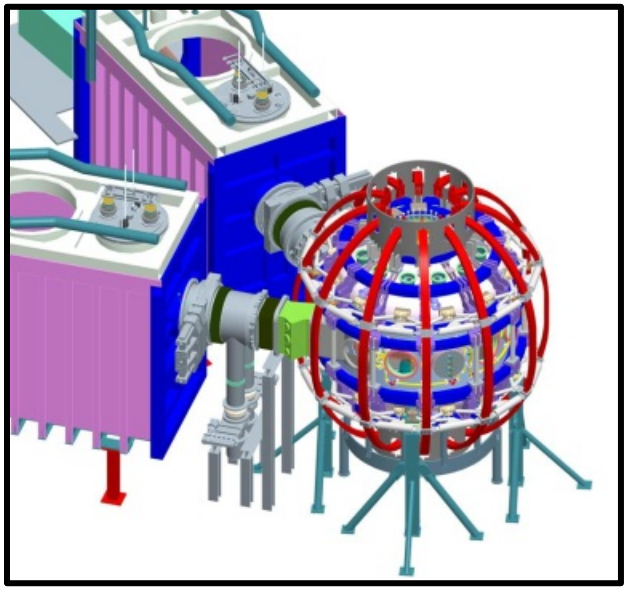
Injecting high-speed neutral atoms into the center of the plasma

Where:

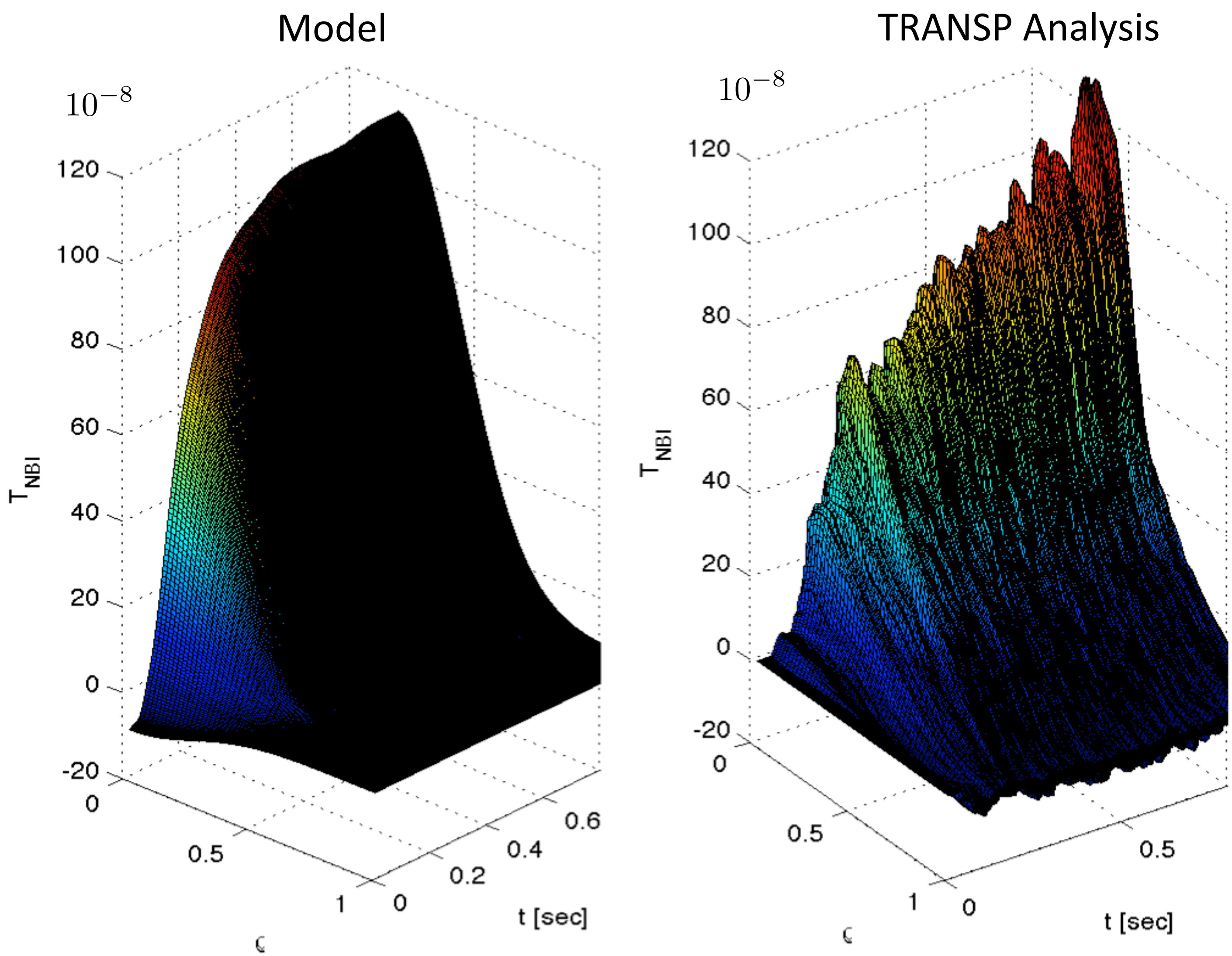
$$\left\{ \begin{aligned} & F_{\mathrm{NBI}}(\rho) = a_{\mathrm{NBI}} \exp\left(-\frac{\rho^2}{2\sigma^2{}_{\mathrm{NBI}}} \right) \\ & \frac{\partial \overline{T}_{\mathrm{NBI}}}{\partial t} + \frac{\overline{T}_{\mathrm{NBI}}}{\tau_{\mathrm{NBI}}} = \kappa_{\mathrm{NBI}} P_{\mathrm{NBI}}(t) \end{aligned} \right\}$$

σ_{NBI} and κ_{NBI} From TRANSP

NSTX-U NBI system

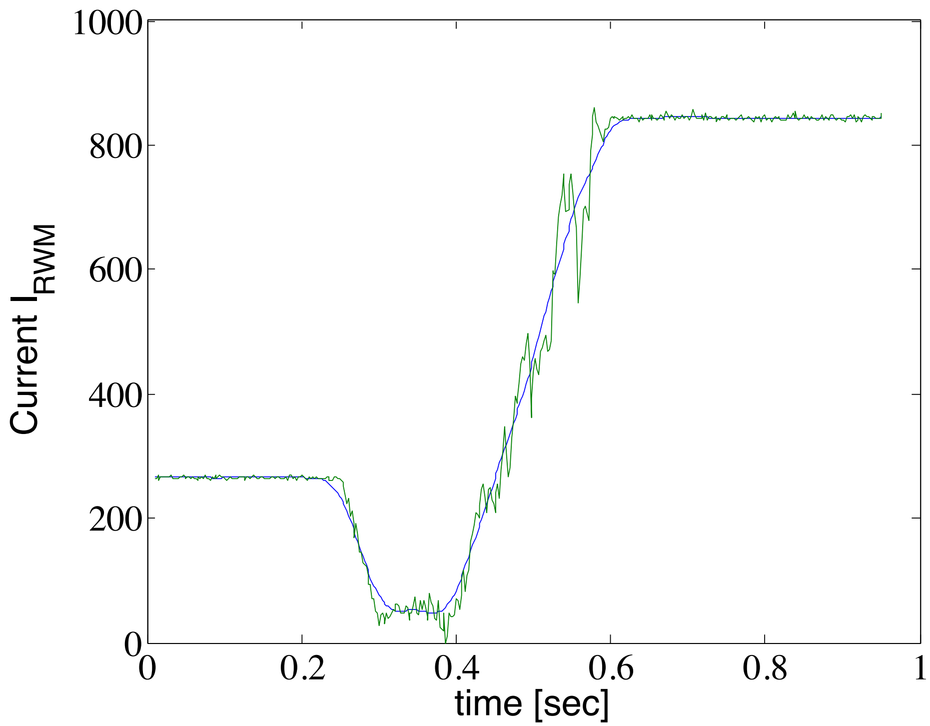


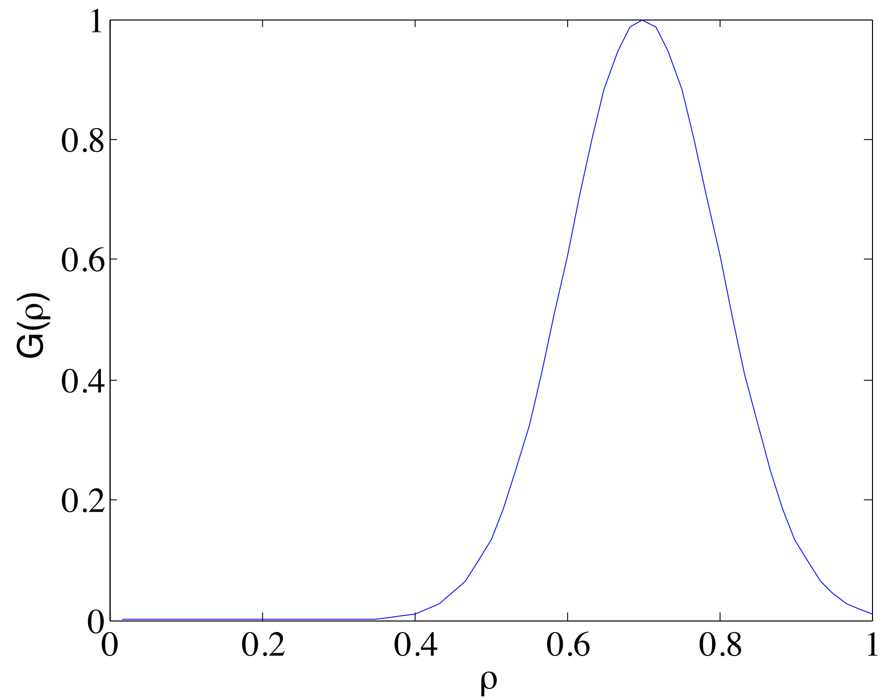
NBI torque from Model similar to the one from TRANSP analysis



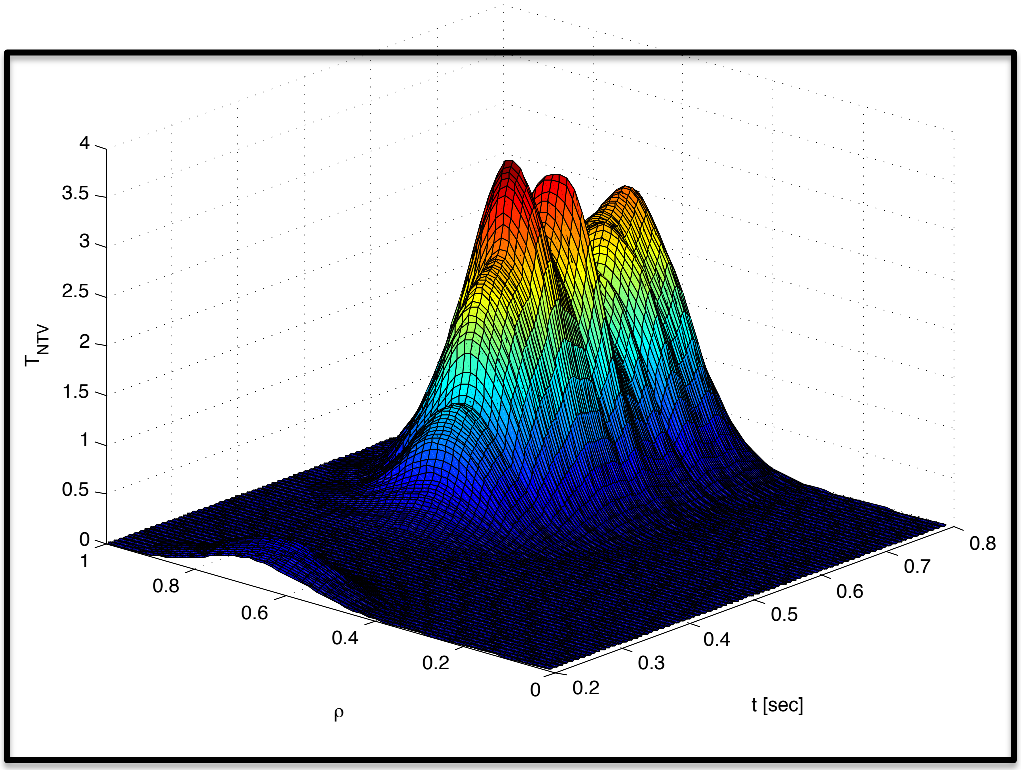
Component 3: Neoclassical Toroidal Viscosity Torque T_{NTV}

$$T_{NTV}(t,\rho) = KG(\rho)\left\langle R^2 \right\rangle I^2(t)\omega(t,\rho)$$



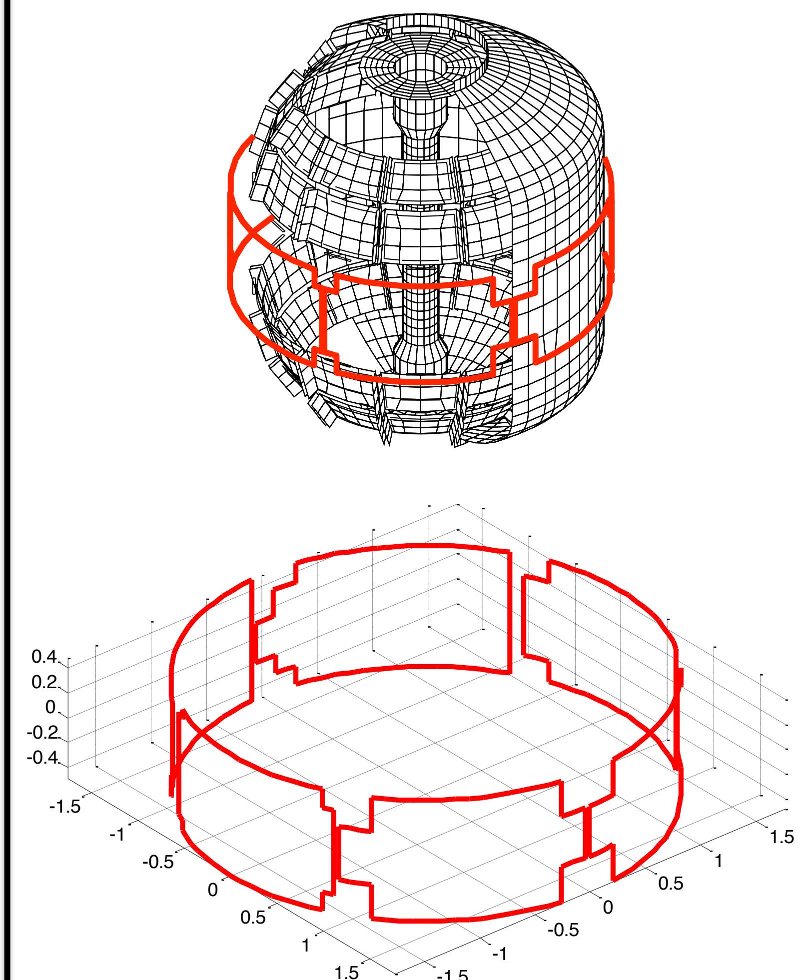


NTV torque obtained from Model is peaked toward the edge



T_{NTV} is peaked toward the edge

[NSTX-U 3D coils (used to generate NTV)]

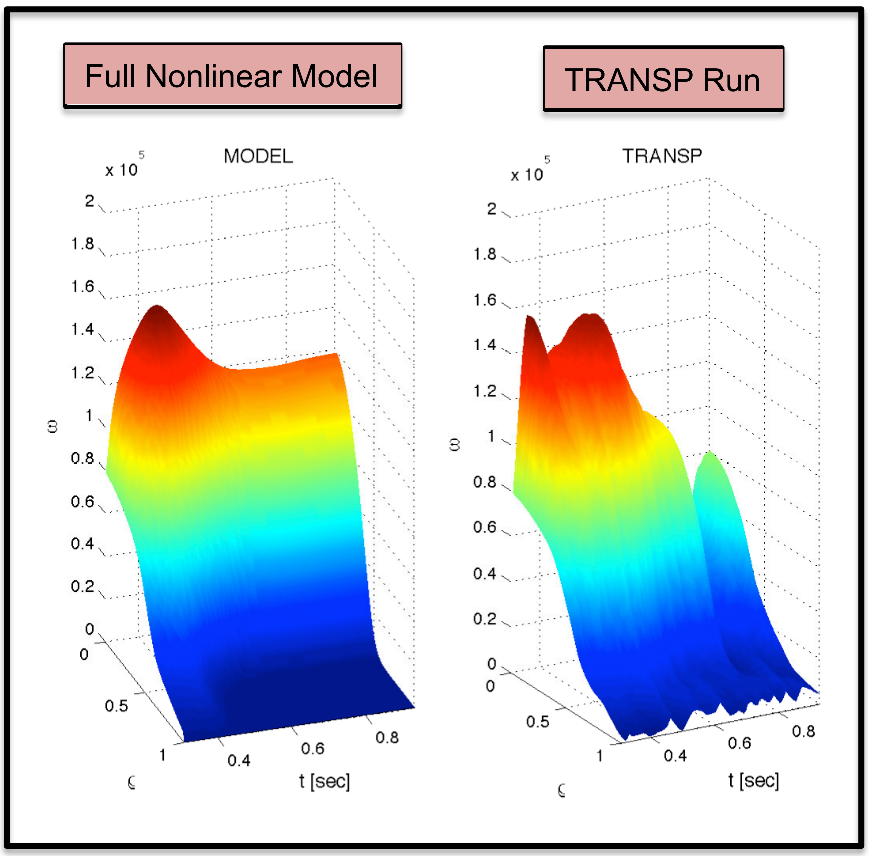


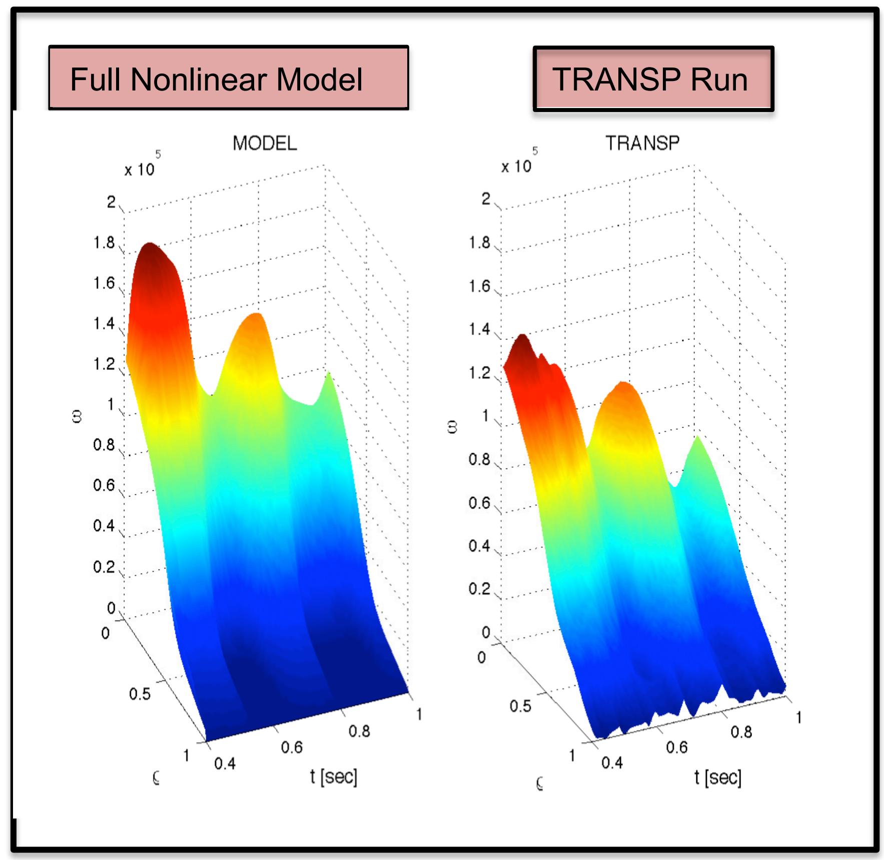
Model of rotational frequency ω captures main dynamics of TRANSP Run

- Momentum force balance equation (Simplified Model)

$$nm\left\langle R^2\right\rangle \frac{\partial \omega}{\partial t} = \left(\frac{\partial V}{\partial \rho}\right)^{-1} \frac{\partial}{\partial \rho}\left[\frac{\partial V}{\partial \rho} nm\chi_\phi \left\langle R^2(\nabla\rho)^2\right\rangle \frac{\partial \omega}{\partial \rho}\right] + T_{\mathrm{NBI}} + T_{\mathrm{NTV}}$$

- Model kept the same, Density and (NTV + NBI) Torques proper adapted to another shot





Bessel Spectral decomposition of ω

- Bessel functions (r = 10 states are used)

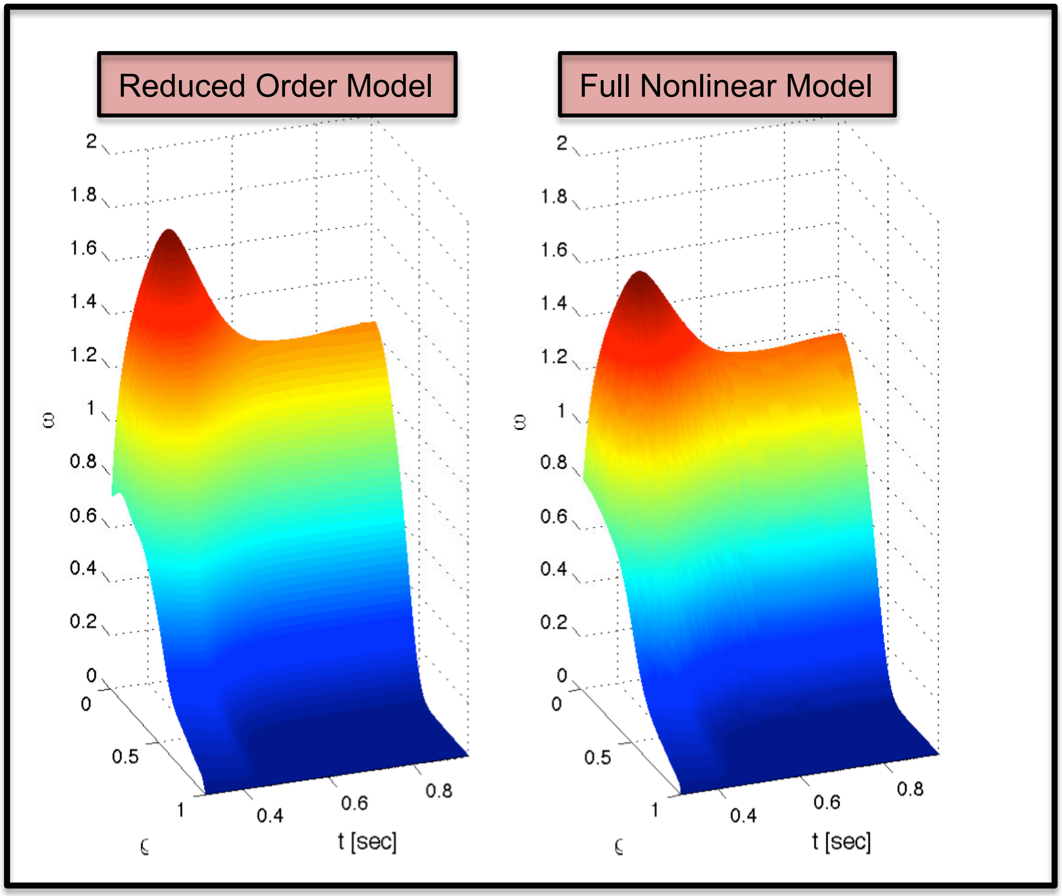
$$\omega(\rho, t) = x_0(t) + \sum_{n=1}^{r} x_n(t) J_0(\alpha_{0,n}\rho)$$

Where :

J_0 Bessel function of the first kind

$\alpha_{0,n}$ is a root, numbered n associated with the Bessel Function J_0

- The states to control will be a row of coefficients x_i of the Bessel Functions



Control Design strategy

Design a full state feedback controller + feedforward

(From the Reduced Order Model)

Combine to obtain a **Compensator**

(*For the Full Nonlinear Model*)

Design an Observer

(For the Full Linear Model)

Control Design Using One Actuator: Coil Current (NTV)

- Simplified Model with Fixed NBI

$$(nm)\left\langle R^2\right\rangle \frac{\partial \omega}{\partial t} = \left(\frac{\partial V}{\partial \rho}\right)^{-1} \frac{\partial}{\partial \rho}\left[\frac{\partial V}{\partial \rho}(nm)\chi_\phi \left\langle R^2 (\nabla\rho)^2\right\rangle \frac{\partial \omega}{\partial \rho}\right] + T_{\mathrm{NBI}} - T_{\mathrm{NTV}}$$

Where :

$$T_{\mathrm{NBI}}(\rho) = T_{\mathrm{NBI0}}(\rho)$$ ***(Fixed in time)***

$$T_{\mathrm{NTV}}(t,\rho) = K\, G(\rho) \left\langle R^2\right\rangle I^2(t)\, \omega(t,\rho)$$ ***(Bilinearity)***

- Linearization

$$\omega(t,\rho) = \omega_0(\rho) + \omega_1(t,\rho)$$

$$I^2(t) = I_0^2 + I_1^2(t)$$

$$T_{\mathrm{NBI}}(\rho) = T_{\mathrm{NBI0}}(\rho)$$

Where : ω_0 is the steady state reached for a given NBI torque T_{NBI0} and current I_0^2
ω_1 and I_1 are the respective perturbations to the equilibriums ω_0 and I_0^2

Reducing the system into a state space realization

- State Space Realization

In Bessel Basis

$$\dot{\omega}_1 = A\,\omega_1 + B\,u_1 \qquad y_1 = C\omega_1$$

$$\dot{x}_1 = A_r\,x_1 + B_r\,u_1 \qquad y_1 = C_r x_1$$

Where :

$$A = \frac{1}{nm\left\langle R^2\right\rangle}\left[\left(\frac{\partial V}{\partial \rho}\right)^{-1}\frac{\partial}{\partial \rho}\left[\frac{\partial V}{\partial \rho}(nm)\chi_\phi\left\langle R^2(\nabla\rho)^2\right\rangle\frac{\partial}{\partial \rho}\right] - KG(\rho)\langle R^2\rangle I_0^2\right]$$

$$B = -\frac{1}{nm\left\langle R^2\right\rangle}KG(\rho)\langle R^2\rangle\omega_0$$

u_1 is the **control input** variable which represents the current perturbation I_1^2
y_1 is the **output** variable (sensor measurements)

Control Design Using One Actuator: Feedback law

- Non-Zero Target state: another steady state

$$\boxed{\begin{bmatrix} x_{1d} \\ u_{1d} \end{bmatrix} = \begin{pmatrix} A & B \\ C & 0 \end{pmatrix}^{-1} \begin{bmatrix} 0 \\ I \end{bmatrix} r_{1d} = \begin{bmatrix} N_x \\ N_u \end{bmatrix} r_{1d}}$$

Where :

$$x_{1d} = x_d - x_0$$

$$u_{1d} = u_d - u_0$$

$$r_{1d} = r_d - r_0$$

- Control Law

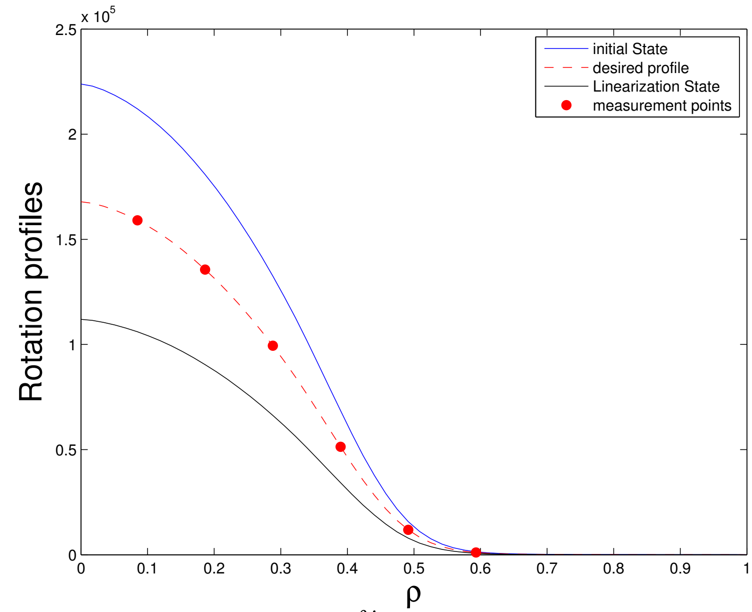
$$\boxed{u_1 = u_{1d} - K(x_1 - x_{1d}) = -Kx_1 + Fr_{1d}}$$

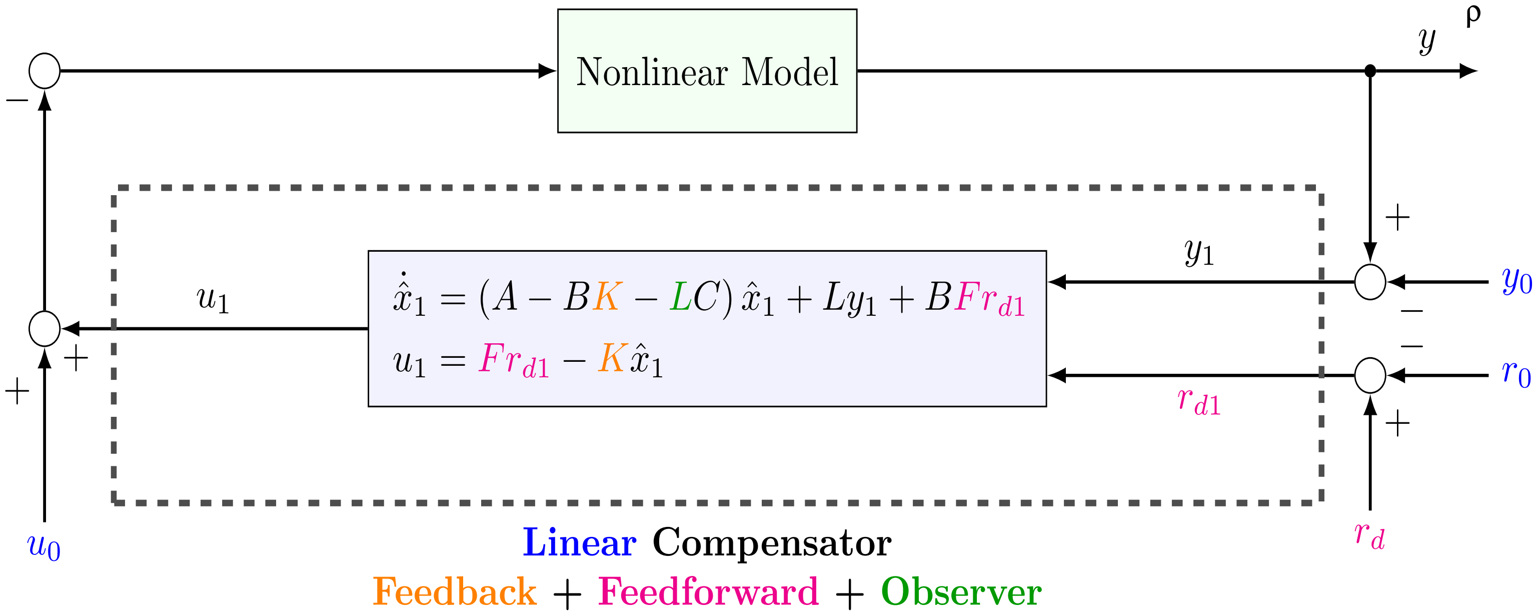
Where : $F \equiv N_u + KN_x$

- Design of K $\longrightarrow$ Linear Quadratic Regulator (LQR)

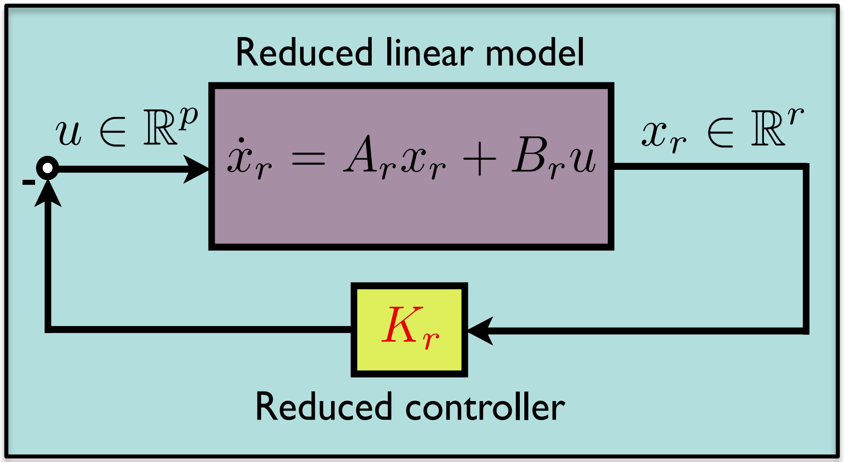
Control Design Using One Actuator: Coil Current (NTV)

- One Actuator : Coil current (scalar)
- 6 sensors: CHERS
  (Charge exchange recombination spectroscopy)
- Need to have **Desired** profile below **Initial** profile (*Active **Drag** only*)





Full State Feedback Control Design



$r =$ dim. of reduced state

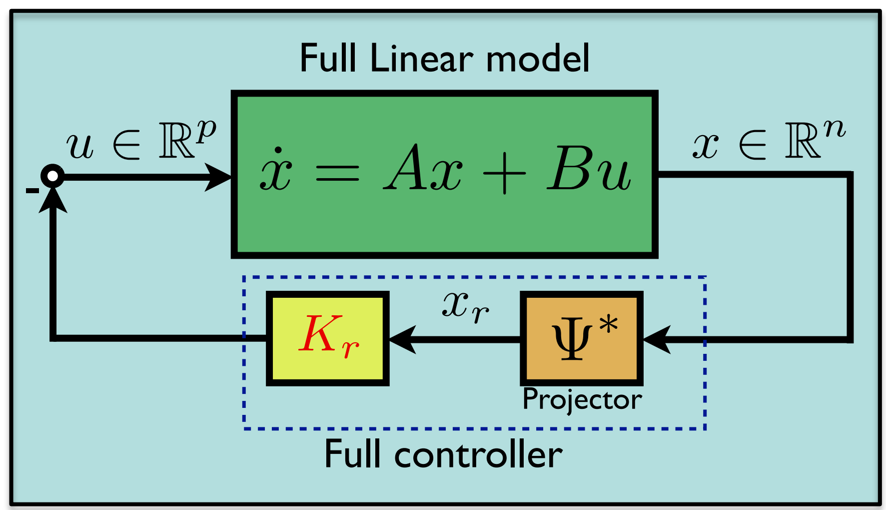
$n =$ dim. of full state

$p =$ dim. of the input

1. Design a *controller* for the reduced order model by using Linear Quadratic Regulators (LQR)

2. Use *controller* combined with a projector: *new controller* of the *full linear model*

3. Replace Full linear model by *Full Nonlinear Model without* changing the design of new controller



Gain matrix Q for the LQR is investigated to determine the impact on the controller

Minimize Quadratic Cost Function : $\mathcal{J} = \int_{t_j}^{\infty} \left(x^T Q x + u^T R u\right) dt$

Where :

$Q \geq 0 \quad R > 0$ are symmetric, positive (semi-)definite weight matrices

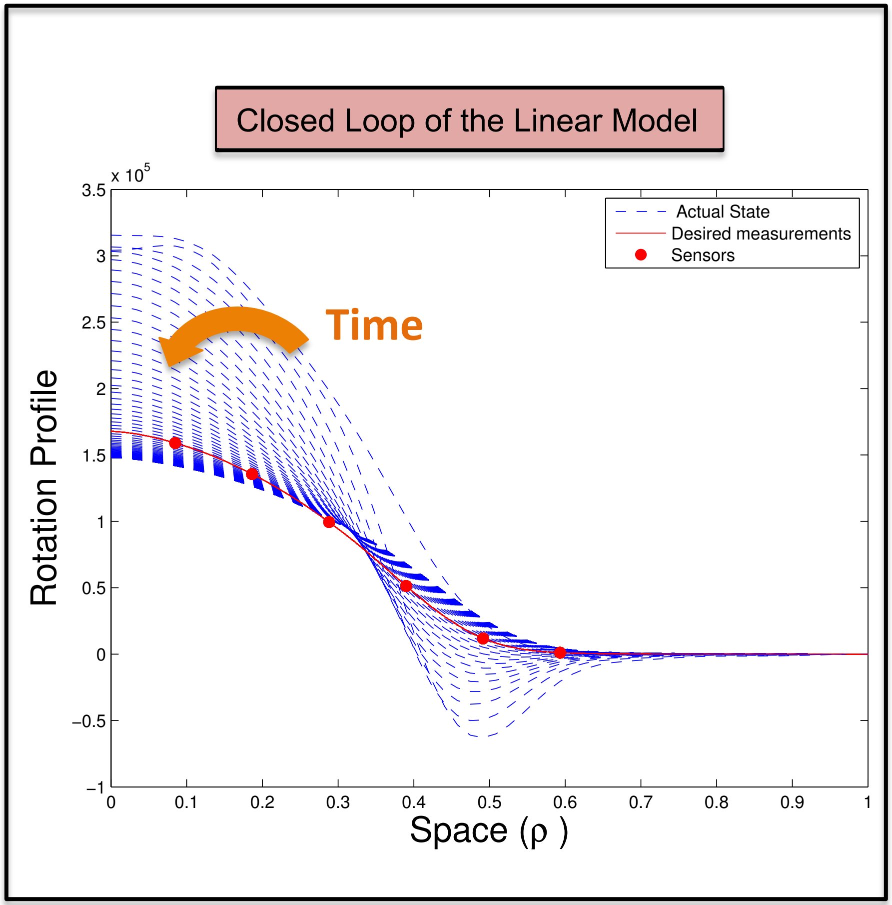
$$\boxed{K = -R^{-1} B^T P}$$

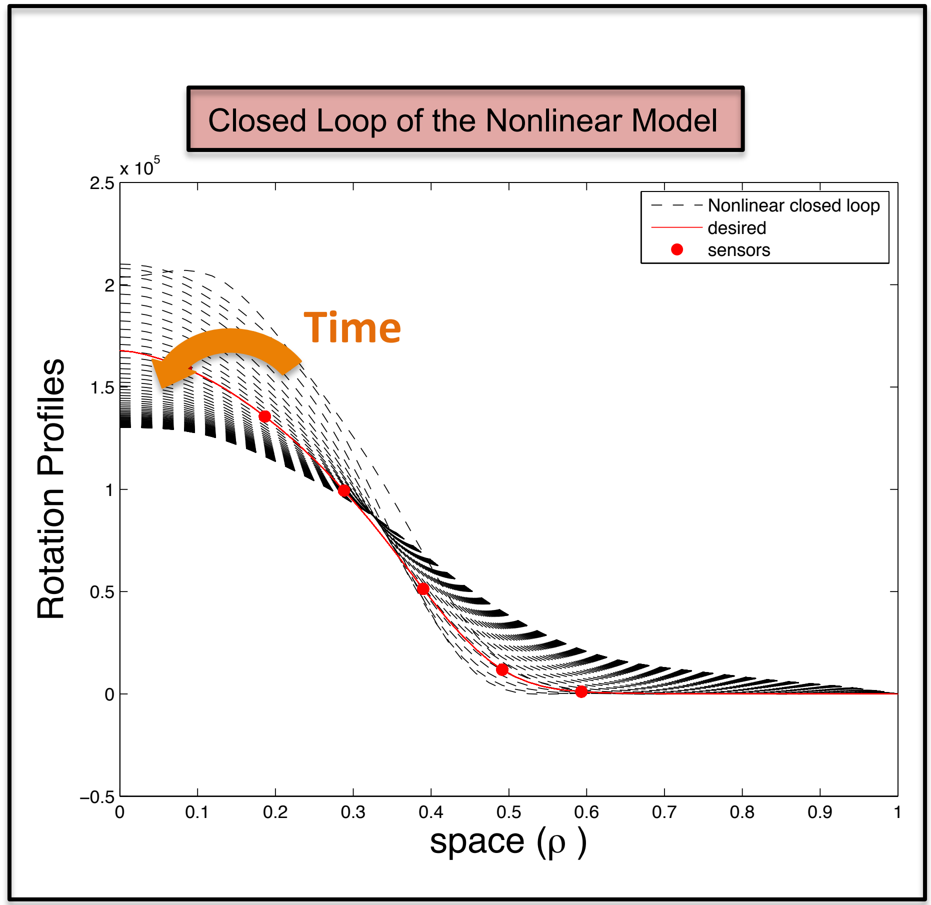
Where : P is a solution of the ***Algebraic Riccati Equation***

$$PA + A^T P - PBR^{-1} B^T P + Q = 0$$

Defining Q as the output gain yields to overshoot

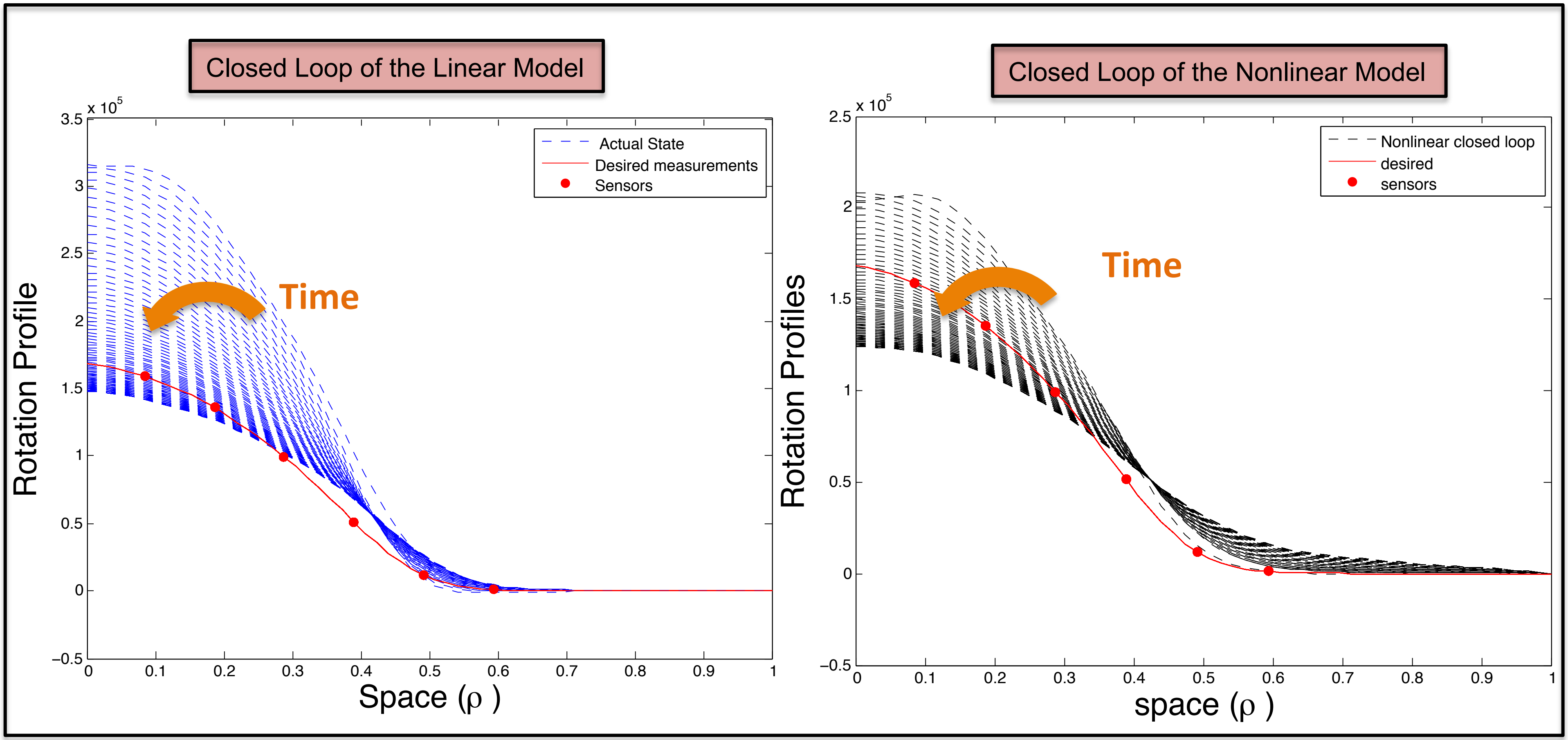
$$Q = a\,C^T C \qquad R = 1$$



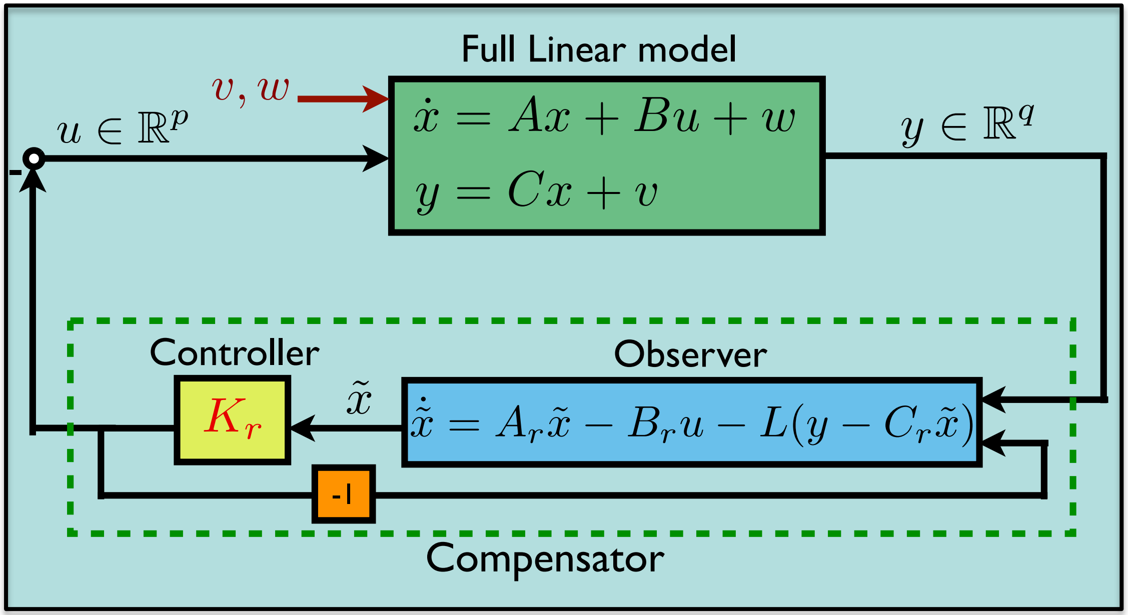


Defining Q as the whole state gain yields a better controller

$$Q = a\,I \qquad R = 1$$



Output Feedback Control Design



$p =$ dim. of the input

$q =$ dim. of the output

$v =$ sensor noise

$w =$ process noise

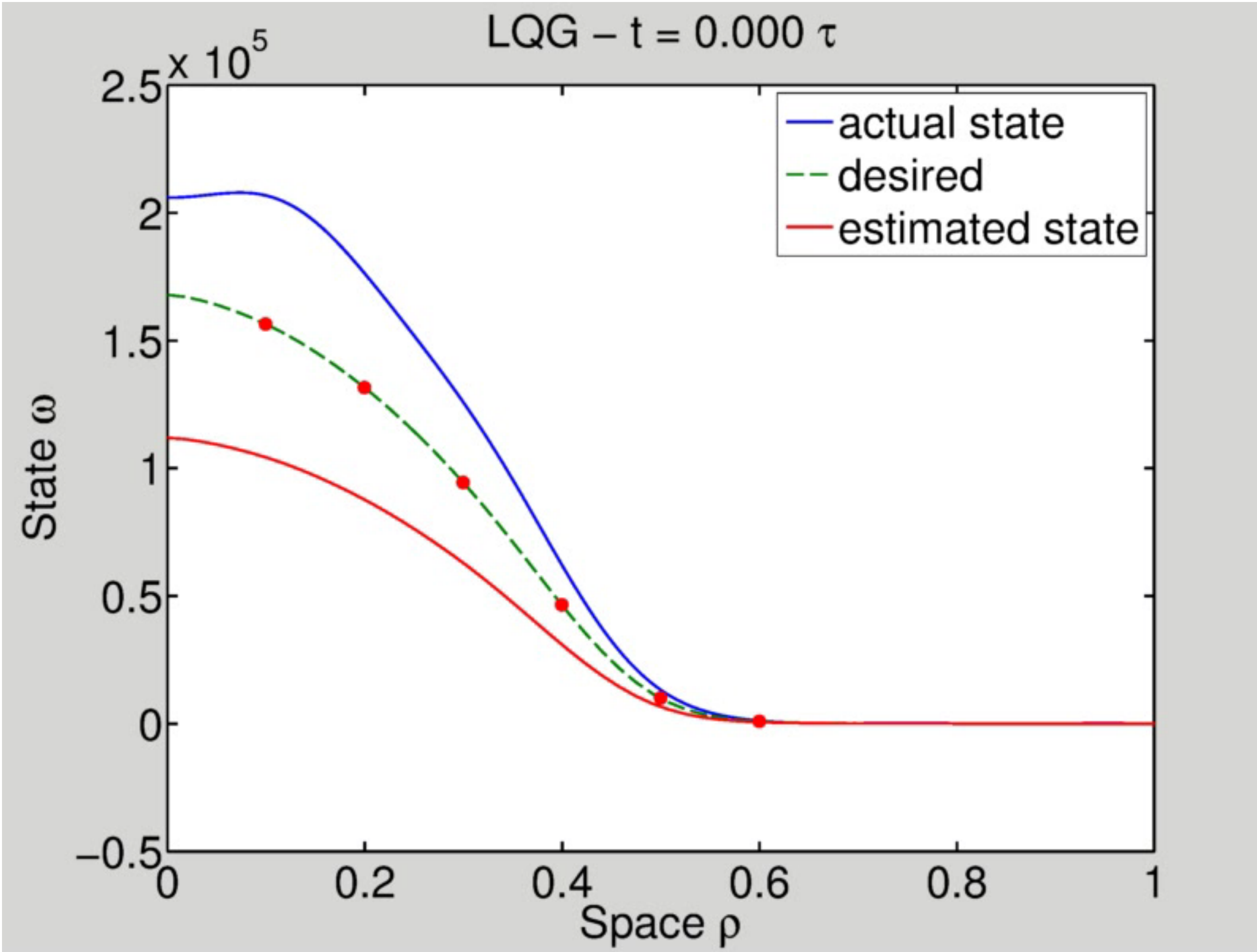
1. Design ***observer*** for the linearized model using a *Kalman filter*

2. Combine it with controller to obtain a ***compensator***

3. Full linear model replaced by *Full Nonlinear Model*, without changing the design of the *compensator*

4. Sense 6 points spread inside the tokamak in the radial direction

Results: Output feedback control applied on the Full Nonlinear Model successfully allow reaching the desired profile



Conclusions

- Developing of a simplified model of rotation momentum that captures most of the dynamics and validating it
- Successfully built low order linear controller for large order nonlinear system
- Control rotation profile with only one actuator and stabilizing it around a desired shape

This work will be used to control the plasma rotation profile in NSTX-U

- Adding more actuators (NBI) and improving their design will provide better control

- Methodology can be applied to real tokamak (NSTX-U) through real time control

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