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Energy Transport via Compressional Alfven Eigenmodes Coupling to Kinetic Alfven Waves

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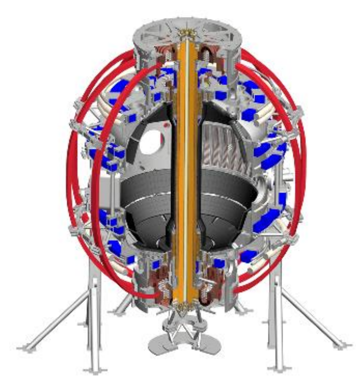
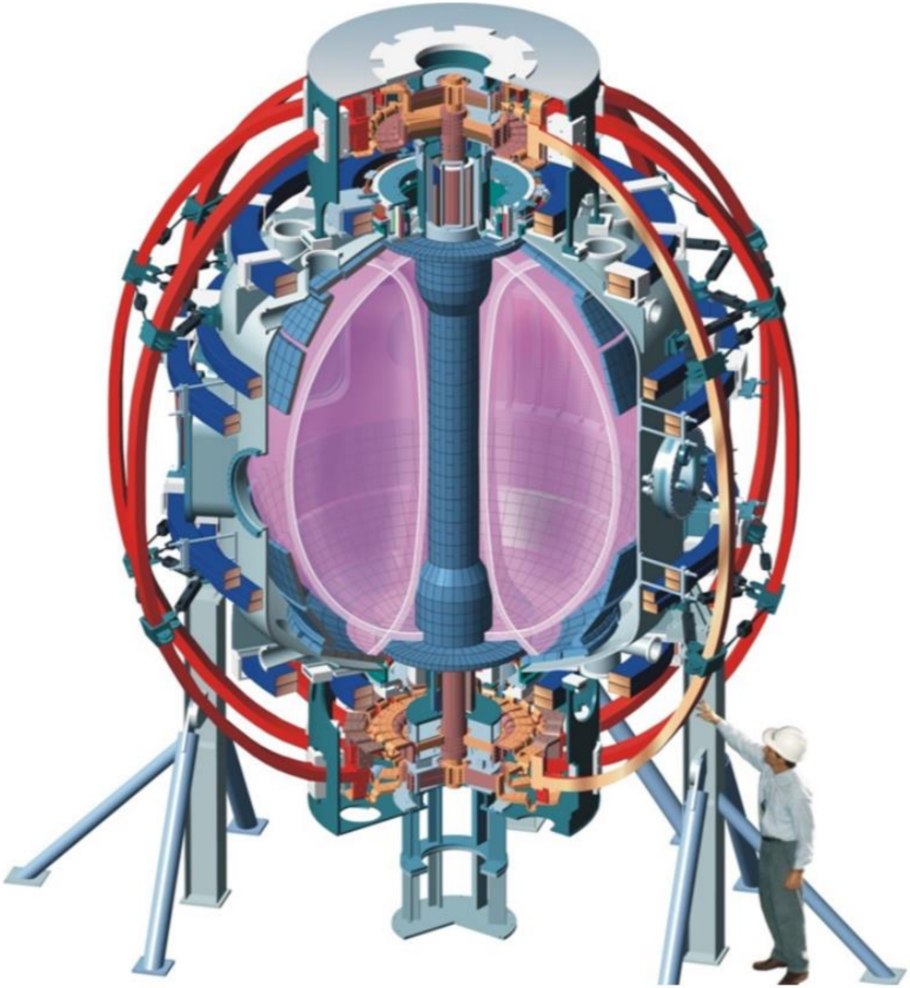


Energy confinement is important in fusion

- Confinement is the challenge
  - Tokamaks are hot (150 million degrees)
    - Need magnetic confinement
  - Heat/Energy is escaping
- Loss mechanisms are an area of intense investigation
- Tokamaks often are heated by beams which excite coherent waves contributing to loss of energy

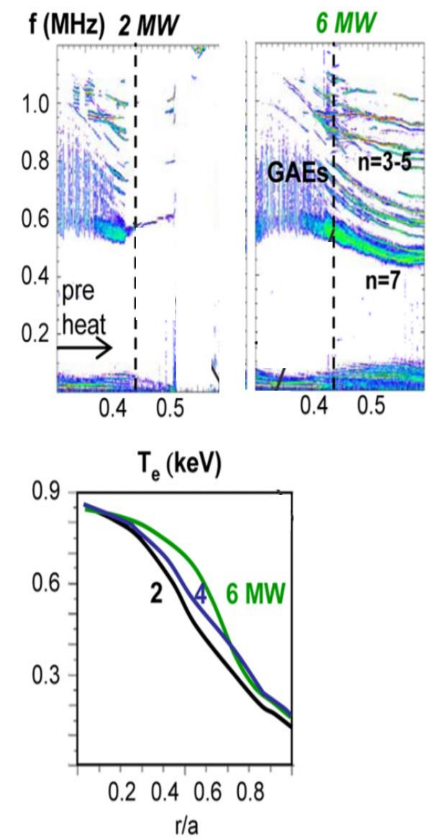
NSTX investigates fusion physics

- Spherical Tokamak
- D+ plasmas
- Temperatures ~ 1 KeV
- B ~ 0.35 - 0.55 T
- Current ~ 1 Mamp
- Density ~ $10^{20}\ m^{-3}$



Compressional Alfven Eigenmodes(CAEs) could play a significant role in plasma energy confinement

- Beams excite long wavelength, high frequency $(\omega \lesssim \omega_{ci})$ Alfven Eigenmodes (AE) including Compressional AE (CAE) in NSTX
- Anomalous temperature profile flattening observed to correlated with high frequency activity in CAEs in the core of National Spherical Torus eXperiment (NSTX) plasma
- Theory suggest CAEs could contribute to the heat loss
  - Multiple mechanisms have been proposed
  - One candidate is CAE-KAW coupling
    - This theory still requires experimental validation



Stutman, PRL 2009

CAE-KAW coupling may explain anomalous flattening

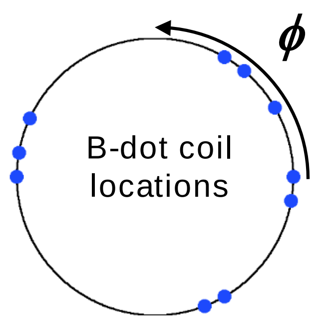
- CAE takes energy from core localized beam ions.
- CAE couple to kinetic Alfven waves (KAW) at resonance location
  - CAE dispersion relationship: $\omega_{cae}^2 = \boldsymbol{k}^2 v_A^2$
  - Resonance location is where $\mathbf{k} \rightarrow \mathrm{k}_\parallel \Rightarrow$ KAW excitation
  - Energy dissipated during KAW excitation
  - KAW energy absorbed during propagation
- KAW are short wavelength (in contrast to CAE)
  - $\omega_{kaw}^2 = k_\parallel^2 v_A^2 [1 + k_\perp^2 \rho_s^2 (1 + \frac{3T_I}{4T_E})]$
- Theory applies for Global Alfven Eigenmodes as well [Kolesnichenko, PRL 2010]

Magnetic diagnostic sensitive to CAEs

- $\dot{B}$ measured from conducting loop at edge
- Faraday's law

$$V_B = -\frac{d}{dt}\oint \boldsymbol{B}(t)\cdot d\boldsymbol{s}$$

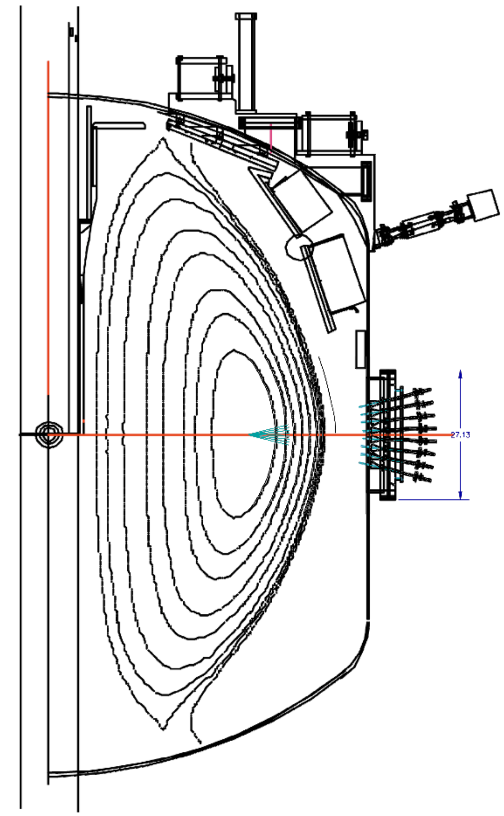
- Toroidal mode number (n) via 10 toroidally distributed coils



Top Down view of NSTX

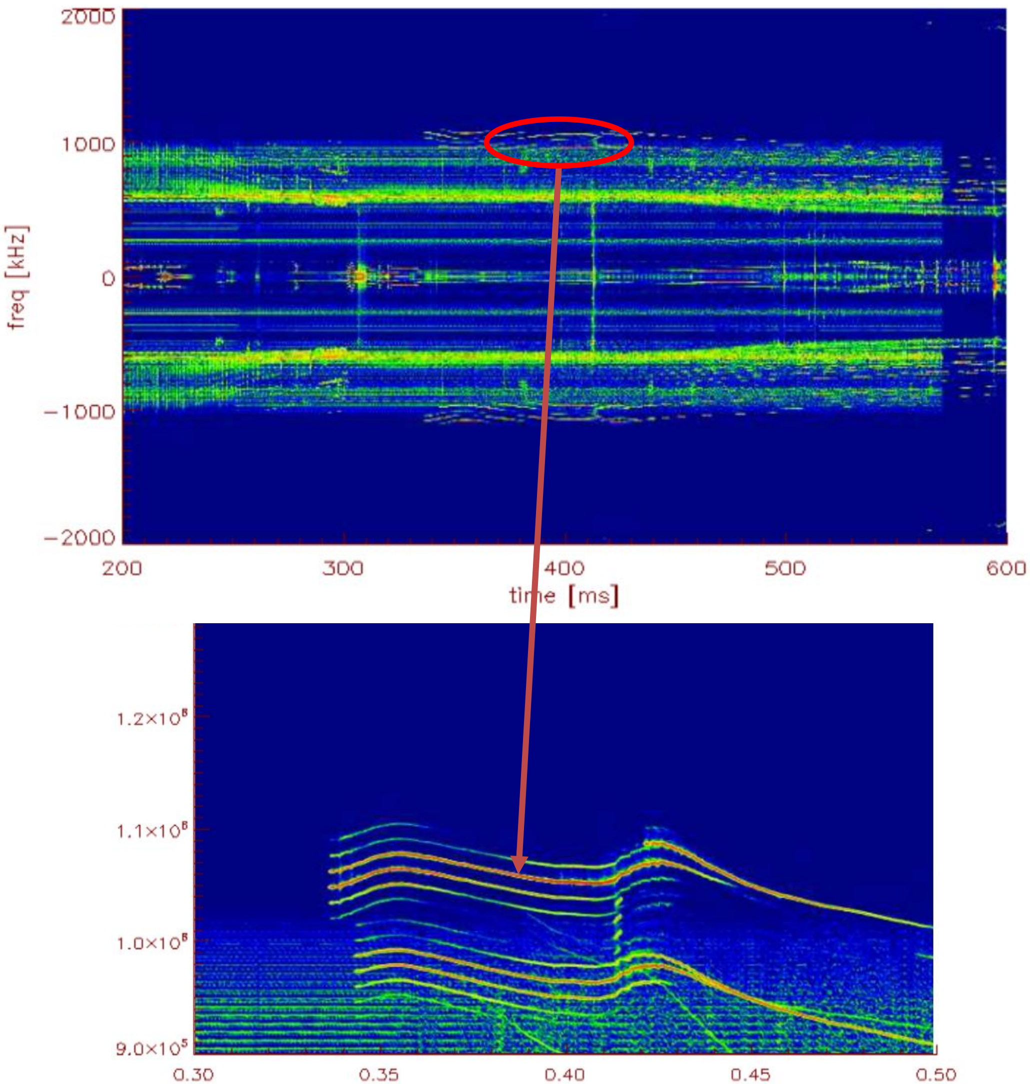
$$\chi^2 \equiv 1 - \left|\sum\nolimits_{\forall\phi} \delta b e^{-in\phi}\right|^2 \Big/ \left(N_\phi \sum\nolimits_{\forall\phi} |\delta b|^2\right)$$

NSTX cross-section

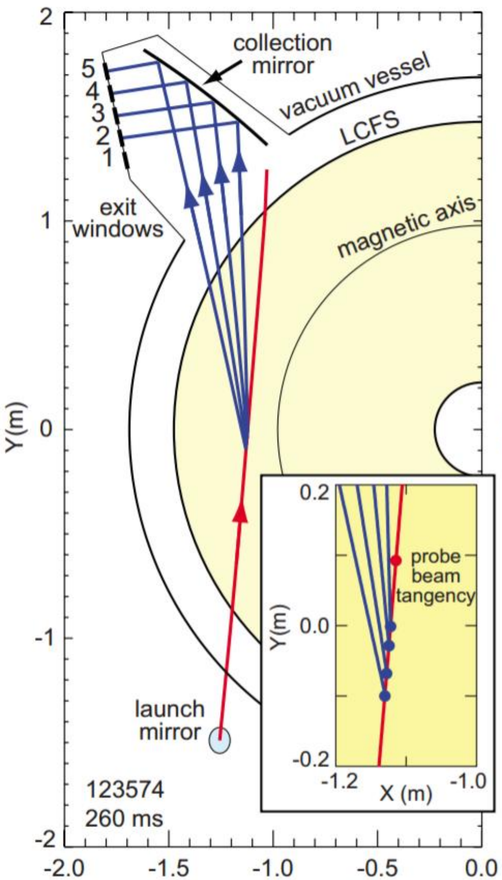


Magnetic diagnostic sensitive to CAEs

- Multiple peaks observed in magnetic spectrum at $f \gtrsim 1MHz$
- Identified as CAE because f at large fraction of f_{ci}



High K-Scattering diagnostic potentially detects KAWs from CAE-KAW coupling



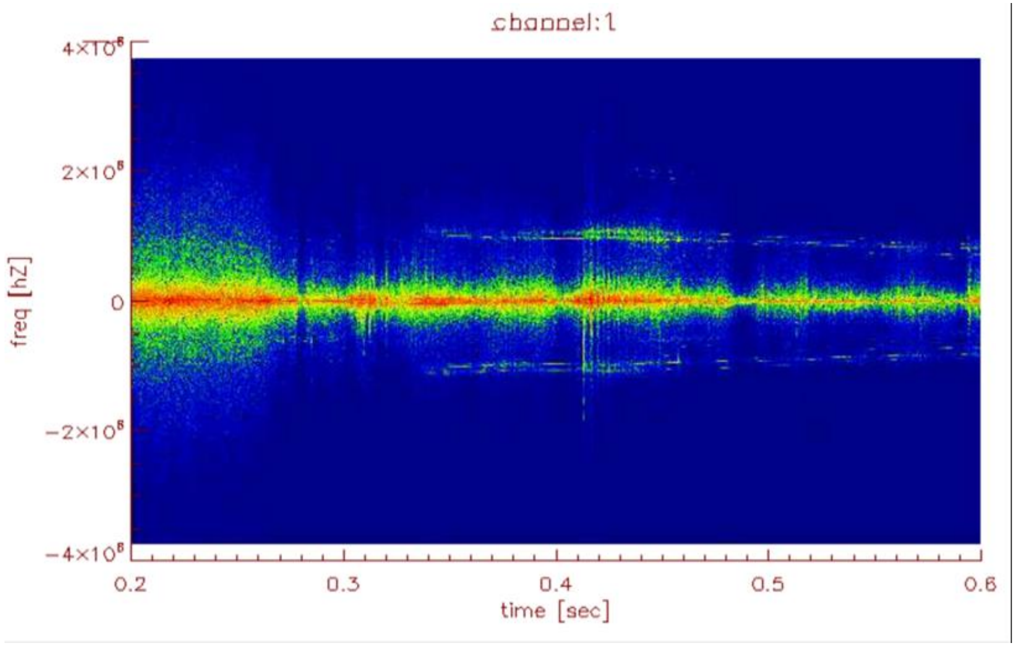
- 280 GHz millimeter wave probe beam scatters into 5 receivers
- Each channel is sensitive to a different **K**

Bragg Scattering

$$\omega_i + \omega_{\delta n} = \omega_{scatter}$$
$$\boldsymbol{k}_i + \boldsymbol{k}_{\delta n} = \boldsymbol{k}_{scatter}$$

- δn is density fluctuation of KAW or turbulence
- beams refracted that changes over time, it has been raytraced, and K's for different channels are determined.

High K-Scattering diagnostic potentially detects KAWs from CAE-KAW coupling



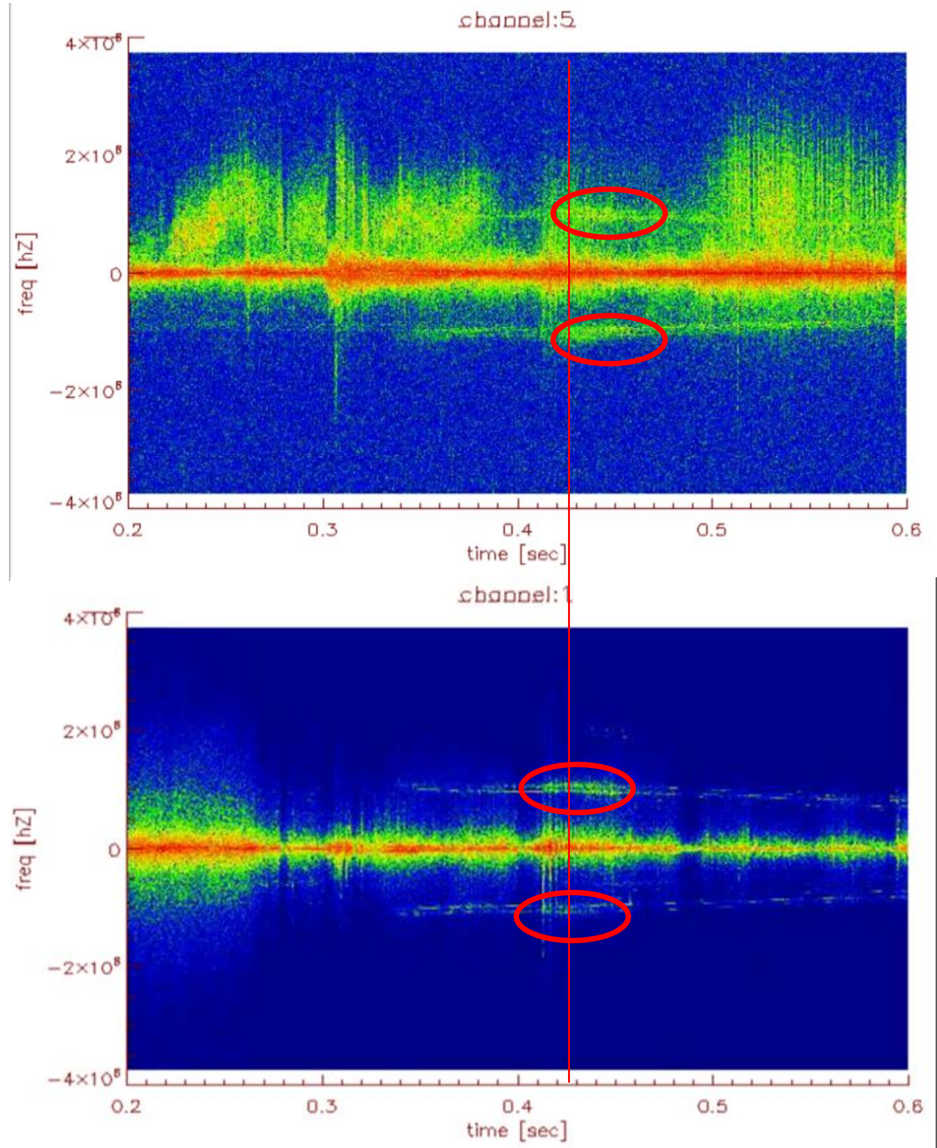
$$E_{hk}(t) \sim e^{i(\omega_s t + \epsilon \sin(\omega_{\text{CAE}} t))} + \text{R}e^{i(\omega_{KAW} t)}$$

– Two contributions to scattered electric field: Turbulence and KAW

- ω_s - frequency of turbulence, $\omega_s \ll \omega_{CAE}$
- ω_{CAE} - CAE frequency; large wavelength fluctuations (amplitude ϵ) modulate E-field scattered from turbulence.
- $\omega_{KAW} = \omega_{CAE}$
- R – relative scattering power

Scattering diagnostic probes KAW wavelengths

- $t_e \sim 780\ eV$
- $t_i \sim 920\ eV$
- $n_e \sim 5.1e13$
- $\boldsymbol{B} \sim 0.36$ T
- Channel 1: $\boldsymbol{k} \sim 7.4\ cm^{-1}$
- Channel 2: $\boldsymbol{k} \sim 7.6\ cm^{-1}$
- Channel 3: $\boldsymbol{k} \sim 8.9\ cm^{-1}$
- Channel 4: $\boldsymbol{k} \sim 11.5\ cm^{-1}$
- Channel 5: $\boldsymbol{k} \sim 14.6\ cm^{-1}$
- $\boldsymbol{\rho_s} \sim 1.1\ cm \rightarrow \boldsymbol{k\rho_s} > 1$
- T = 0.4483 sec



High K combined with magnetics allows detection of CAE-KAW coupling

- Both diagnostic show peaks at similar frequency in CAE range → CAE-KAW coupling?
- Expect coupling to produce KAW coherent with CAE→ must be verified with analysis
- Verify peak in high k spectrum results from scattering.
- Verify if detected k consistent with KAW theory.
  - Some Channels see peaks, other's don't.

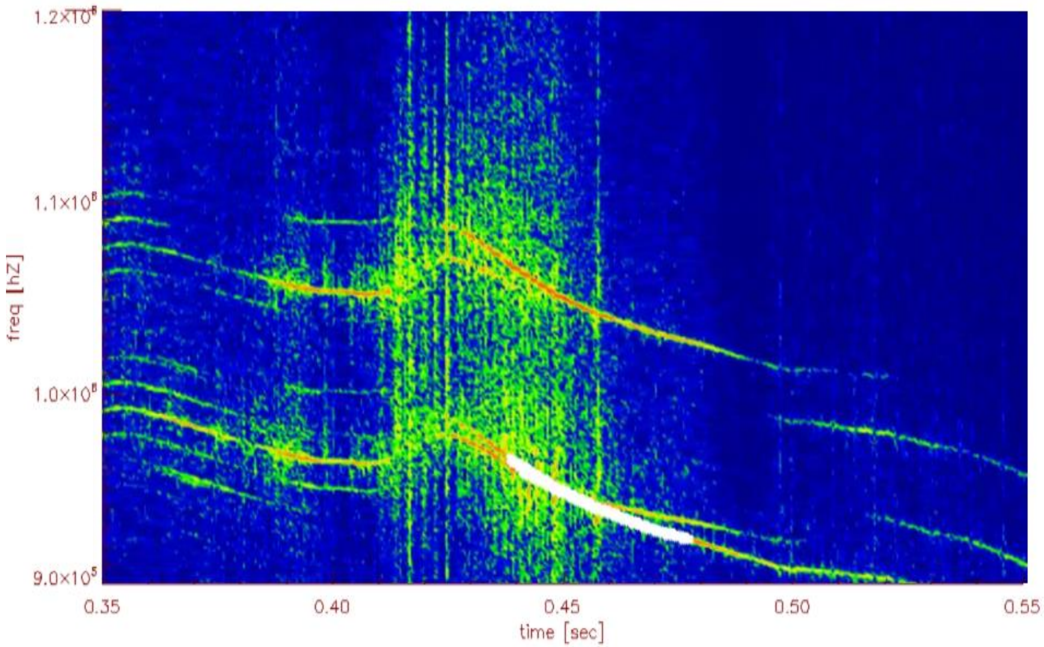
Establishing coherence of high k spectrum peak with magnetic peak is a multistep process

- Isolates mode oscillation from each signal.
  - Determine *f(t)* for peak → filter signals to isolate mode
- Using SVD, isolate global mode oscillation (from mag)
- Calculate coherence of global signal with mode oscillation in scattering signal
  - Test statistical significance for each high-k channel.

Filtering: Determine times and frequencies with image feature analysis

- Track time evolution of mode by choosing a time, frequency range, and a frequency bandwidth.

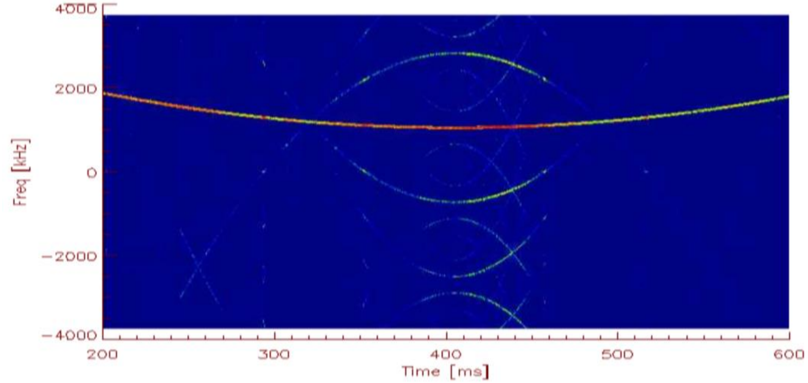
- Identified *f(t)* can be used to create filter to isolate mode.

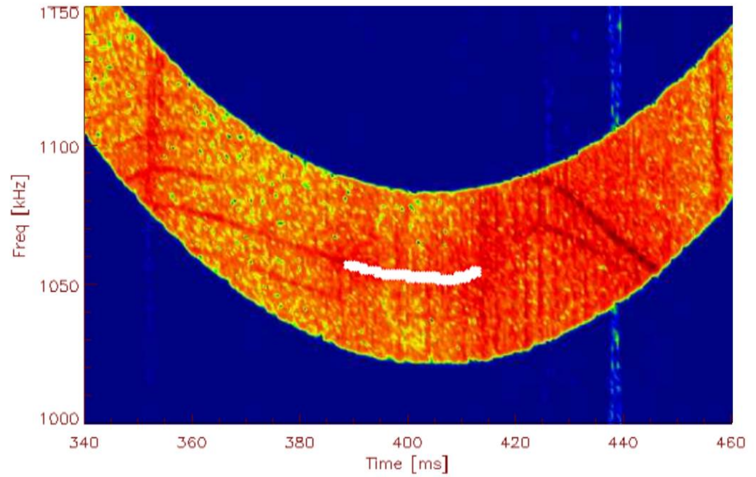


Feature analysis finds mode in spectrum.

Filtering: Construct time dependent bandpass filter from times and frequencies

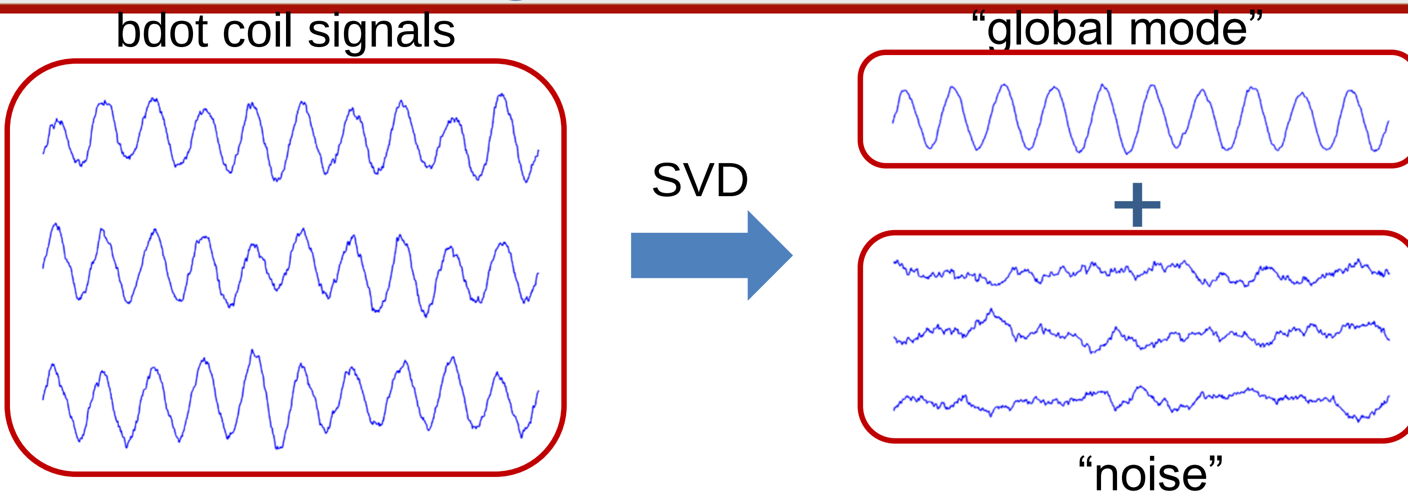
- Fit an analytic (polynomial) *f(t)* to (t,f) from image analysis
- Integrate *f*(t) to get Φ(t)
- $e^{-i\Phi(t)}$ is used to Dopplershift to DC
- Low pass filter used to isolate mode.
- Un-Dopplershift with $e^{i\Phi(t)}$





$$S(t) = e^{i\phi(t)} \int_{-\Delta\omega}^{\Delta\omega} d\omega e^{i\omega t} \int_{t}^{t+T} dt' e^{-i\omega t'} e^{-i\phi(t')}$$

SVD: Singular value decomposition gives better "global mode" δb



- global mode observed by 10 bdot coils (HN array)
- "filter" signals with SVD ⇒ global mode w/reduced noise
  - SVD factors space & time dependence of signal matrix:

$$b_{jk} = \tilde{b}_j(t_k) \rightarrow \tilde{b}_{0\,j}\tilde{b}_{global}(t_k) + \epsilon_j(t_k)$$

- Steps *before* SVD …

1) bandpass filter coil signals to isolate mode
2) make signals complex ⇒ spatial phase (e.g. $n\phi_j$) factors out automatically:

$$\tilde{b}_j(t) = A(t)\cos\big(\theta(t) + \theta_{0j}\big) \rightarrow \hat{\tilde{b}}_j(t) = \frac{1}{\sqrt{2}}A(t)e^{i\left((\theta(t)+\theta_{0j})\right)} = \frac{1}{\sqrt{2}}\int_0^{\infty} d\omega e^{i\omega t}\int_{-\infty}^{\infty} dt'\tilde{b}(t')e^{-i\omega t'}$$

SVD: finds global mode from eigenvector of signal correlation matrix

- SVD solves factoring problem

$$\hat{\tilde{b}}_j(t_k) = \hat{\tilde{b}}_{0\,j}\hat{\tilde{b}}_{global}(t_k) + \hat{\epsilon}_j(t_k)$$

- by minimizing χ^2:

$$\chi^2 = \sum_{j,k} \left| \hat{\tilde{b}}_j(t_k) - \hat{\tilde{b}}_{0\,j}\hat{\tilde{b}}_{global}(t_k) \right|^2$$

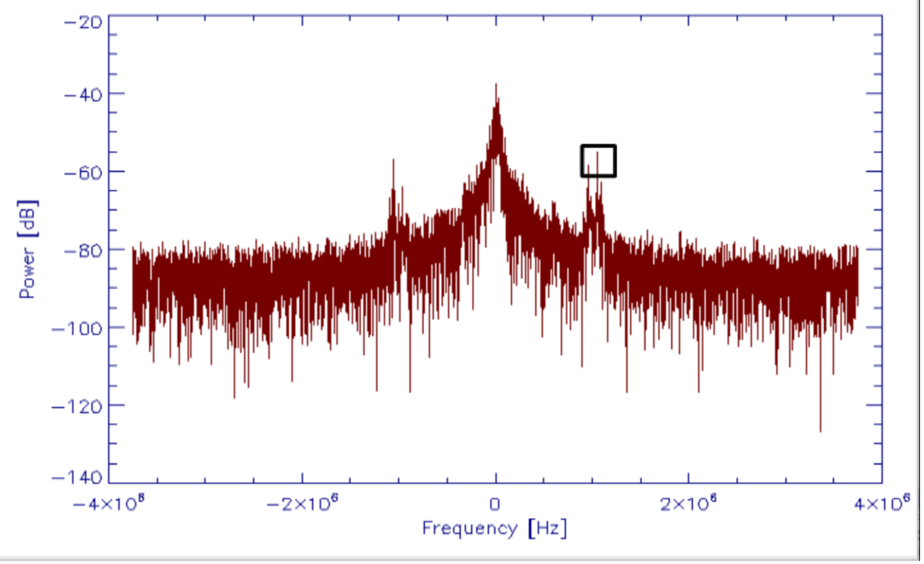
- $\Rightarrow$ spatial coefficients ($\hat{\tilde{b}}_{0\,j}$) of global mode from ***eigenvector*** of correlation matrix ***with largest eigenvalue***:

$$\mathbf{C}\hat{\tilde{\mathbf{b}}}_0 = \lambda\hat{\tilde{\mathbf{b}}}_0$$

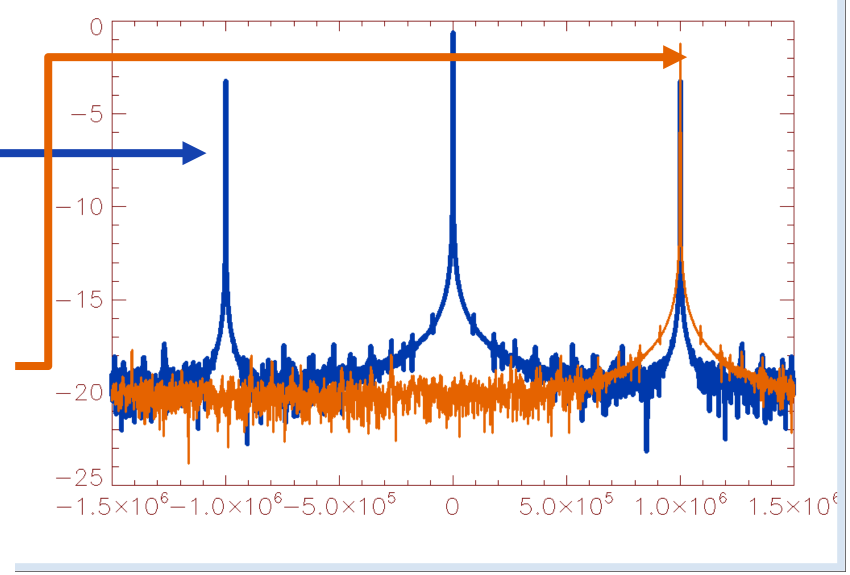
$$[\mathbf{C}]_{ij} = \left\langle \hat{\tilde{b}}_i(t)\hat{\tilde{b}}_j^*(t) \right\rangle, \left[\hat{\tilde{\mathbf{b}}}_0\right]_j = \hat{\tilde{b}}_{0\,j}$$

Scattering Verification: Scattering produces an asymmetric spectrum

- Peaks at CAE frequency observed for 2 reasons.
  - Sidebands of scattering from low frequency turbulence
  - Scattering from KAW

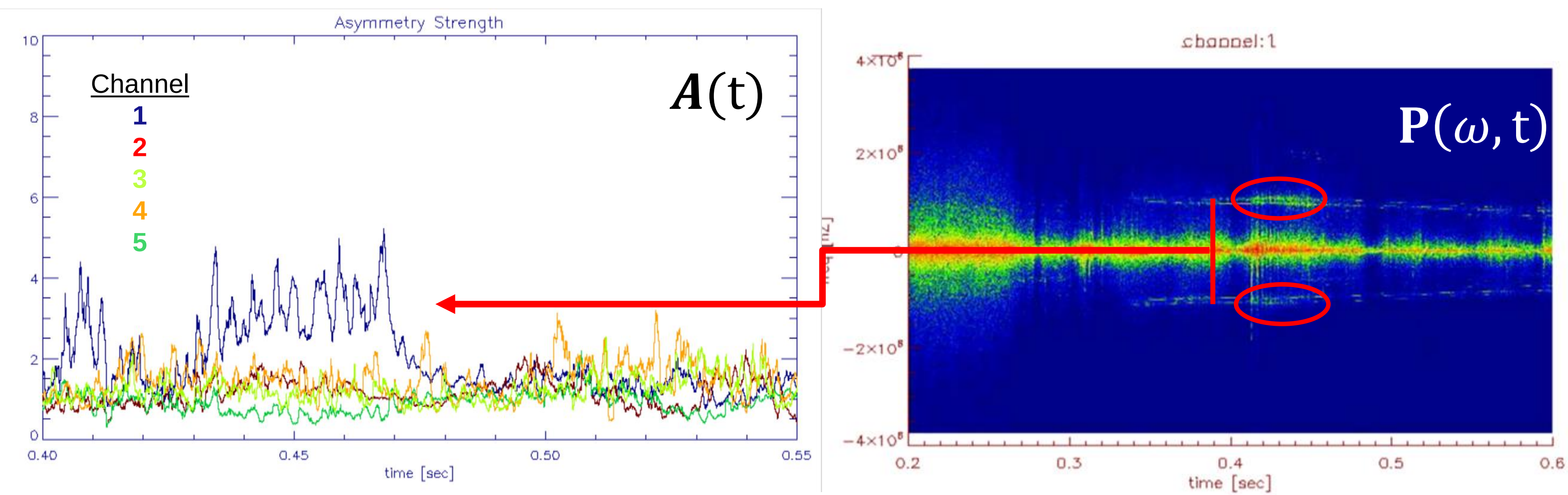


$$E_{hk}(t) \sim e^{i(\omega_s t + \epsilon \sin(\omega_{cae} t))} + R e^{i(\omega_{cae} t)}$$



Scattering Verification: Scattering produces an asymmetric spectrum

- Asymmetry strength across different channels is determined by dividing power at positive mode frequency by power by negative mode frequency
- $\boldsymbol{A}(\mathrm{t}) = \mathbf{P}(\omega_{CAE}(\mathrm{t}), \mathrm{t}) / \mathbf{P}(-\omega_{CAE}(\mathrm{t}), \mathrm{t})$

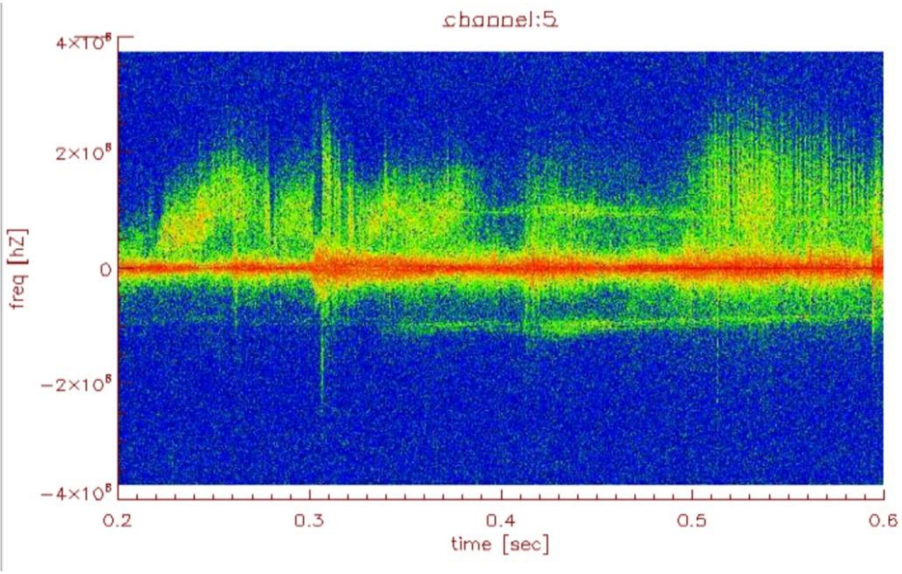


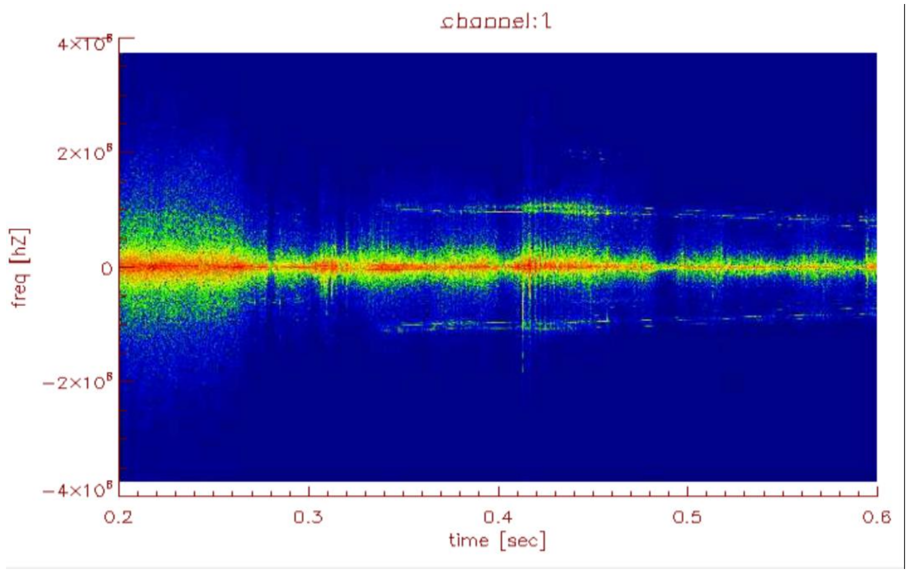
Coherence: Is scattering peak is coherent with magnetic peak as expected?

- Outgoing KAW from mode conversion should be coherent with incoming CAE
- $Coherence = \gamma = \frac{|\langle F_1(t) F_2^*(t) \rangle|}{(\langle |F_1|^2 \rangle \langle |F_2|^2 \rangle)^{1/2}}$
  - $\langle \quad \rangle$ average over time
- Coherence is evaluated with filtered magnetic and scattering spectral peaks
- Resulting coherence is evaluated for statistical significance
  - Monte Carlo Simulation using identically filtered random signals

Comparison of high k channels shows peak at CAE frequency has well defined k

- Expect to see KAW predominantly in only one channel
- Channel sensitive to peak should correspond to k from theory
- Possible to have asymmetry from turbulence with low coherence

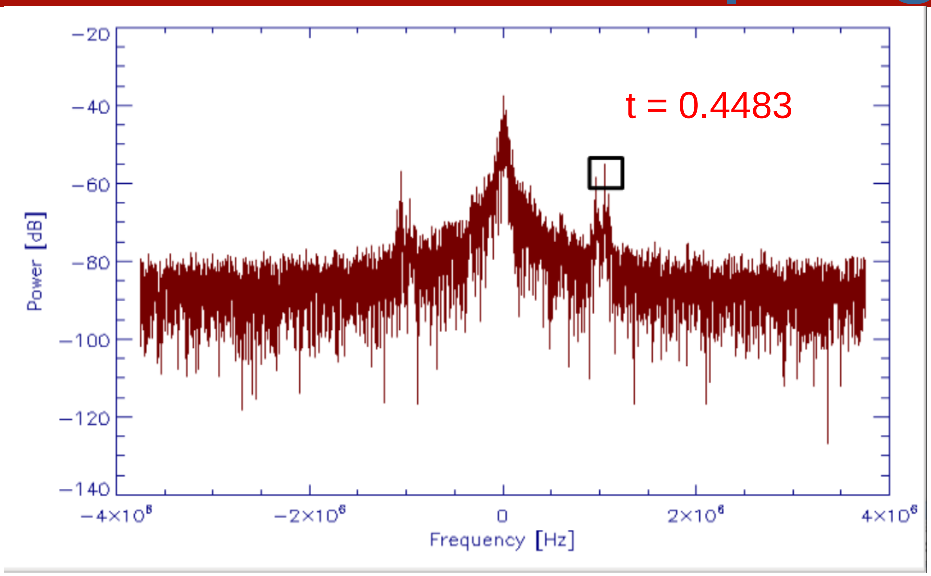


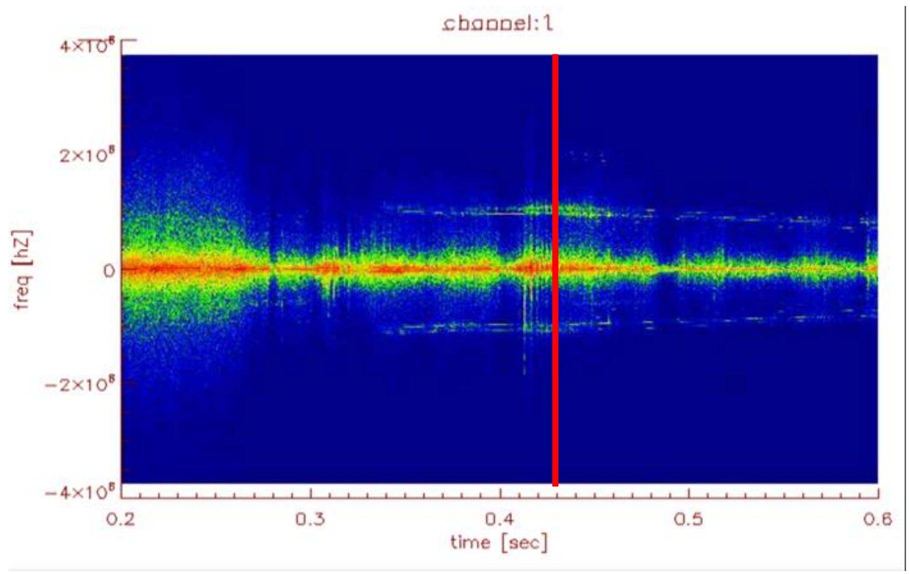


CAEs observed in shot 139395, analyzed for evidence of KAW coupling

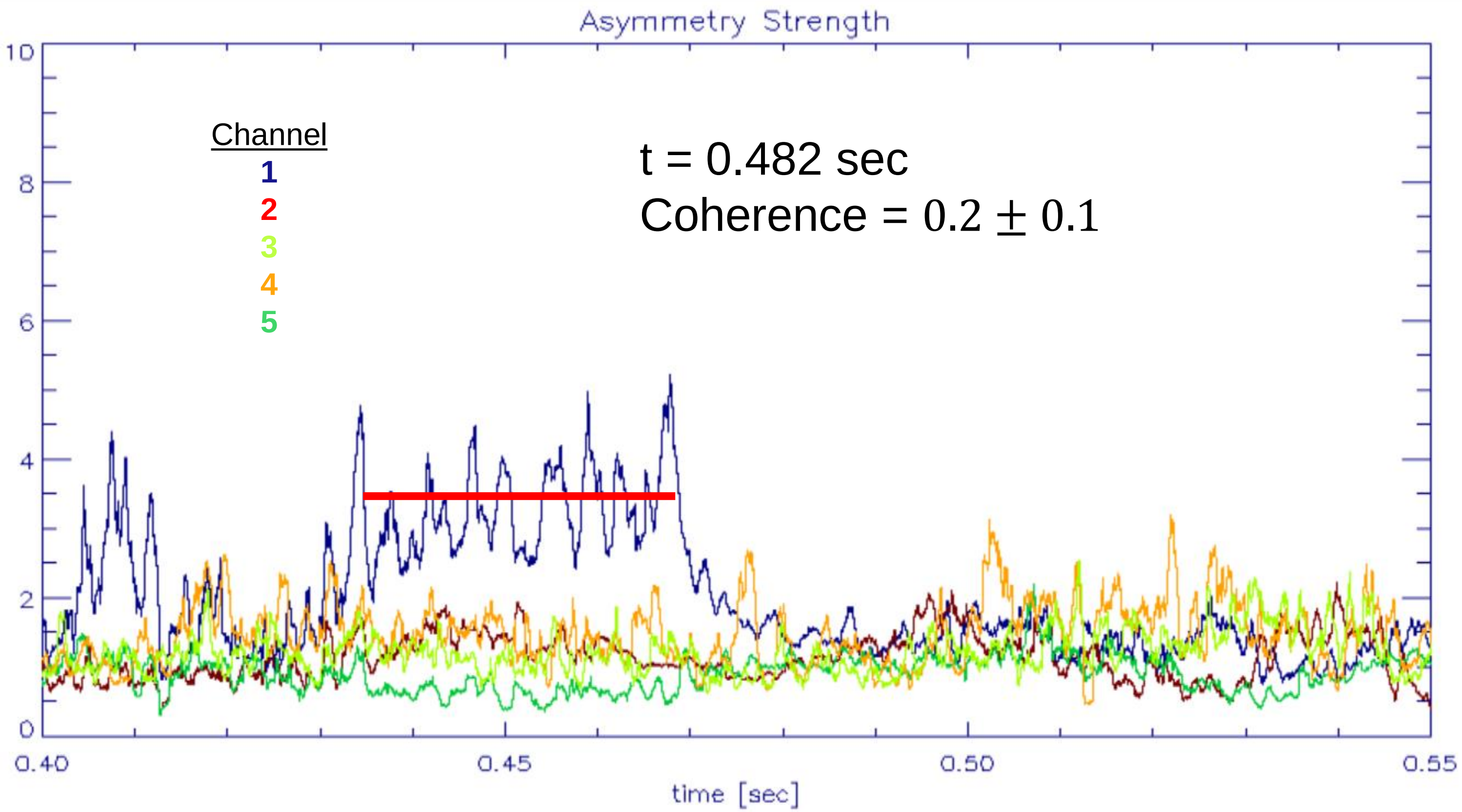
- several modes identified
  - 10KHz bandwidth for filtering

$t\ (sec)$	$f\ (Hz)$	$\Delta t(sec)$
0.448	1.07e6	0.046
0.481	1.06e6	0.035
0.487	1.04e6	0.132
0.498	9.11e5	0.047





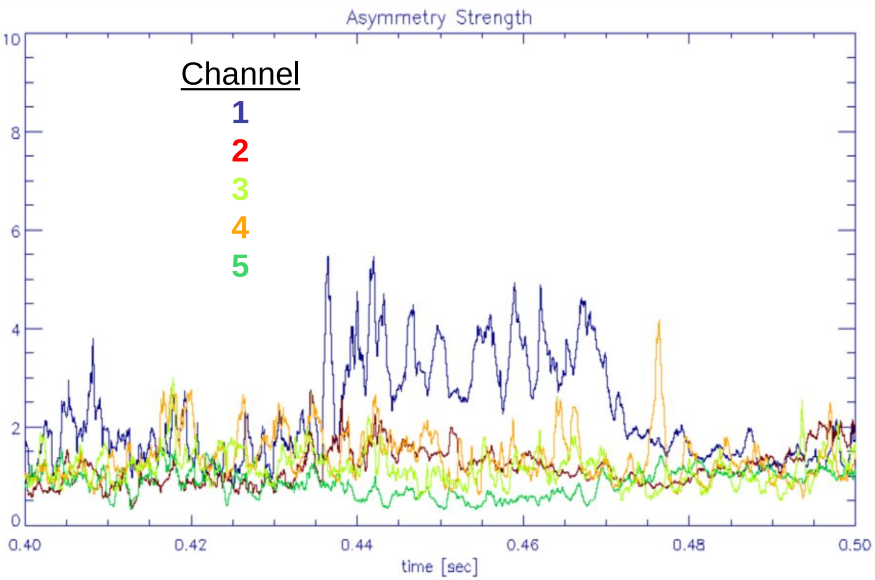
Asymmetry varies with scattered k, as expected

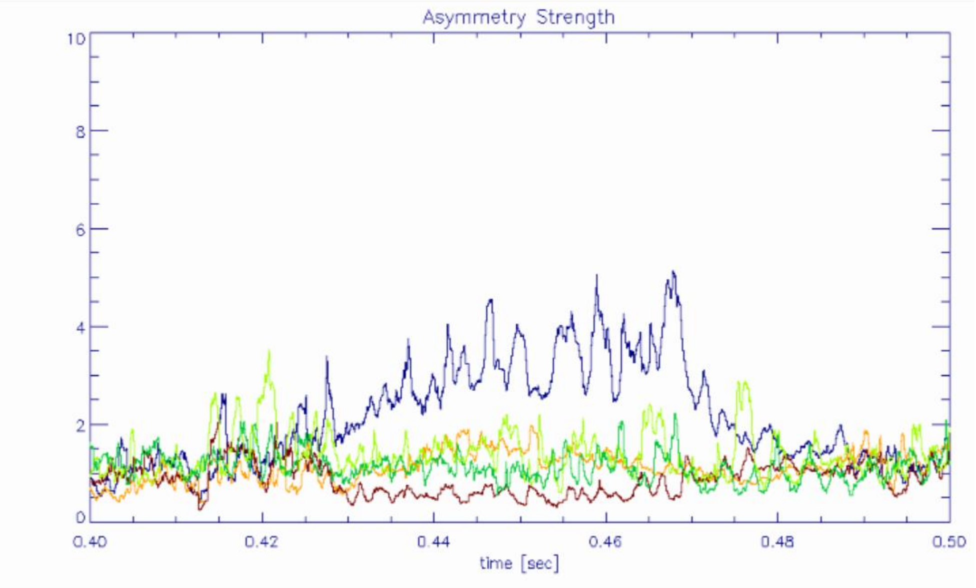


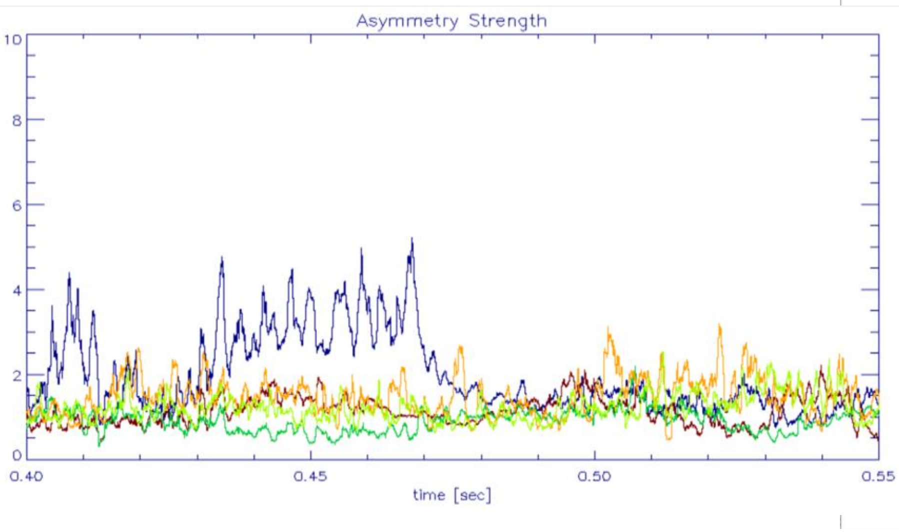
$t(sec)$ $f(Hz)$	A
0.448 1.07e6	6.2
0.481 1.06e6	3.8
0.487 1.04e5	6.3
0.493 9.11e5	4.1

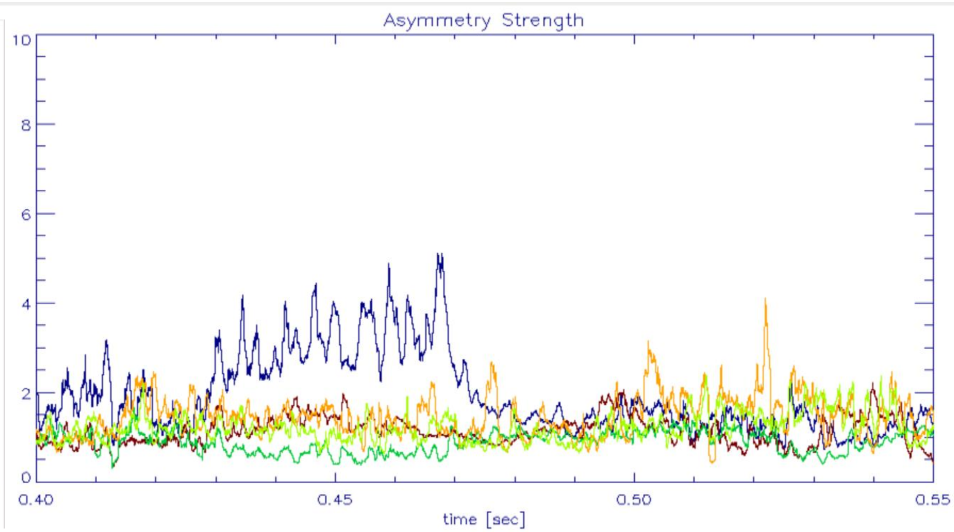
- Channel one ($\mathbf{k} = 760\, m^{-1}$) typically demonstrate a high asymmetry at times where mode is present.

Asymmetry observed in high k channels for multiple modes









Coherence found between magnetics and scattering found for following modes

Coherence	Time
0.1777	0.4483
0.111	0.4817
0.20	0.4871
0.0636	0.4983

Observation consistent with CAE-KAW coupling

- Peaks at same frequency in magnetic and scattering signal are coherent
- Comparison of scattering signals at different k demonstrate well defined k for peaks at CAE freq
- Peaks at lower k spectra demonstrate high asymmetry, in contrast with high k spectra
- Coherence + Asymmetry strongly implies scattering from KAW

Future work

- Compare scattering measurement with expectation from KAW dispersion relationship
  – Predict which channels see KAWs as profiles evolve
  – Assess absorption to determine if significant power can reach measurement location
- Evaluate potential contamination of scattering signal by refraction of probe beam.
  – Beam tracing

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