NSTX-U

NSTX-U is sponsored by the
U.S. Department of Energy Office of Science
Fusion Energy Sciences

Energetic-particle-modified global Alfvén eigenmodes

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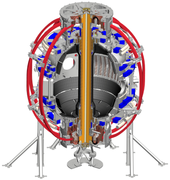
59th APS-DPP Meeting
Milwaukee, WI
October 23 – 27, 2017

PPPL



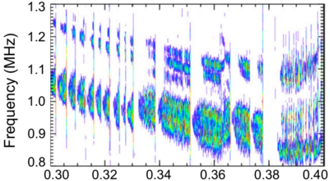
Motivation

Why study GAEs in NSTX-U?

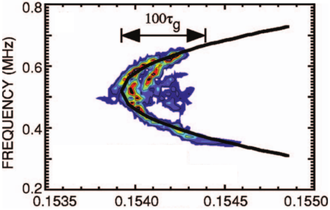
- GAEs (and CAEs) have been linked to anomalous T_e flattening at high beam power in NSTX[1]
- Recently shown to be suppressed by off-axis neutral beam injection[2]

E. Fredrickson Invited Talk
NI3.00005 Wednesday 11:30 AM

- Avalanches and chirping can generate large fast ion losses
  - Presents opportunities to probe nonlinear physics



E. Fredrickson *et al.* (2012) Nucl. Fusion **52**, 043001

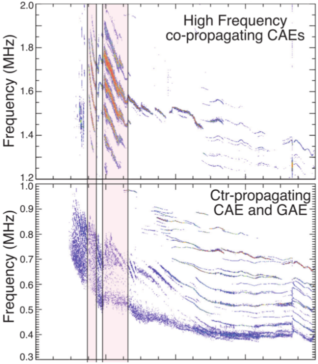


E. Fredrickson *et al.* (2006) Phys. Plasmas **13**, 056109

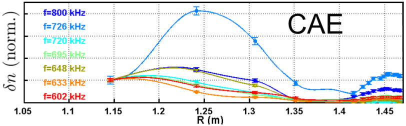
[1] D. Stutman *et al.* Phys. Rev. Lett. **102**, 115002 (2009)
[2] E. Fredrickson *et al.* Phys. Rev. Lett. **118**, 265001 (2017)

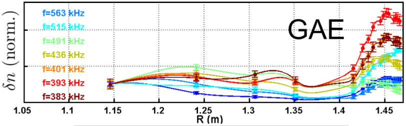
Sub-cyclotron Alfvén Eigenmodes in NSTX

- High frequency Alfvén eigenmodes routinely excited in NSTX(-U) plasmas by neutral beam injection
  - Driven by Doppler-shifted cyclotron resonance with fast ions $\omega - \langle k_{\|} v_{\|} + k_{\perp} v_{\mathrm{Dr}} \rangle = \ell \langle \omega_{ci} \rangle$
- Identified as combination of compressional (CAE) and global (GAE) Alfvén eigenmodes
  - Co-/cntr-propagating $|n| \approx 3 - 14$
  - $\omega / \omega_{ci} \approx 0.1 - 0.7 \; (\gg \omega_{TAE})$



E. Fredrickson *et al.* (2013)
Phys. Plasmas **20**, 042112

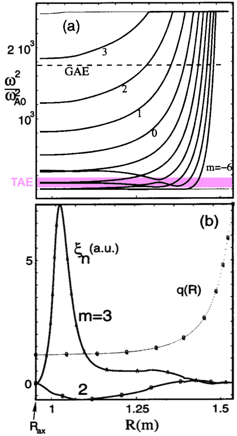




N. Crocker Poster PP11.00044

Perturbative GAE Properties

- Weakly damped GAEs may exist below minimum of Alfvén continuum
  - Approximate dispersion $\omega \leq \left[k_{\|}(r) v_A(r)\right]_{min}$
- Shear Alfvén mode: $\delta B_{\perp} \gg \delta B_{\|}$
  - In NSTX conditions, also have large compressional component $\delta B_{\|} \approx \delta B_{\perp}$ near edge
- **Perturbative assumption**
  Fast ions provide drive but do not modify mode properties



$n = -3$ GAE calculated by NOVA in NSTX[3]

[3] N. Gorelenkov *et al.* Phys. Plasmas **11**, 2586 (2004)

Hybrid Simulation Method

- Hybrid MHD and Particle code (HYM)[4]
  - Single fluid resistive MHD thermal plasma
  - Full orbit kinetic fast ions with δF scheme
- Initial value code in 3D toroidal geometry
- Linear fluid equations and unperturbed particle trajectories
  - Includes nonlinear physics (not used for this study)
- Self-consistent equilibrium including energetic particle effects

$$\frac{\partial^2\psi}{\partial z^2} + R\frac{\partial}{\partial R}\left(\frac{1}{R}\frac{\partial\psi}{\partial R}\right) = -R^2 P' - HH' - GH' + RJ_{b\phi}$$

$$\boldsymbol{B} = \nabla\phi\times\nabla\psi + h\nabla\phi \qquad h(R,z)\equiv H(\psi)+G(R,z) \qquad \boldsymbol{J}_{\boldsymbol{b},\mathrm{pol}} = \nabla G\times\nabla\phi$$

$\longrightarrow$ pressure anisotropy, increased Shafranov shift, more peaked current

[4]E. Belova *et al.* Phys. Plasmas **10**, 3240 (2003)

HYM Physics Model

Fluid thermal plasma

$$\rho\frac{d\boldsymbol{V}}{dt} = -\nabla P + (\boldsymbol{J} - \boldsymbol{J_b}) \times \boldsymbol{B} - en_b(\boldsymbol{E} - \eta\delta\boldsymbol{J}) + \mu\Delta\boldsymbol{V}$$

$$\boldsymbol{E} = -\boldsymbol{V} \times \boldsymbol{B} + \eta\delta\boldsymbol{J}$$

$$\frac{\partial\boldsymbol{B}}{\partial t} = -\nabla \times \boldsymbol{E}$$

$$\mu_0\boldsymbol{J} = \nabla \times \boldsymbol{B}$$

$$\frac{\partial\rho}{\partial t} = -\nabla \cdot (\rho\boldsymbol{V})$$

$$\frac{d}{dt}\left(\frac{P}{\rho^\gamma}\right) = 0$$

Kinetic fast ions

$$\frac{d\boldsymbol{x}}{dt} = \boldsymbol{v}$$

$$\frac{d\boldsymbol{v}}{dt} = \frac{q_i}{m_i}\left(\boldsymbol{E} - \eta\delta\boldsymbol{J} + \boldsymbol{v} \times \boldsymbol{B}\right)$$

δF Scheme

$$F = F_0(\mathcal{E}, \mu, p_\phi) + \delta F(t)$$

$$w \equiv \delta F / F$$

$$\frac{dw}{dt} = -(1 - w)\frac{d\ln F_0}{dt}$$

- ρ, $\boldsymbol{V}$, P are plasma mass density, velocity, and pressure
- n_b, $\boldsymbol{J_b}$ are beam ion density and current
  - Assuming $n_b \ll n_e$ but allowing $\boldsymbol{J_b} \approx \boldsymbol{J_{th}}$

Fast Ion Distribution

- Equilibrium distribution $F_0 = F_1(v)F_2(\lambda)F_3(p_\phi, v)$
  - Energy $\mathcal{E} = \frac{1}{2}m_i v^2$
  - Trapping parameter $\lambda = \mu B_0/\mathcal{E} \approx \mathcal{E}_\perp B_0/\mathcal{E}B$
    - Passing: $0 < \lambda < 1 - \epsilon$
    - Trapped: $1 - \epsilon < \lambda < 1 + \epsilon$
  - Canonical angular momentum $p_\phi = -q_i\psi + m_i R v_\phi$

$$F_1(v) = \frac{1}{v^3 + v_c^3} \quad \text{for } v < v_0$$

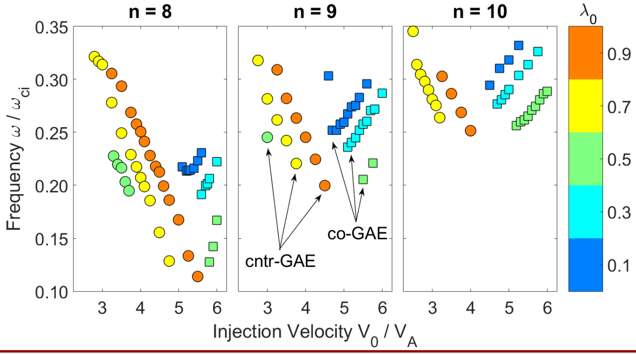
$$F_2(\lambda) = \exp\left(-\left(\lambda - \lambda_0\right)^2 / \Delta\lambda^2\right)$$

$$F_3\left(p_\phi, v\right) = \left(\frac{p_\phi - p_{\min}}{m_i R_0 v - q_i\psi_0 - p_{\min}}\right)^\alpha \quad \text{for } p_\phi > p_{\min}$$

- NSTX: $v_0/v_A \lesssim 5$, $v_c \approx v_0/2$, $\lambda_0 = 0.7$, $\Delta\lambda = 0.3$, $\alpha = 6$
- Simulations expand to $v_0/v_A = 2 - 6$, $\lambda_0 = 0.1 - 0.9$

Self-Consistent Equilibrium Simulations

- GAE frequency changes dramatically with v_0 / v_A for all n
  - Can change by $20 - 50\%$, or $100 - 500$ kHz
- Change is *continuous* to at least $\Delta v_0 = 0.1 v_A$ resolution
  - Uncharacteristic of excitation of distinct MHD modes with discrete frequencies
- In contrast, CAEs do *not* exhibit large frequency changes



EP Effects on Frequency

- Two independent ways EPs can affect the most unstable mode frequency

Equilibrium	Changes to EP distribution can modify self-consistent equilibrium, shifting continuum
Phase Space	Moving EP in phase space changes range of accessible resonances

- $\mathcal{J} \equiv n_b v_0 / n_e v_A \propto J_{\text{beam}} / J_{\text{plasma}}$ characterizes EP influence
- Varying n_b / n_e independently determines frequency dependence due to equilibrium changes alone

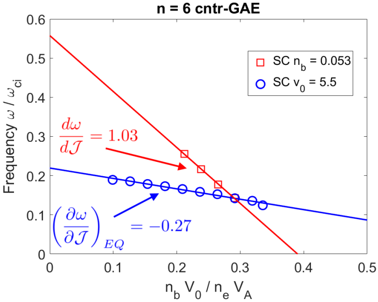
Methods to Separate Frequency Effects

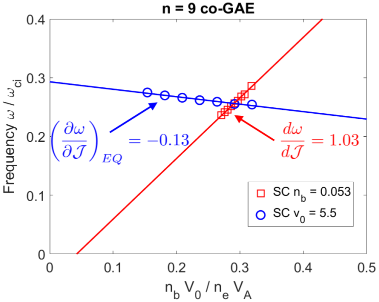
- Calculating "MHD only" equilibrium *without* EP effects and varying v_0/v_A isolates dependence on EP phase space
  - Computed with same total current as self-consistent case
  - EP pressure absorbed into thermal pressure

	Equilibrium Effect	**Phase Space Effect**	**Total Effect**
Equilibrium	Self-consistent	MHD only	Self-consistent
Vary	n_b/n_e	v_0/v_A	v_0/v_A
Fix	v_0/v_A	n_b/n_e	n_b/n_e

Equilibrium Effect on Frequency

- **EP-induced changes to equilibrium not large enough to account for change in frequency**
- Red squares: simulations with $n_b = 0.053$, varying v_0/v_A
- Blue circles: simulations with $v_0/v_A = 5.5$, varying n_b
  - Slope $\left(\frac{\partial\omega}{\partial\mathcal{J}}\right)_{EQ}$ is effect due to changes in equilibrium

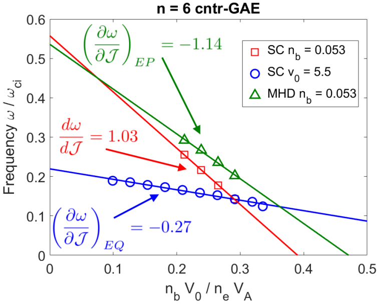


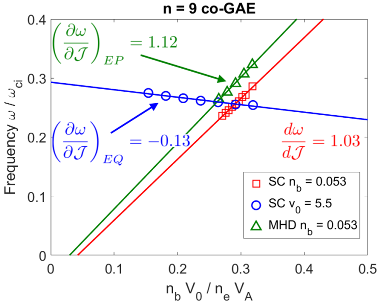


MHD Equilibrium Simulations

- **EP phase space produces most of change in frequency**
- Green triangles: simulations with $n_b = 0.053$, varying v_0 / v_A with a single "MHD" equilibrium
  - Slope $\left(\frac{\partial\omega}{\partial\mathcal{J}}\right)_{EP}$ is effect due to changes in EP phase space
- Equilibrium and EP phase space effects are nearly linear

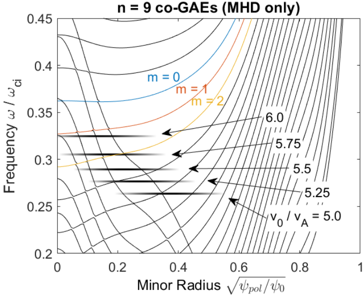
$$\frac{d\omega}{d\mathcal{J}} \approx \left(\frac{\partial\omega}{\partial\mathcal{J}}\right)_{EQ} + \left(\frac{\partial\omega}{\partial\mathcal{J}}\right)_{EP} = n_e v_A \left[\frac{1}{v_0}\frac{\partial\omega}{\partial n_e} + \frac{1}{n_e}\frac{\partial\omega}{\partial v_0}\right]$$





Mode Locations Relative to Alfvén Continuum

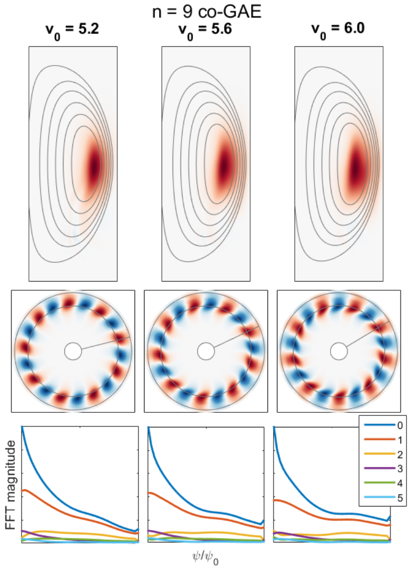
- Modes **are not** localized near minimum of continuum
- Modes intersect continuum, but do not have singular structure
- $m \approx 0$ structure, yet frequencies are far from this branch
- Caveat: continuum does not include kinetic fast ions
  - *e.g.* kinetic Alfvén wave including beam ion FLR $\omega = k_{\parallel} v_A \left(1 + \frac{n_b}{n_e} \frac{k_\perp^2 \rho^2}{1 + k_\perp^2 \rho^2}\right)$
  - For $n_b/n_e = 0.2$, $k_\perp \rho \approx 2$, kinetic ion term can be 15%.



$n = 9$ continuum for the "MHD only" equilibrium, including EP pressure as part of total plasma pressure. Calculated by `NOVA` with $|m| \leq 22$

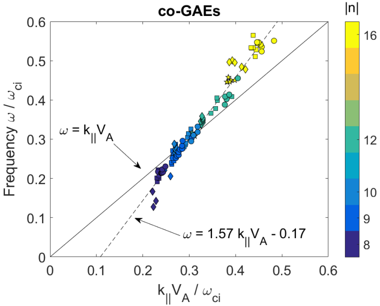
Mode Structure of co-GAEs

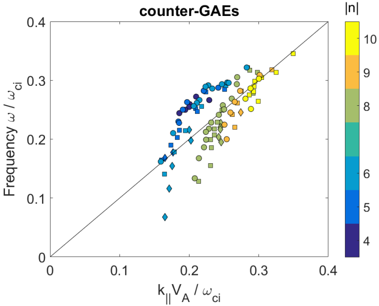
- Mode structure does not change qualitatively with frequency
  - Slight changes: peak location moves gradually inwards, mode becomes slightly elongated
- Frequency changes $\approx 20\%$ from $\omega/\omega_{ci} = 0.24$ to 0.29 ($\Delta\omega = 125$ kHz) due to 15% change in v_0/v_A
- Remains dominated by $m = 0$ harmonic



Inferred Dispersion

- Frequencies are near *local* shear Alfvén dispersion
  - $\omega \approx k_{\|} v_A \pm 20\%$ evaluated at mode location
  - $k_{\|}$ calculated as peak of FT of flux surface average of $\delta B_{\perp}$
    - $k_{\|} \approx (n - m/q)/R_0$ is an unreliable approximation here
- co-GAEs are well-fit by dispersion $\omega = 1.57 k_{\|} v_A - 0.17$
- cntr-GAEs have larger spread, likely due to larger range of m
- Accurate dispersion must include EP nonperturbatively



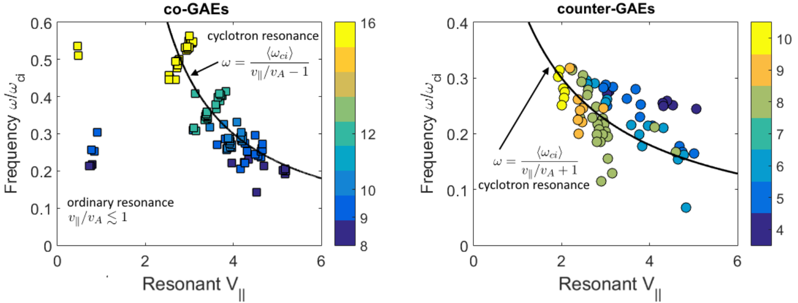


Resonance Implies Frequency Change

- Time evolution of particle weights identifies resonant particles
- Combination of dispersion and resonance yields frequency dependence on $v_{\|}^{res}$

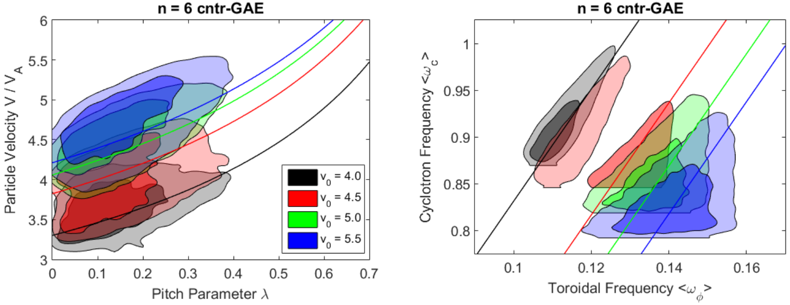
$$\omega = \frac{\langle \omega_{ci} \rangle}{\ell + \langle v_{\|} \rangle / v_A} \quad \text{where } \ell = -\text{sign } k_{\|}$$

- Unresolved: how does $v_{\|}^{res}$ depend on v_0 / v_A quantitatively?



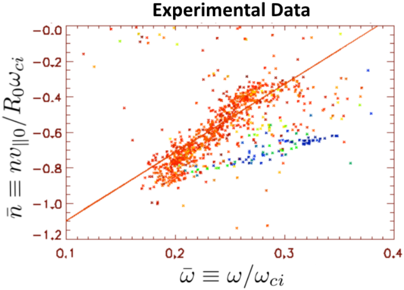
Frequency Determined by Resonant Particles

- Resonant particles move to higher energy as v_0/v_A increases
- In $\langle\omega_\phi\rangle$, $\langle\omega_{ci}\rangle$ coordinates, particles with largest weights cluster around resonances: $\omega - n\langle\omega_\phi\rangle - p\langle\omega_\theta\rangle = \ell\langle\omega_{ci}\rangle$
- Frequency of mode is being set by location of resonant particles in phase space – characteristic of EPMs



Experimental Clues

- Experimental analysis[5] of many NSTX discharges shows cntr-GAE frequency *decrease* with increasing $|n|$
  - Opposite trend expected from dispersion $\omega = -k_{\|} v_A \propto -n$
  - Resonance condition more important than dispersion for frequency?
- Provides clues but not confirmation without measurements of m



With above normalizations, resonance condition implies $\bar{n} \approx \bar{\omega} - 1$. Experimental fit is $\bar{n} = 0.8\bar{\omega} - 1.2$

[5] S. Tang *et al.* EU/US Transport Task Force , Williamsburg, VA (2017)

Other Nonperturbative Solutions

- Many examples of *low* frequency nonperturbative modes exist
  - Fishbone, E-GAM, RSAE cascades, RTAE, EPMs, *etc*
- Numerical results may indicate first high frequency solution
  - Key difference: excitation through Doppler-shifted cyclotron resonance vs Landau ($\ell = 0$) resonance
- EP-GAE shares characteristics of low frequency cases
  - Frequency tracks characteristic frequencies of EP motion
  - Prone to large frequency variation with robust mode structure

	MHD GAE	**EP-GAE**
Dispersion	$\omega \leq \left[k_{\parallel}(r) v_A(r)\right]_{\text{min}}$	$\omega = k_{\parallel}(r) v_A(r) \pm 20\%$
Resonance	$\omega - \langle k_{\parallel} v_{\parallel} \rangle = \ell \langle \omega_{ci} \rangle$	$\omega - \langle k_{\parallel} v_{\parallel} \rangle = \ell \langle \omega_{ci} \rangle$
Structure	Large $\Delta\omega \iff \Delta m \neq 0$	Robust to large $\Delta\omega$
Polarization	$\delta B_{\perp} \gg \delta B_{\parallel}$	$\delta B_{\perp} \gg \delta B_{\parallel}$

Summary

- Self-consistent hybrid simulations reveal strong EP modifications to GAEs in NSTX conditions
  - In contrast, CAEs in simulations are not strongly modified
- Frequency changes significantly and continuously with v_0/v_A
  - Change in frequency is consistent with the resonance condition
  - Mostly due to changes in EP phase space, *not* EP-induced changes to equilibrium
- Mode structure does not change substantially with frequency
  - MHD eigenmodes would have different poloidal mode numbers (m, s) associated with large frequency changes
- Results may indicate a new, high frequency EPM
  - *Energetic-particle-modified* global Alfvén eigenmode (EP-GAE)

Open Questions and Future Work

- Derive EP-GAE dispersion nonperturbatively
  - Requires calculation of fast ion corrections to Alfvén continuum
- Demonstrate transition from perturbative GAE to EP-GAE
- Experimental identification in NSTX-U
  - Are the high frequency shear Alfvén modes routinely excited in NSTX actually EP-GAEs or GAEs?
- Explore impact on T_e flattening theories
  - How sensitive are existing theories of anomalous electron heat transport[6,7] to properties of EP-GAE vs GAE modes?

Paper submitted to Phys. Plasmas
Contact jlestz@pppl.gov for preprint

[6] Y. Kolesnichenko *et al*. Phys. Rev. Lett. **104**, 075001 (2010)
[7] N. Gorelenkov *et al*. Nucl. Fusion **50**, 084102 (2010)