

ORBIT modelling of fast particle redistribution induced by sawtooth instability

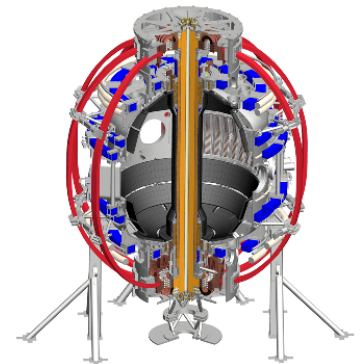
D. Kim, M. Podestà, D. Liu¹ and F. M. Poli

Princeton Plasma Physics Laboratory

¹University of California, Irvine

59th Annual Meeting of the APS Division of Plasma Physics
October 23 - 27, 2017 • Milwaukee, Wisconsin

UCIrvine
University of California, Irvine



Abstract

Initial tests on NSTX-U show that introducing energy selectivity for sawtooth (ST) induced fast ion redistribution improves the agreement between experimental and simulated quantities, e.g. neutron rate. Thus, it is expected that a proper description of the fast particle redistribution due to ST can improve the modelling of ST instability and interpretation of experiments using a transport code. In this work, we use ORBIT code [1] to characterise the redistribution of fast particles. In order to simulate a ST crash, a spatial and temporal displacement is implemented as $\xi(\rho, t, \theta, \phi) = \sum \xi_{mn}(\rho, t) \cos(m\theta + n\phi)$ [2] to produce perturbed magnetic fields from the equilibrium field \vec{B} , $\delta\vec{B} = \nabla \times (\vec{\xi} \times \vec{B})$, which affect the fast particle distribution. From ORBIT simulations, we find suitable amplitudes of ξ for each ST crash to reproduce the experimental results. The comparison of the simulation and the experimental results will be discussed as well as the dependence of fast ion redistribution on fast ion phase space variables (i.e. energy, magnetic moment and toroidal angular momentum). [1] White R.B. and Chance M.S. 1984 Phys. Fluids 27 2455 [2] Farengo R. et al 2013 Nucl. Fusion 53 043012

¹Work supported by the U.S. Department of Energy, Office of Science, Office of Fusion Energy Sciences under contract number DE-AC02-09CH11466

Long Sawtoothed Discharges on NSTX-U with the new 2nd NBI & Central Solenoid

Major radius: 0.95 m

Aspect ratio: 1.5

Elongation: 2.7

Triangularity: 0.8

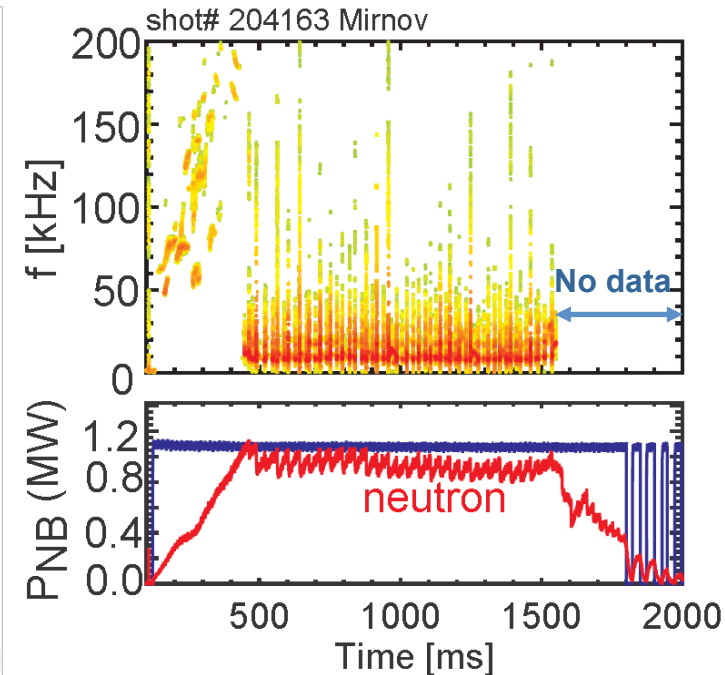
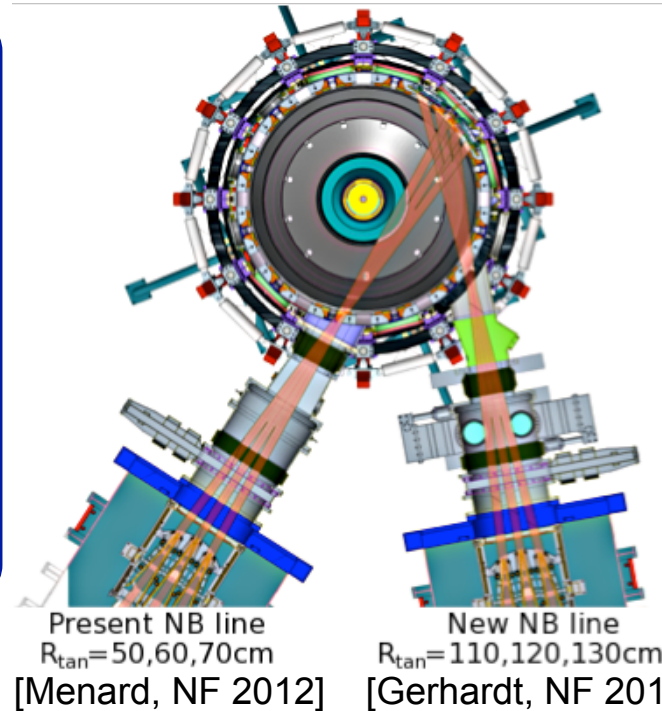
Plasma current: < 2 MA

Toroidal field: < 1.0 T

Pulse length: ~1 - 5 s

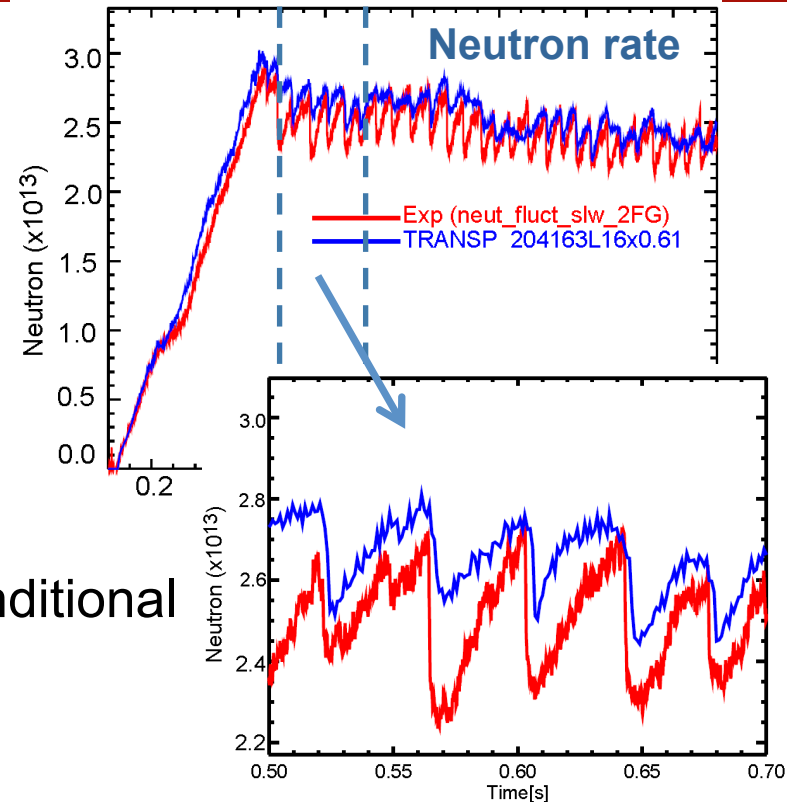
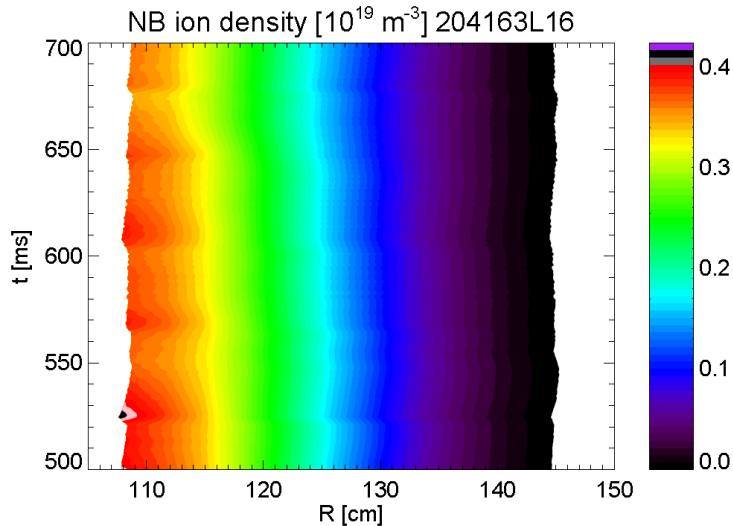
6 Neutral Beam sources:

$P_{\text{NBI}} \leq 12 \text{ MW}$, $E_{\text{injection}} \leq 95 \text{ keV}$



- Reproducible sawtoothed L-mode discharges are obtained on NSTX-U
 - Neutron rate can drop as large as ~15% at sawtooth crash
 - Flattening of T_e in the core indicated by MPTS and Soft x-ray diagnostics
 - No MSE measurement is available for these discharges

TRANSP results indicate significant effect of thermal profile evolution on neutron rate



- TRANSP run using thermal profiles from conditional average reconstruction
 - Sawtooth model ON for q -profile evolution
 - Sawtooth model OFF for fast ions
 - Little/no modulation of fast ion profile
- Neutron rate drops caused by thermal plasma profile evolution are significantly different from the measured ones
 - Motivates improved analysis strategy to unfold fast ion effects

See D. Liu's poster (PP11.00048)
for more experimental
and simulation results

Can effect of fast particle transport improve sawtooth model?

- Sawtooth modelling in TRANSP [transpweb.pppl.gov]
 - Several free parameters (e.g. partial reconnection fraction) in the model
→ need to find an optimum set for each case
 - Unable to determine self-consistent parameter setting
 - Cannot reproduce experimental results with a fixed parameter set
- Improvement of sawtooth model for fast ion redistribution in phase space?
 - ORBIT simulations can be a guidance to develop a more comprehensive model for fast particle transport by sawteeth
 - Implementation of a fast particle model into TRANSP is expected to enable self-consistent modelling including more physics
 - NSTX-U data can guide model development and be used for validation

Simulation setting

- Simulation code: ORBIT [White PoF84]
 - Hamiltonian guiding center particle motion code
 - Use of numerical equilibrium, field perturbations in flux coordinates (Boozer coordinate in this work)
 - Analyzing test particle transport (especially energetic ions)
- Target discharge: NSTX-U #204083
 - Equilibrium data (eqdsk file from TRANSP) from one time slice before sawtooth crash (1093ms)
 - Initial distribution of fast particles from TRANSP (NUBEAM)
 - Number of particle for calculation: 10,000
 - Radial displacement for the application of linear perturbation from sawtooth instability (constant in time)

Displacement from sawtooth crash induces perturbed magnetic field δB

- Perturbation magnetic field from displacement

$$\delta \vec{B} = \nabla \times (\vec{\xi} \times \vec{B})$$

- Radial displacement model [Farengo NF13]

- $\xi(\rho, t, \theta, \zeta) = \sum \xi_{m,n}(\rho, t) \cos(n\zeta - m\theta - \omega t)$
- (m, n) : poloidal and toroidal mode numbers
- (θ, ζ) : poloidal and toroidal angles
- $(1,1)$ mode is applied and temporal change of displacement is not considered
- Tested on cylindrical coordinate in Farengo NF13
 - ➔ toroidal geometry effect, e.g. toroidal mode coupling, may need to be considered

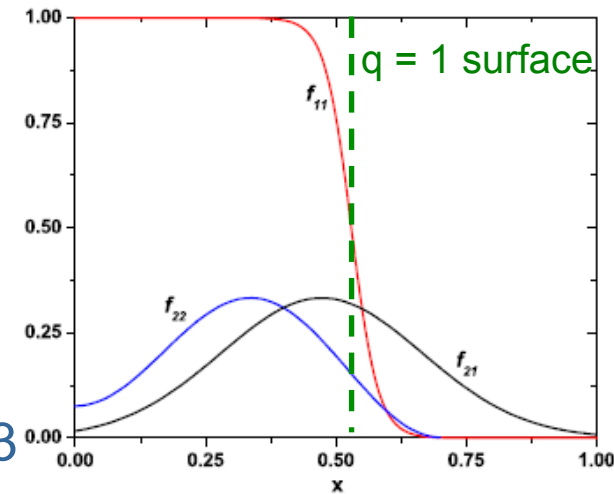
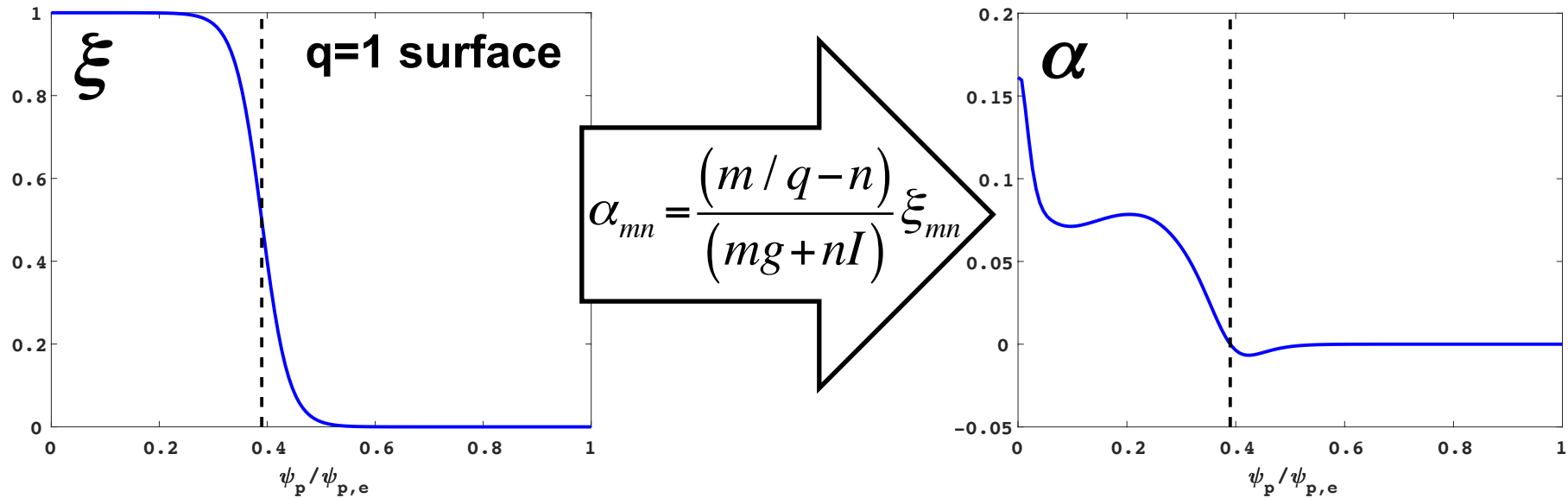


Figure from [Farengo NF13]

In ORBIT code, perturbation is defined using α , not ξ



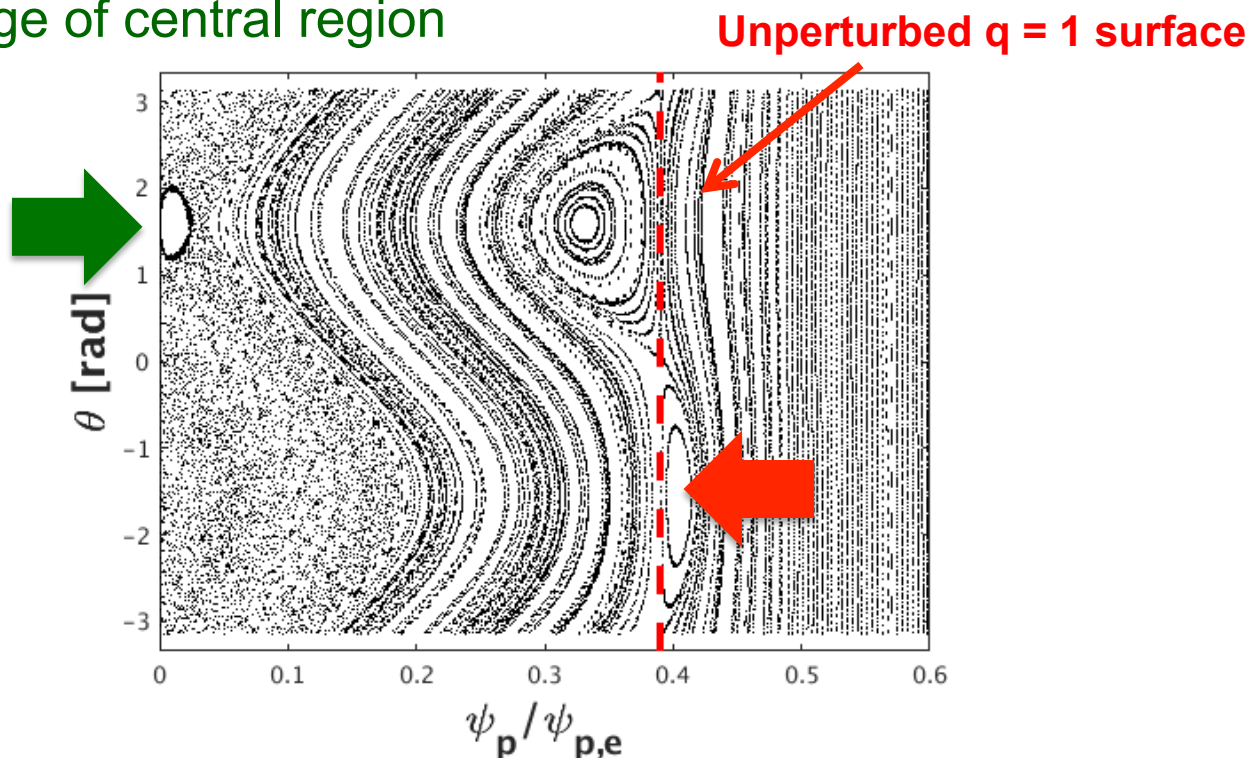
- Perturbation in ORBIT: $\delta \vec{B} = \nabla \times (\alpha \vec{B})$
 - ξ can be transformed into α [White PoP13]
 - Radial component of resultant perturbation is equivalent to ξ model

$$\delta \vec{B} \cdot \nabla \psi_p = \sum_{m,n} \frac{mg + nI}{J} \alpha_{m,n} \cos(n\xi - m\theta - \omega t)$$

- Mode amplitude is prescribed for ξ and is used for α after normalisation
- Only first order term is considered

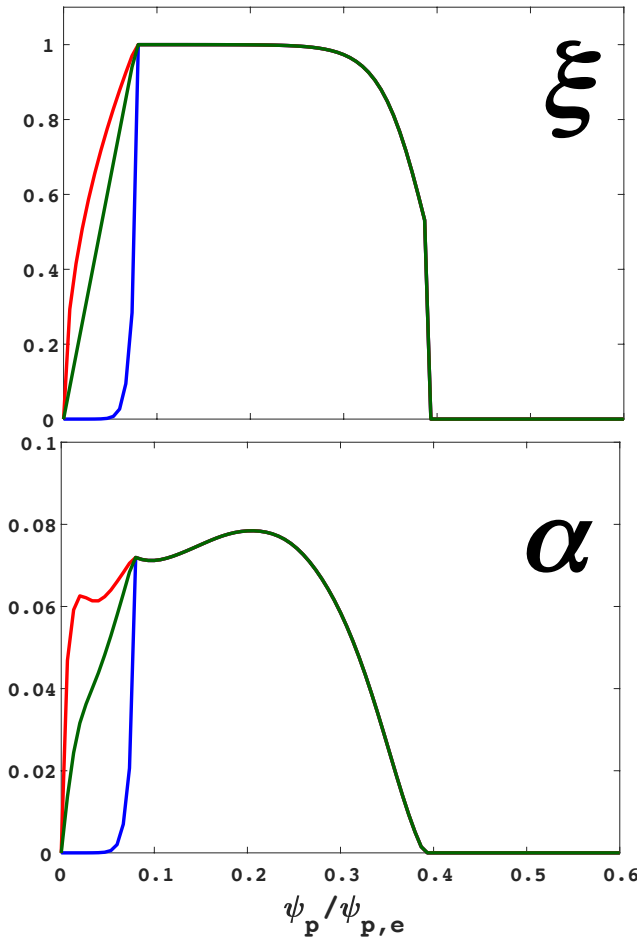
Assumed (1,1) mode perturbation rises to (2,2) mode like island structure

- Need to modify perturbation form?
 - To have a consistent island structure with more accurate physics models
 - Shape : cut-off outside $q=1$ surface, modification of central part
 - Central value
 - Modified range of central region



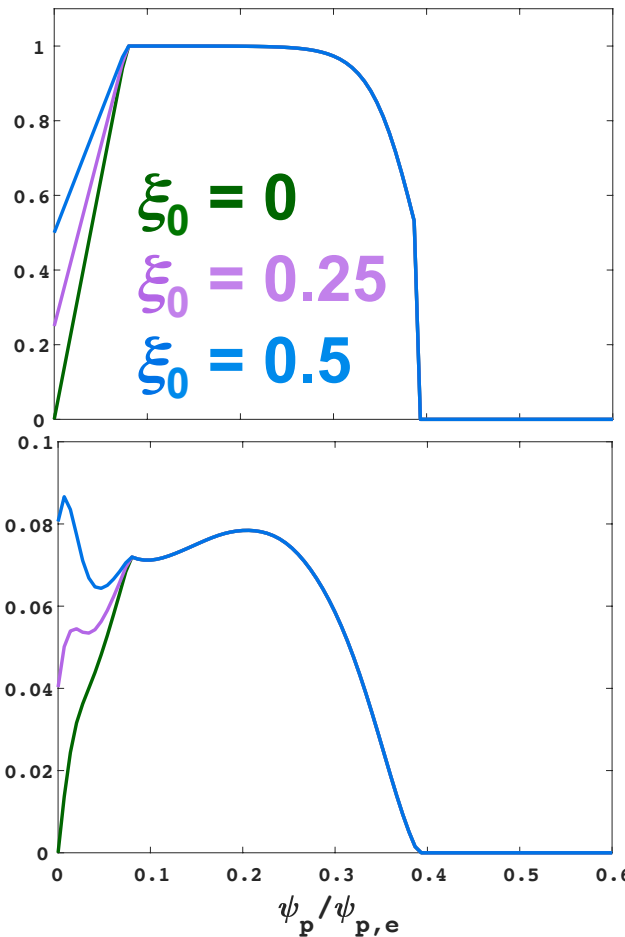
Modification of perturbation profile

Central shape



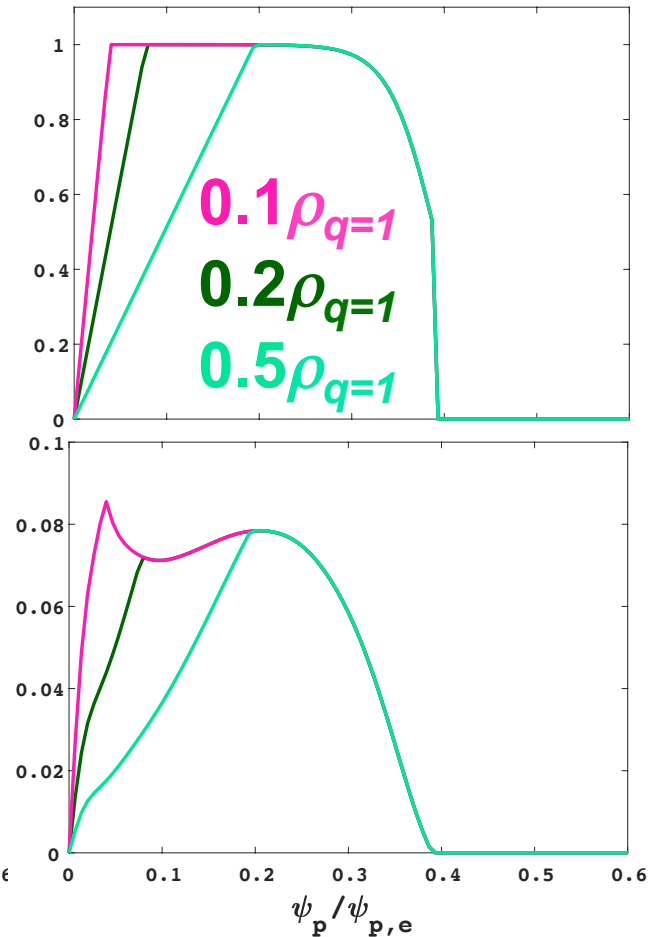
$\xi_0 = 0, 0.2\rho_{q=1}$

Central value



shape, $0.2\rho_{q=1}$

Shape change position

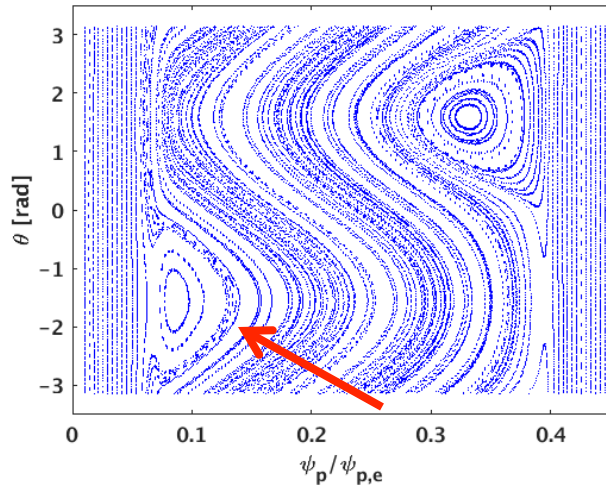


shape, $\xi_0 = 0$

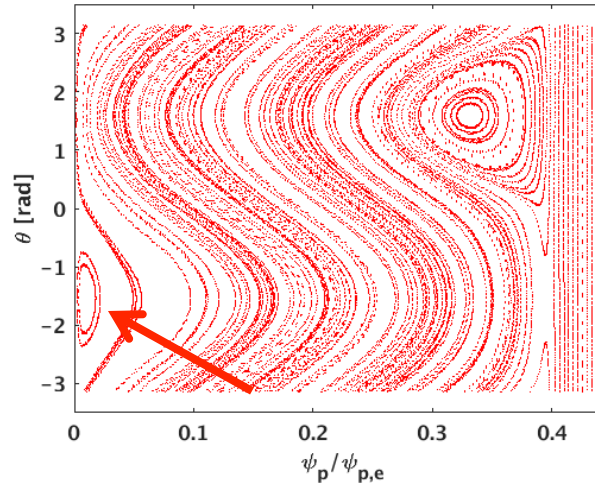
Modification of perturbation profile

- Central region shape test (biggest effect on result)

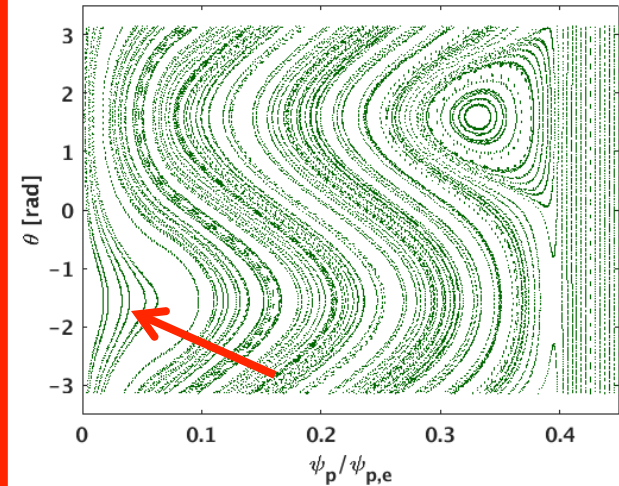
Shape 1



Shape 2

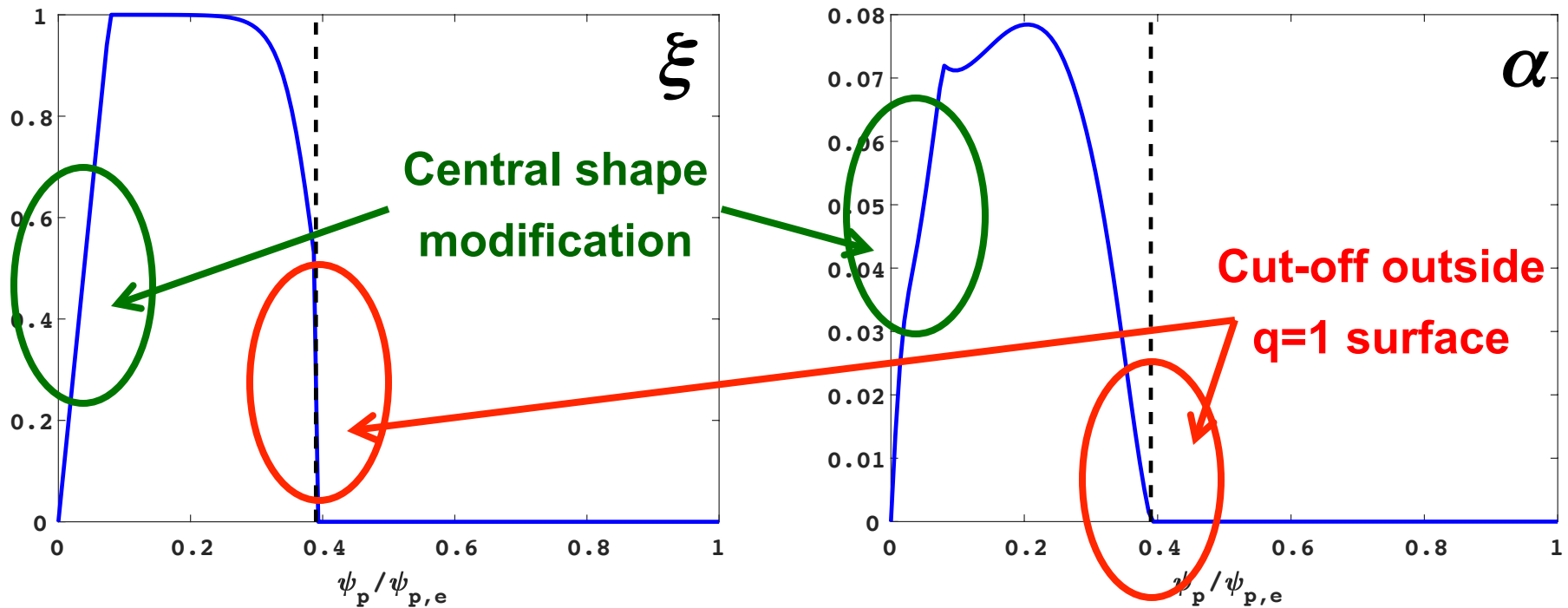


Shape 3



- Looking for a perturbation shape that eliminates undesirable island structures (center and $q=1$ surface)
- ξ decreases inside $0.2\rho_{q=1}$, $\xi_0 = 0$
- Island is formed where α has large gradient
- Central shape case 3 (linear change) brings perturbation at the center but not enough to form an island

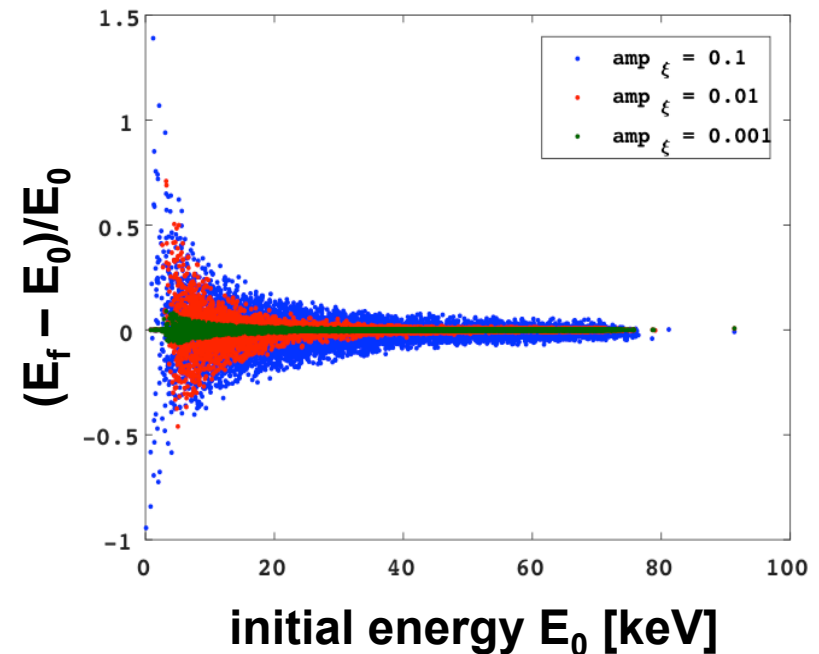
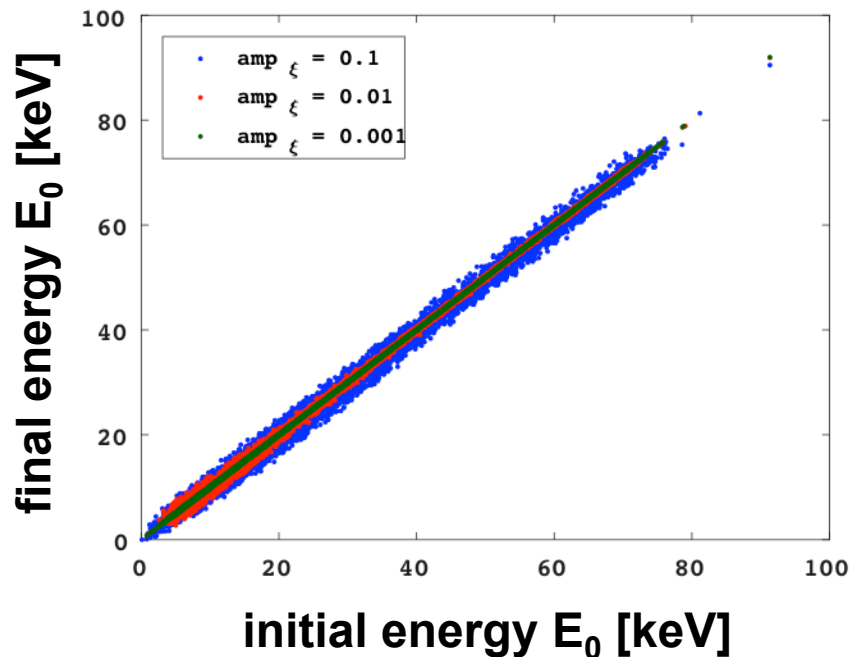
Modified perturbation shape



- The resultant α shape is similar to that from [Zhao PoP97]
- Set as the default perturbation shape for the further analysis with different mode amplitude (default: 0.01)

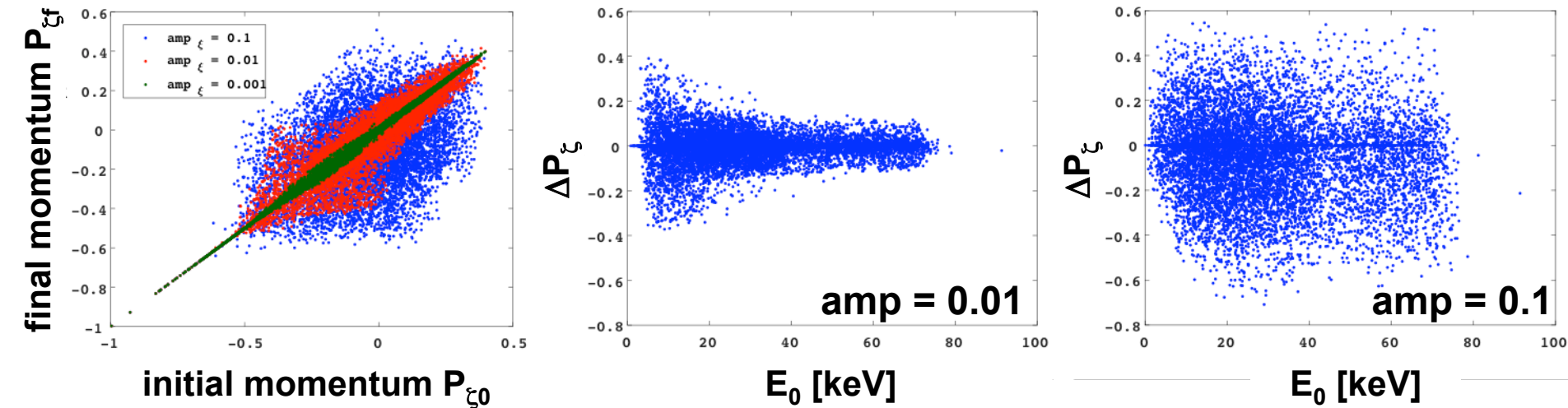
Fast ion energy is not changed significantly by sawtooth

- ORBIT calculation shows the final energy is not deviated much from the initial energy
 - With 100 times different amplitude (0.001 to 0.1), the change is little
 - Relative change can show some variation among low energy particles



Fast ions are redistributed in phase space depending on the mode amplitude

- As perturbation amplitude increases, particle redistribution in phase space becomes more significant
- Smaller amplitude case, initially low energy particles are more redistributed while particles in all energy level are affected by larger amplitude sawtooth

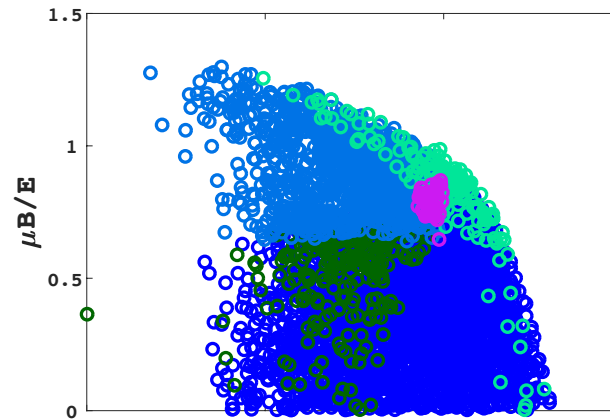


Redistribution of different orbit types and energy level particles in phase space

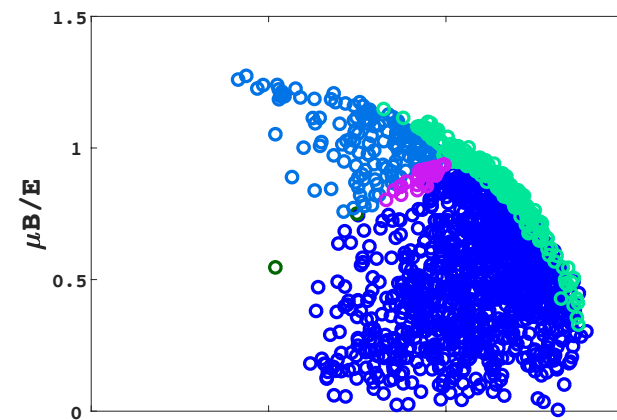
- In smaller amplitude case (0.01), particle redistribution is weak

- Lower energy level [10 ~ 30keV] particles have more redistribution than higher energy [50 ~ 70keV] case

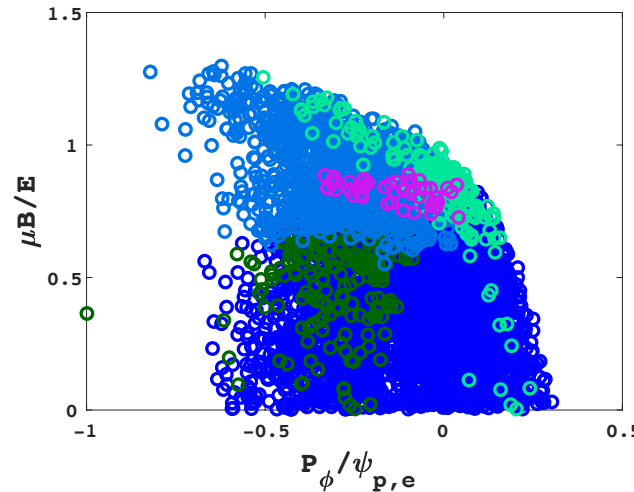
Initial distribution E = [10, 30] keV



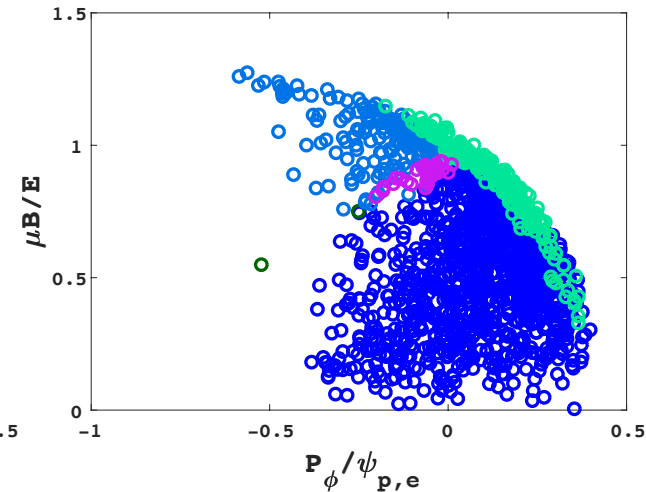
Initial distribution E = [50, 70] keV



Final distribution E = [10, 30] keV



Final distribution E = [50, 70] keV



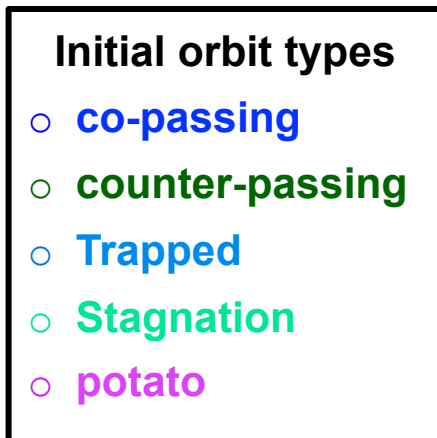
Initial orbit types

- co-passing
- counter-passing
- Trapped
- Stagnation
- potato

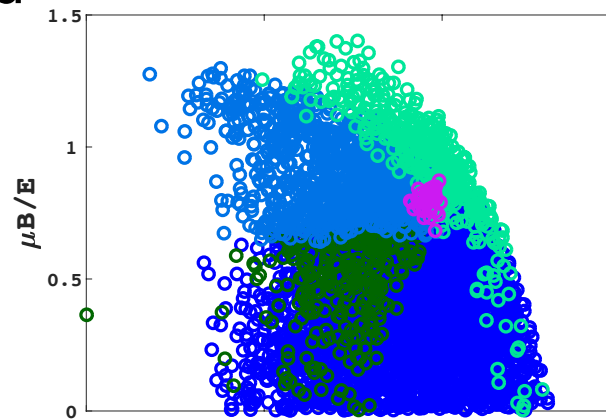
Redistribution of different types and energy level particles in phase space

- In larger amplitude case (0.1), particles with low and high energy redistributed

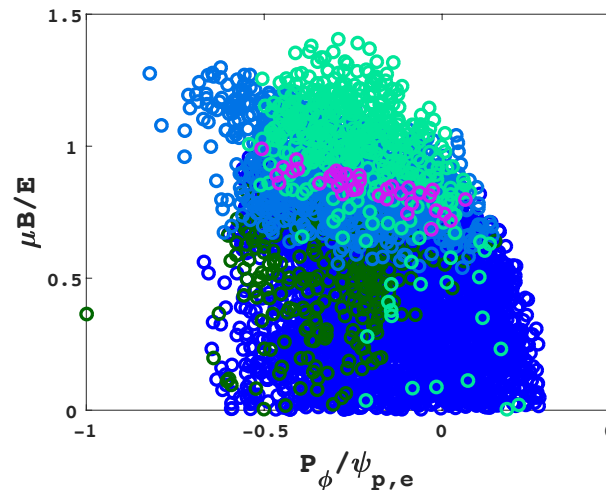
- For both low and high energy level, particles are strongly affected by sawtooth and redistributed



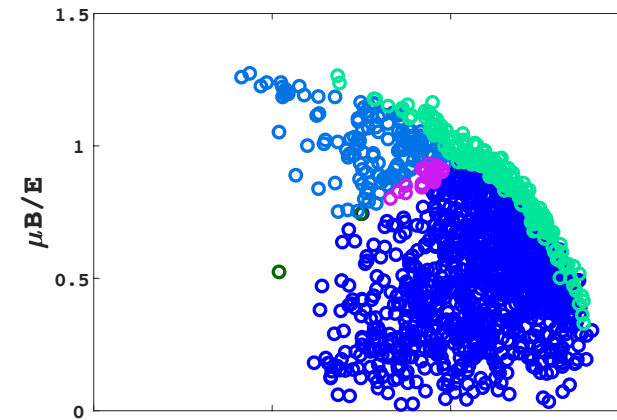
Initial distribution E = [10, 30] keV



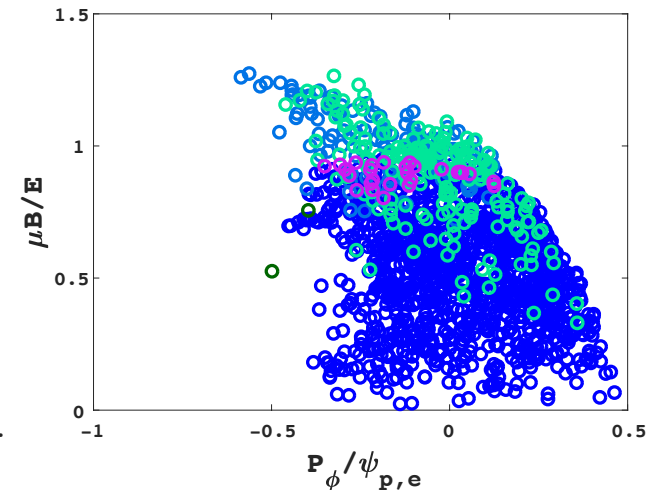
Final distribution E = [10, 30] keV



Initial distribution E = [50, 70] keV

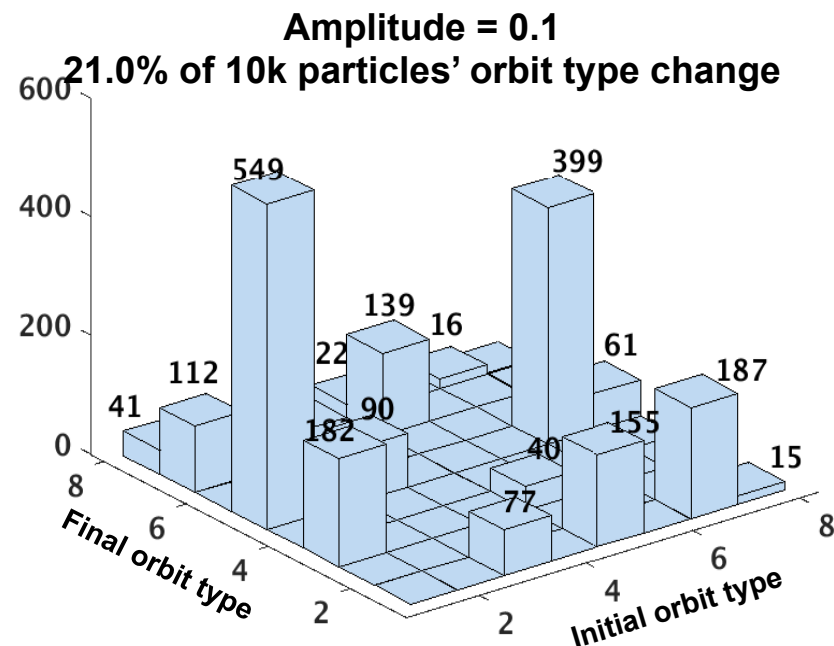
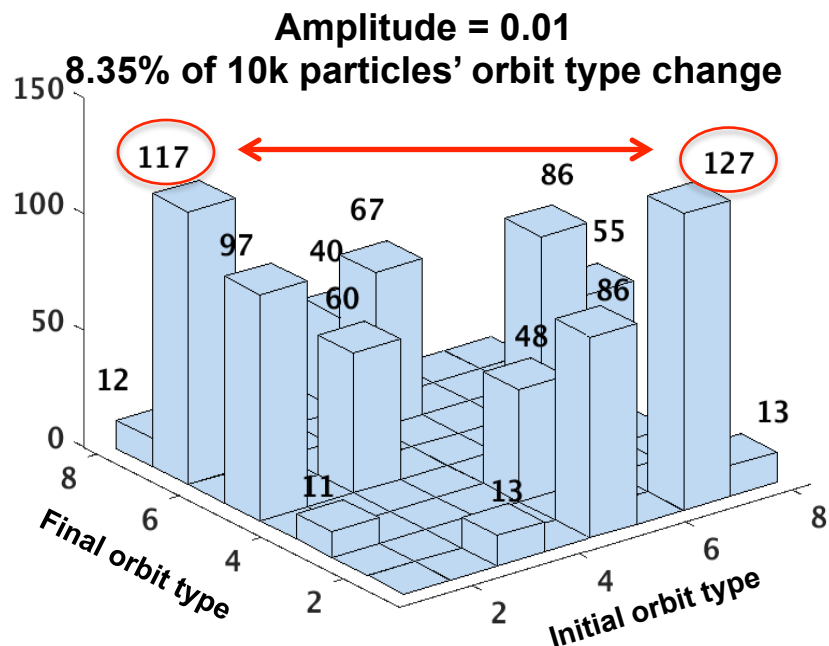


Final distribution E = [50, 70] keV



During redistribution, particles experience the change of orbit type

Orbit type 1: co-pass confined	Orbit type 3: ctr-pass confined	Orbit type 5: trapped confined	Orbit type 7: stagnation
Orbit type 2: co-pass lost	Orbit type 4: ctr-pass lost	Orbit type 6: trapped lost	Orbit type 8: potato confined



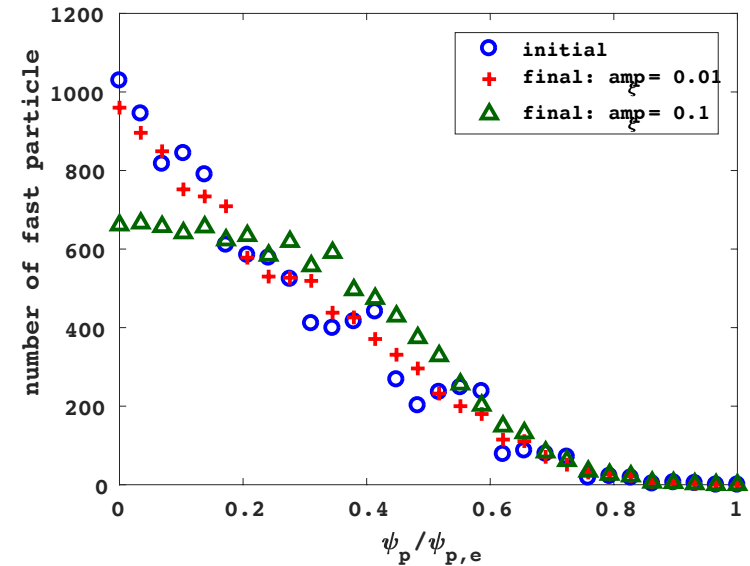
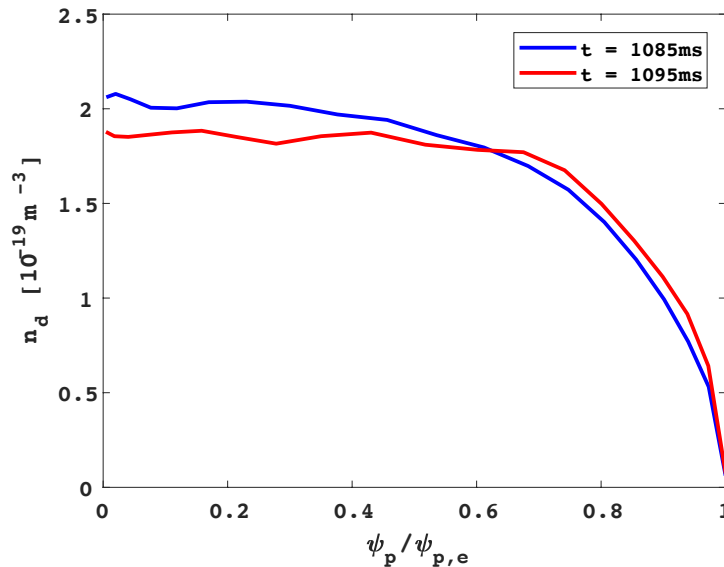
$(i,f) = (a,b)$: initially a orbit type turns into b type

- Orbit types swapping brings almost no change in total number of each type
- Co-passing and stagnation types particles mostly turn into trapped particles

Relative change of neutron rate – comparison with experimental result

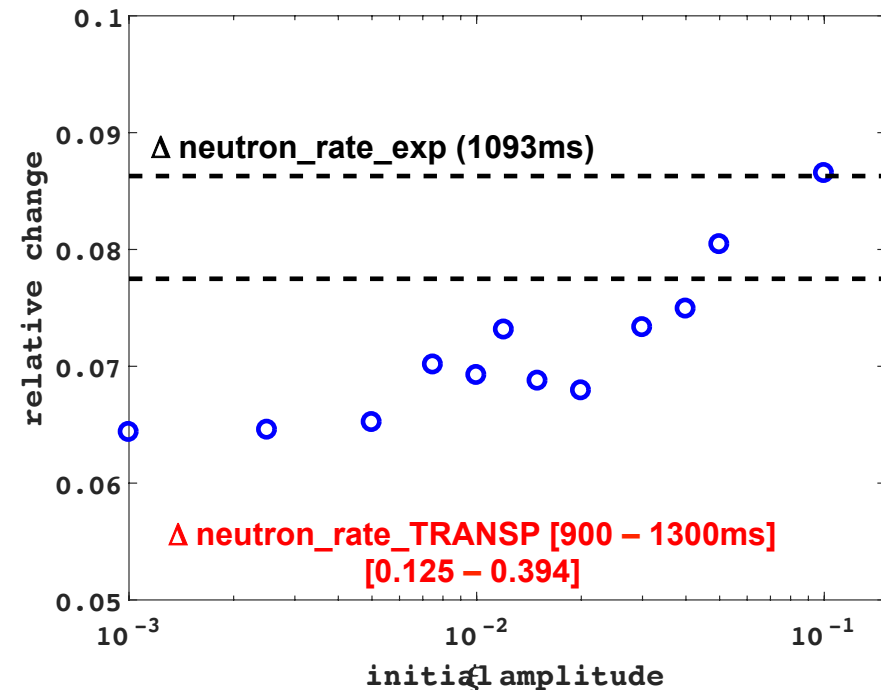
- Estimate relative neutron rate change
 - Using deuterium density at each fast particle position, cross section and particle energy of each particle energy

$$\Delta = \frac{\sum_k^{N_f} n_{df,k} S_{f,k} \sqrt{E_{f,k}} - \sum_k^{N_0} n_{d0,k} S_{0,k} \sqrt{E_{0,k}}}{\sum_k^{N_0} n_{d0,k} S_{0,k} \sqrt{E_{0,k}}} \quad (k = \text{fast particle index})$$



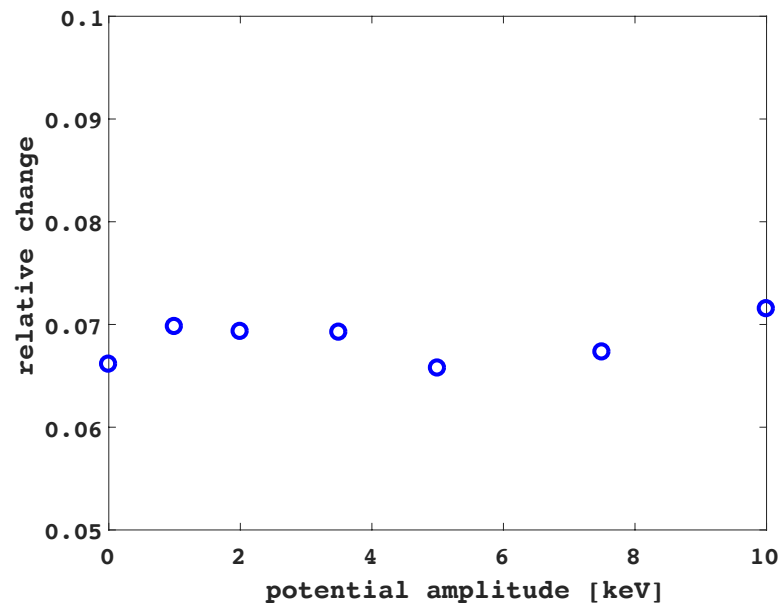
Relative change of neutron rate – comparison with experimental result

- Relative neutron rate change from experiment
 - Difference between neutron rate post and pre crash is normalized by neutron rate before sawtooth crash
 - Depending on calibration parameter, values are slightly different
- Perturbation amplitude scan
 - Δ calculation using ξ amplitude of [0.001 to 0.1]
 - In lower amplitude case, Δ is almost constant since fast particles are not redistributed much
 - Amplitude of between [0.045, 0.1] can reproduce experimental values



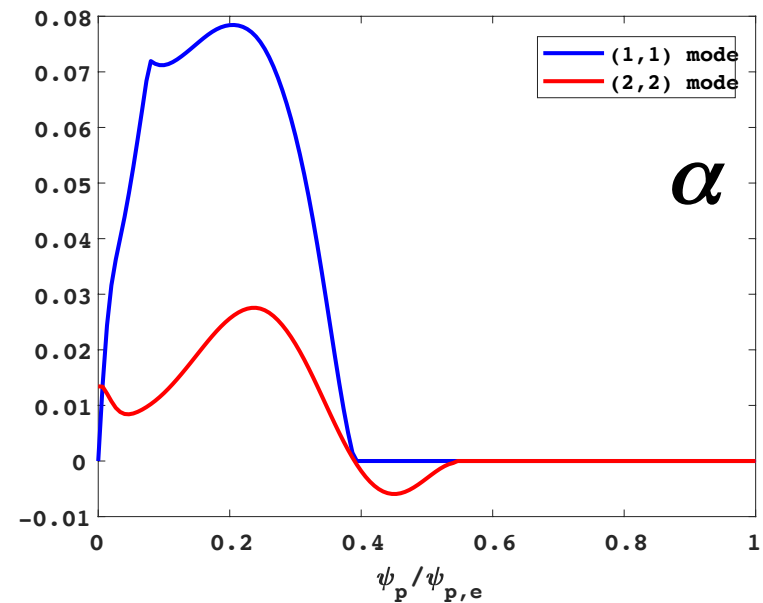
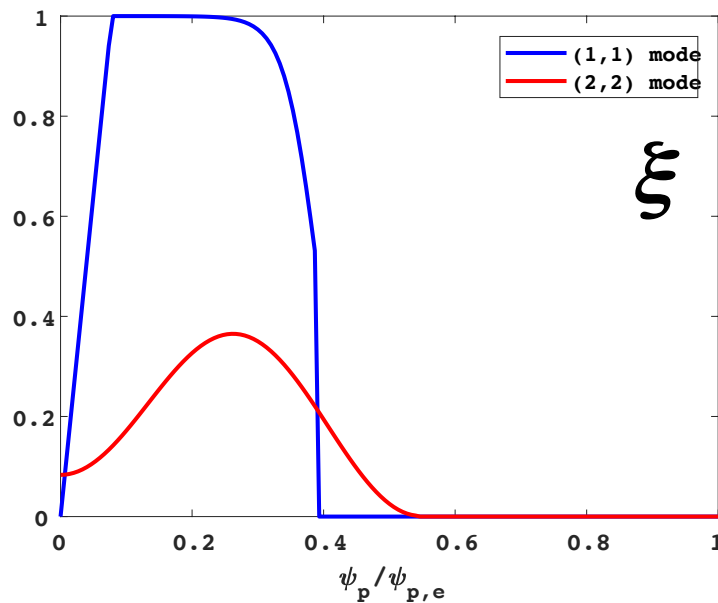
Relative change of neutron rate – comparison with experimental result

- Radial potential amplitude scan
 - Δ calculation using potential amplitude of [0 to 10] keV
 - With finite potential amplitude, a simple exponential radial potential profile is added
 - With fixed perturbation amplitude of 0.01, Δ does not vary with potential
 - Mode frequency is fixed to 10 kHz



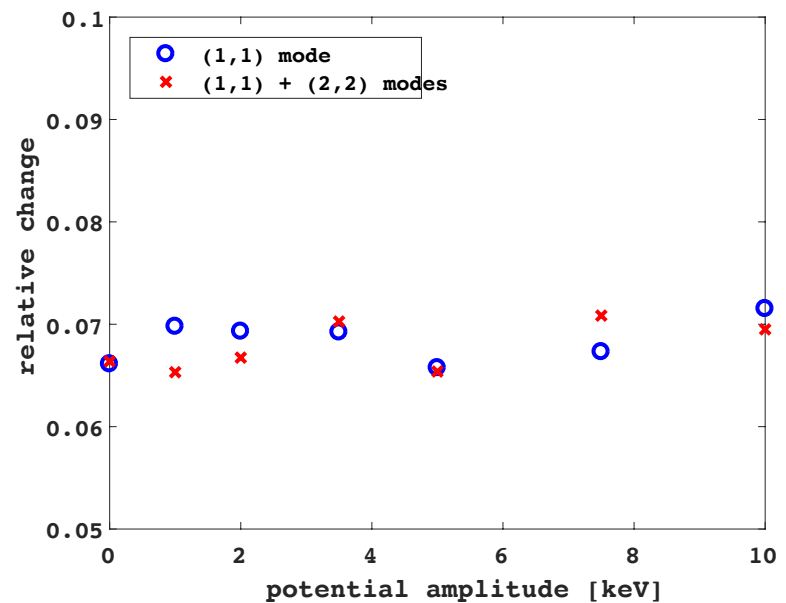
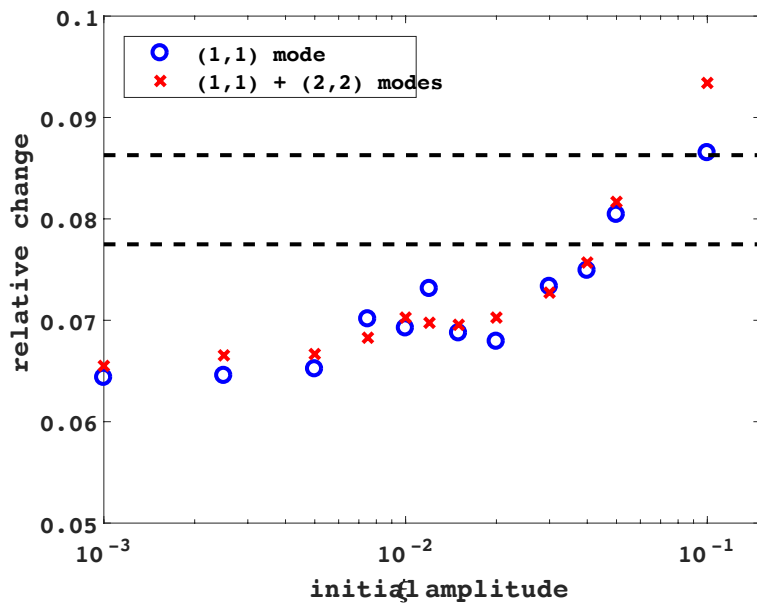
Test of two mode case: (1,1)+(2,2) modes - perturbation profile

- (2,2) mode based on [Farengo] is added to (1,1) mode
 - Mode amplitude is set to 1/3 of (1,1) mode and twice mode frequency
 - Modification of profile is not applied to (2,2) mode perturbation profile
 - ➔ additional island structure is found from Poincaré plot



Test of two mode case: $(1,1) + (2,2)$ modes - relative neutron rate change

- Perturbation amplitude/potential scan
 - Two modes case has the same trend as $(1,1)$ mode case
 - Since the mode amplitude of $(2,2)$ mode is only $1/3$ of $(1,1)$ mode amplitude, the effect seems small



ORBIT simulation has been tested for sawtooth induced fast ion redistribution in phase space

- Linear perturbation α is implemented into ORBIT code
 - Transformation from ξ is applied
 - Perturbation shape is modified to induce (1, 1) mode island at $q=1$ surface
- Energy and particle redistribution due to sawtooth crash is investigated
 - Regardless the amplitude of perturbation, E is not changed significantly while P_ξ variation becomes significant with increase of mode amplitude
 - For low amplitude perturbation, low initial energy particles are redistributed more while all energy level are affected by high amplitude perturbation
- Relative change of neutron rate induced by sawtooth crash is compared with experimental value
 - Δ value increases as mode amplitude grows from amplitude of 0.02 (below Δ stays almost similar level)
 - Experimental value can be reproduced by amplitude range of [0.045, 0.1]
 - Modification of potential does not have significant effect on Δ value
 - (2, 2) mode is added to (1, 1) mode but amplitude and potential scan results are almost the same as (1, 1) only mode case

On going work

Redistribution of particles

– comparison with theory [Kolesnichenko PoP97]

- Criteria for sawtooth induced fast particle redistribution

Trapped particle

$$\Delta r_b \ll r_{crit} = \frac{r_{mix}}{(\epsilon_{mix} \tau_{cr} \omega_B / 4\pi)^{1/2}}$$

Passing particle

$$\Delta r_b \ll r_{crit} = r_{mix}$$

Δr_b = orbit width, r_{mix} = sawtooth mixing radius assuming full reconnection
 ϵ_{mix} = inverse aspect ratio at r_{mix} , τ_{cr} = crash time, ω_B = ion cyclotron frequency

- Radial position change vs initial orbit width – Comparison with critical value for different particle energy level and orbit type
 - Passing particles: criterion is well satisfied
 - Radial position change occurs only for orbit width smaller than the critical one
 - Stagnation and potato particles: marginal
 - Inside and near the critical value
 - Trapped particle: criterion is not applied
 - Particles with orbit width bigger than the critical one experiences radial position change

Redistribution of particles – comparison with theory

