NSTX-U

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Study on Neoclassical Transport and Kinetic Stability in Perturbed Tokamaks with Particle Simulation

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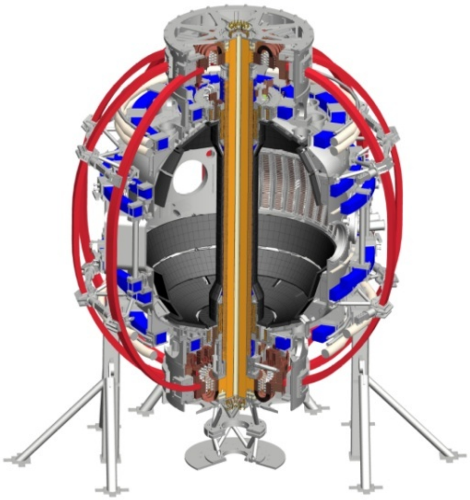


Motivation

- Accurate description of guiding-center particle motions is essential for neoclassical transport in the nonaxisymmetric magnetic perturbations
  - Drift-kinetic δf particle code: POCA (Particle Orbit Code for Anisotropic pressures)
    - Guiding-center orbit motions on flux coordinates
    - Fokker-Plank equation with pitch-angle scattering conserving toroidal momentum
  - Benchmarked for neoclassical transport in axisymmetry
  - Verify basic NTV physics and analyze NTV torque in magnetic braking experiments
- 3D neoclassical theory found equivalence of NTV and kinetic potential energy
  - Mathematically proved, numerically verified
  - Identical form but different resonant conditions depending on rotation and collision
    - Different feature can be captured by particle motion
- Neoclassical particle simulation can be a useful tool for kinetic stability analysis
  - Benchmarked with semi-analytic theory for NTV study
  - Kinetic potential energy can be calculated from NTV torque in a fully numerical method

Outline

- Numerical verification of NTV physics with particle simulation
- Unification of NTV and kinetic potential energy
- A potential application of particle code to kinetic stability study
- Summary and Future Work

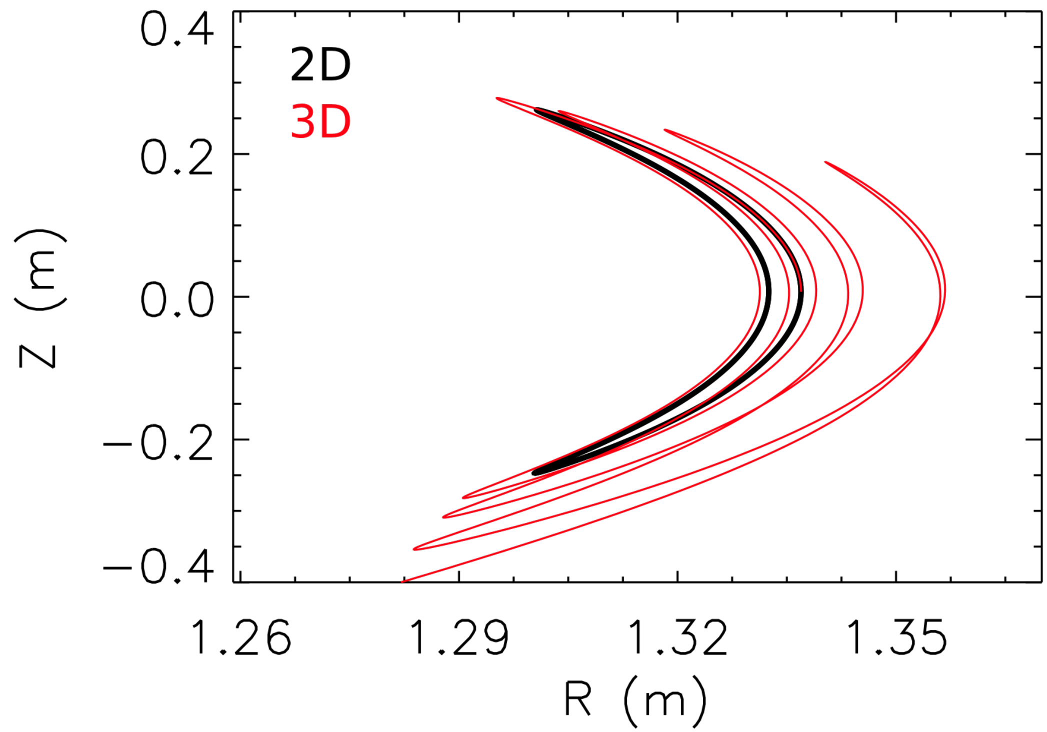
Fundamental NTV physics can be described by drift-kinetic particle motions in perturbed tokamaks

- Semi-analytic NTV theory represents the fundamental NTV physics

$$\left\langle \hat{e}_\phi \cdot \nabla \cdot \vec{\Pi} \right\rangle_l = \frac{\varepsilon^{-1/2} P}{\sqrt{2}\pi^{3/2} R_0} \underbrace{\int_0^1 d\kappa^2 \delta_{\omega,l}^2}_{\delta B} \underbrace{\int_0^\infty dx R_{1l}}_{\text{Resonance}} \underbrace{\left[u^\varphi + 2.0\sigma \left| \frac{1}{e} \frac{dT}{d\chi} \right| \right]}_{\text{Neoclassical offset}}$$

[J.-K. Park, PRL (2009)]

Banana orbits in 2D and 3D w/o collisions



[K. Kim, POP (2012)]

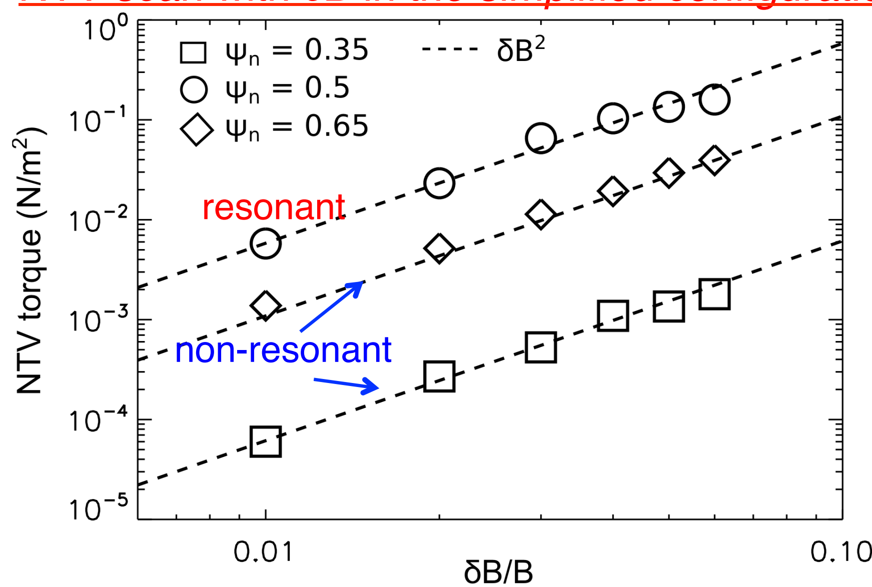
Quadratic δB Dependency of NTV

- Semi-analytic NTV theory represents the fundamental NTV physics

$$\left\langle \hat{e}_\phi \cdot \nabla \cdot \vec{\vec{\Pi}} \right\rangle_l = \frac{\varepsilon^{-1/2} P}{\sqrt{2}\pi^{3/2} R_0} \underbrace{\int_0^1 d\kappa^2 \delta_{\omega,l}^2}_{\delta B} \int_0^\infty dx R_{1l} \left[u^\varphi + 2.0\sigma \left| \frac{1}{e} \frac{dT}{d\chi} \right| \right]$$

  - Particle simulation shows the quadratic δB dependency of NTV, on the both resonant and non-resonant flux surfaces
  - Useful to particle simulation, enable NTV scaling by δB to improve statistics

<u>*NTV scan with δB in the simplified configurations*</u>



[K. Kim, POP (2012)]

Rotational resonances can enhance radial particle transport by suppressing random phase-mixing

- Semi-analytic NTV theory represents the fundamental NTV physics

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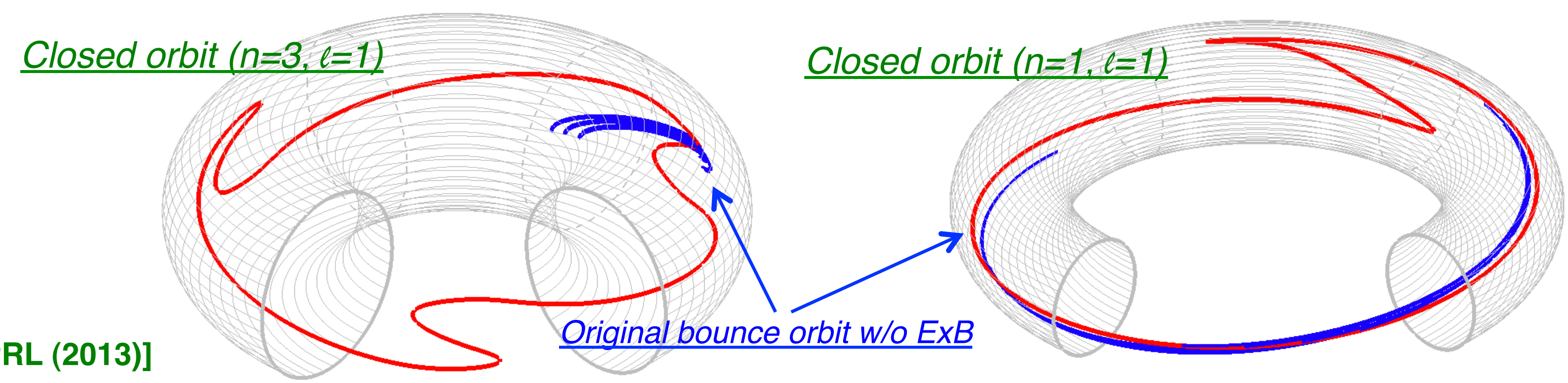
  - Resonant field condition shifted to follow path of closed orbit

$$F^y_{nml} \equiv \int_{-\vartheta_t}^{\vartheta_t} d\vartheta\left(\kappa^2 - \sin^2(\vartheta/2)\right)^y \underbrace{\cos(m - nq \pm l)\vartheta}_{\textit{Phase shift of resonant surface by ExB precession}}$$

  - Resonance frequency condition by particle's bouncing class ℓ

$$R_{yl} = \frac{1}{2}\frac{n^2\left(1+(l/2)^2\right)\frac{\nu}{2\varepsilon}x\left(x-\frac{5}{2}\right)^y e^{-x}}{\left(\underbrace{l\omega_{l,bounce} - n\omega_{E\times B} - n\omega_{\nabla B}}_{=0}\right)^2 + \left(\left(1+(l/2)^2\right)\frac{\nu}{2\varepsilon}\right)^2 x^{-3}}$$



[K. Kim, PRL (2013)]

Rotational resonances can enhance radial particle transport by suppressing random phase-mixing

- Semi-analytic NTV theory represents the fundamental NTV physics

$$\left\langle \hat{e}_\phi \cdot \nabla \cdot \vec{\Pi} \right\rangle_l = \frac{\varepsilon^{-1/2} P}{\sqrt{2}\pi^{3/2} R_0} \int_0^1 d\kappa^2 \delta_{\omega,l}^2 \int_0^\infty dx \underbrace{R_{1l}}_{\text{Resonance}} \left[u^\varphi + 2.0\sigma \left| \frac{1}{e} \frac{dT}{d\chi} \right| \right]$$

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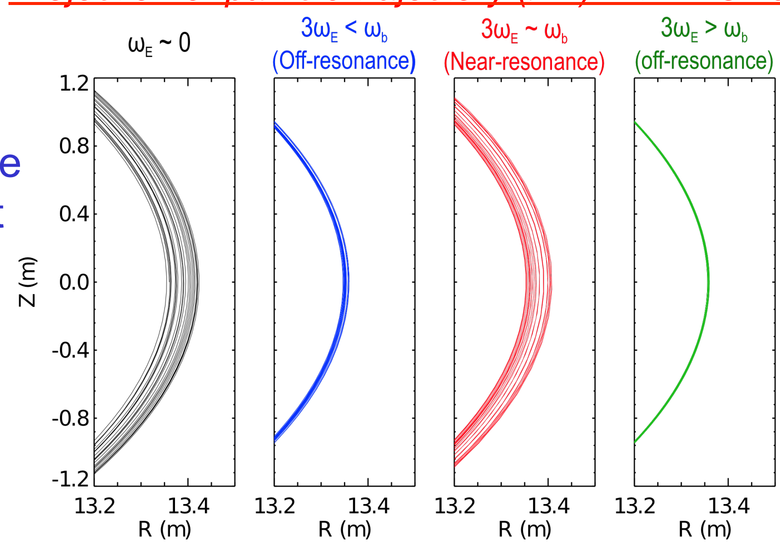
- 3D particle transport can be dominated by rotational resonances
  - Enhanced transport by precession resonance at $\omega_E \sim 0$, and bounce-harmonic resonance at $3\omega_E \sim \omega_b$
  - Off-resonance rotations reduce the particle transport by random phase-mixing

Projection of particle trajectory (ℓ=1) with n=3 fields



[K. Kim, PRL (2013)]

Neoclassical offset can be identified at zero-torque

- Semi-analytic NTV theory represents the fundamental NTV physics

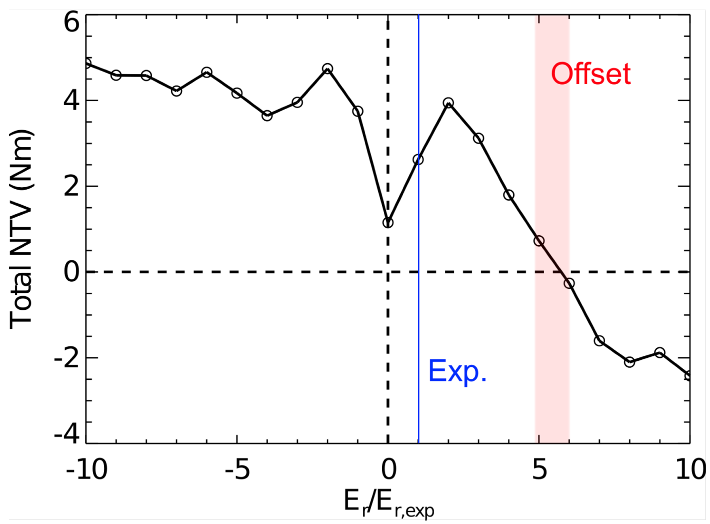
$$\left\langle \hat{e}_\phi \cdot \nabla \cdot \vec{\Pi} \right\rangle_l = \frac{\varepsilon^{-1/2} P}{\sqrt{2}\pi^{3/2} R_0} \int_0^1 d\kappa^2 \delta_{\omega,l}^2 \int_0^\infty dx R_{1l} \left[\underline{u^\varphi + 2.0\sigma \left| \frac{1}{e} \frac{dT}{d\chi} \right|} \right]$$

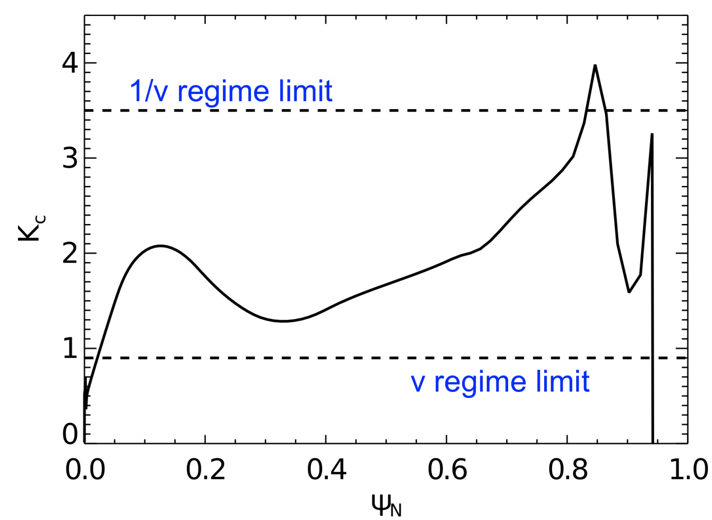
Neoclassical offset

- Neoclassical offset rotation is theoretically predicted as

$$V_{\phi,offset} = K_c \frac{1}{Z_i e B_\theta} \frac{dT_i}{dr} \qquad K_c = \frac{V_{\phi,offset}}{\frac{1}{Z_i e B_\theta} \frac{dT_i}{dr}} \approx 3.5\ (1/\nu \text{ regime}) \quad \text{or} \quad 0.9\ (\nu \text{ regime})$$

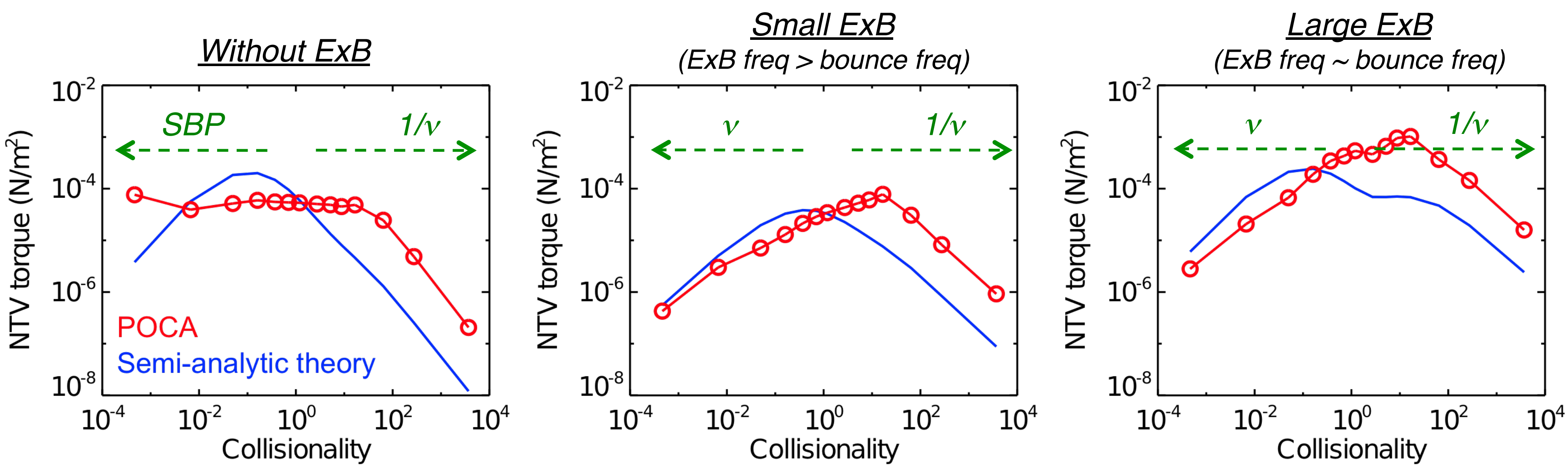
- Numerically obtained K_c is consistent with theory prediction, indicating the plasma is in the connected regime





Particle simulation reproduces predicted NTV physics

- Consistent features of NTV dependency on the collisionality are reproduced with drift-kinetic particle simulations
  - Clear 1/ν-regime in the high collisionality with passing particle effects
  - Superbanana-plateau (SBP) resonance in the low collisionality without ExB
  - ν−regime in the low collisionality with small ExB
  - Enhanced NTV by bounce-harmonic resonance in the large ExB



Equivalence of NTV and kinetic energy has been mathematically proved and numerically verified

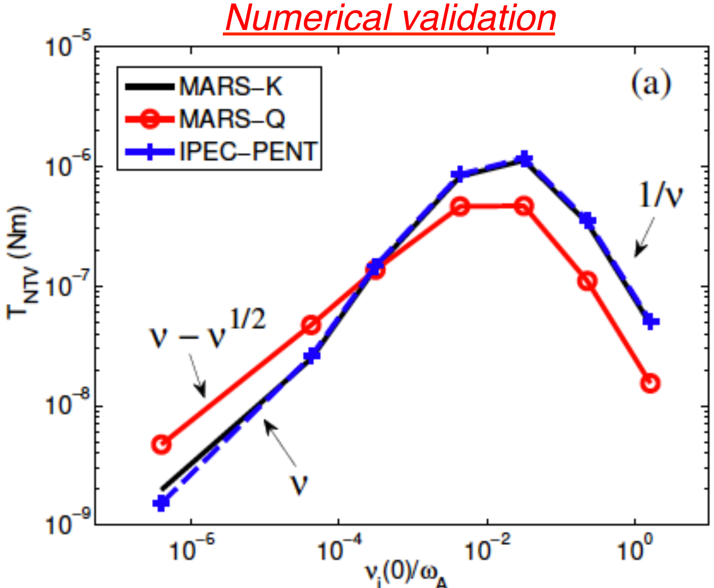
- 3D neoclassical theory shows kinetic potential energy drives NTV force
  - Imaginary part of kinetic energy is equivalent to NTV torque

$$T_\varphi = 2in\delta W_K$$

[J.-K. Park, POP (2011)]

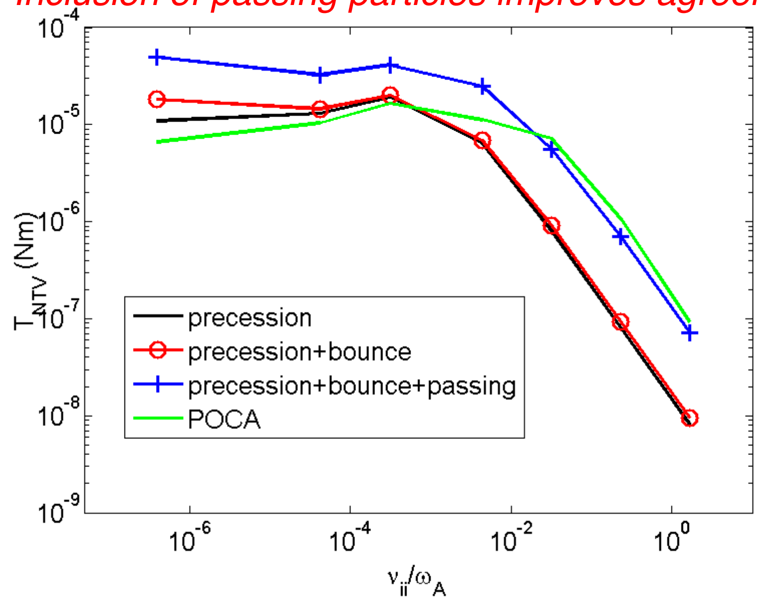
$$T_\varphi = \sum_{\ell nmm'} \int dV \int_0^1 d\kappa^2 \int_0^\infty dx \frac{\varepsilon^{1/2} p}{\sqrt{2}\pi^{3/2}} \delta_{w\ell}^2 \mathcal{R}_{NTV} \omega_{t\ell}^\varphi \qquad \delta W_K = \sum_{\ell nmm'} \int dV \int_0^1 d\kappa^2 \int_0^\infty dx \frac{\varepsilon^{1/2} p}{2\sqrt{2}\pi^{3/2}} \delta_{w\ell}^2 \mathcal{R}_{\delta W_K} \omega_{e\ell}^\varphi$$

Numerical validation



[Z. Wang, Submitted]

Comparison of POCA with MARS-K:
Inclusion of passing particles improves agreement



  - Lead to unification of NTV theory and RWM kinetic theory

Equivalence of NTV and kinetic energy has been theoretically proved and numerically verified

- 3D neoclassical theory shows kinetic potential energy drives NTV force
  - Imaginary part of kinetic energy is equivalent to NTV torque

$$T_\varphi = 2in\delta W_K$$ **[J.-K. Park, POP (2011)]**

$$T_\varphi = \sum_{\ell nmm'} \int dV \int_0^1 d\kappa^2 \int_0^\infty dx \frac{\varepsilon^{1/2} p}{\sqrt{2}\pi^{3/2}} \delta_{w\ell}^2 \mathcal{R}_{NTV} \omega_{t\ell}^{\varphi} \qquad \delta W_K = \sum_{\ell nmm'} \int dV \int_0^1 d\kappa^2 \int_0^\infty dx \frac{\varepsilon^{1/2} p}{2\sqrt{2}\pi^{3/2}} \delta_{w\ell}^2 \mathcal{R}_{\delta W_K} \omega_{e\ell}^{\varphi}$$

- Perturbed magnetic field is expressed as Fourier series

$$\delta B_{mn}(\psi_n) = \sum_m a_{mn}(\psi_n)\cos(m\vartheta - n\varphi) + b_{mn}(\psi_n)\sin(m\vartheta - n\varphi)$$

- POCA calculates NTV using perturbed field spectrum

$$T_\varphi = \left\langle \frac{\delta P}{B}\frac{\partial B}{\partial \varphi} \right\rangle \longrightarrow T_\varphi = \sum_m n \left\langle \frac{\delta P}{B}\left[a_{mn}\sin(m\vartheta - n\varphi) - b_{mn}\cos(m\vartheta - n\varphi)\right] \right\rangle$$

- *Kinetic potential energy can be calculated from imaginary term of NTV*

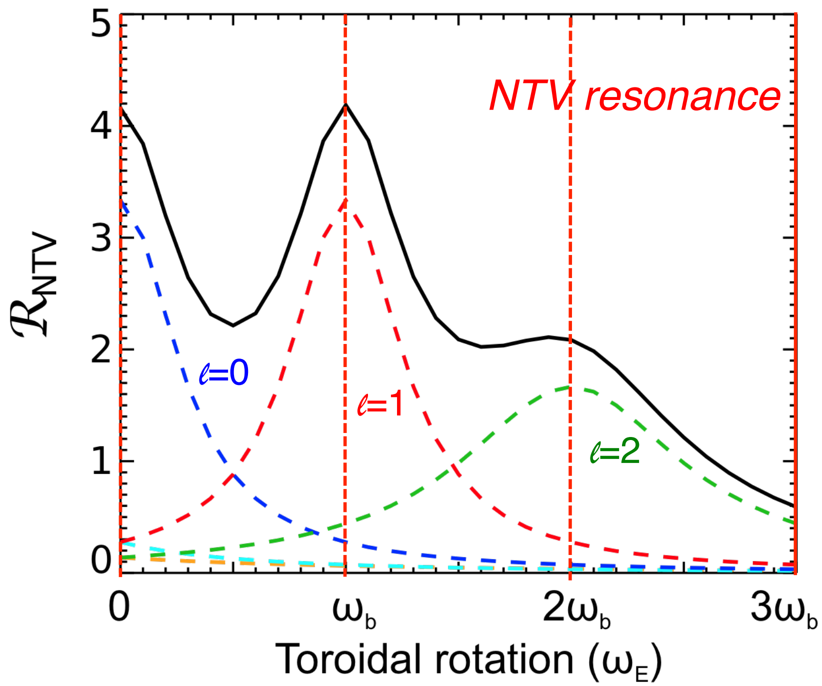
$$\delta W_K = -\frac{1}{2}\sum_m \left\langle \frac{\delta P}{B}\left[a_{mn}\cos(m\vartheta - n\varphi) + b_{mn}\sin(m\vartheta - n\varphi)\right] \right\rangle$$

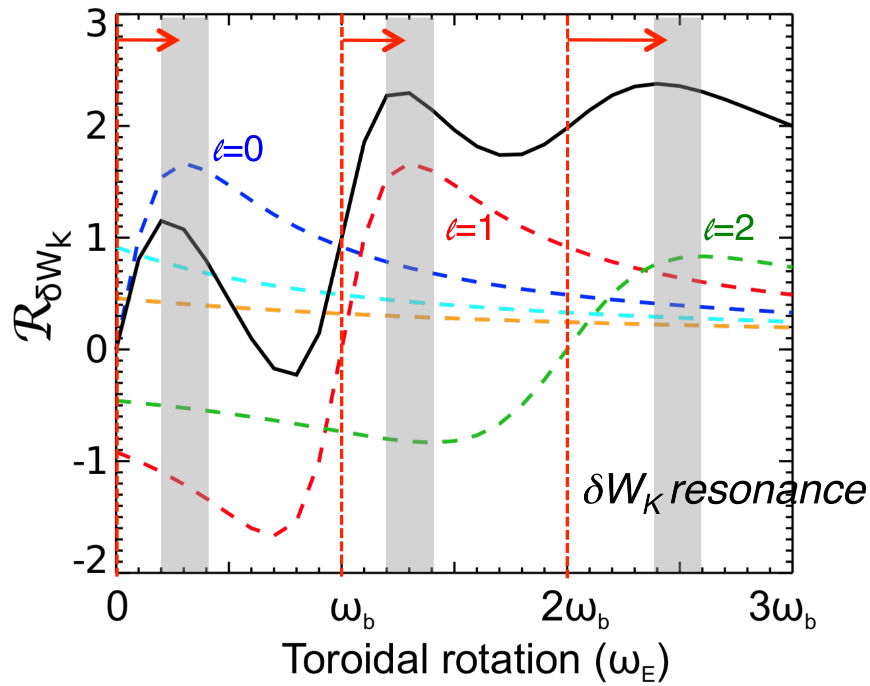
NTV and δW_K are almost identical in formalism, but differ in resonant operators due to phase change

- Plots of resonant operators show different feature in the rotational resonances

$$\mathcal{R}_{NTV} = \frac{n^2 \nu_{Dl}\left(xe^{-x}\right)}{\left[l\omega_b - n\left(\omega_E + \omega_B\right)\right]^2 + \nu_{Dl}^2}$$

$$\mathcal{R}_{\delta W_K} = \frac{n\left[n\left(\omega_E + \omega_B\right) - l\omega_b\right]\left(x^{5/2}e^{-x}\right)}{\left[l\omega_b - n\left(\omega_E + \omega_B\right)\right]^2 + \nu_{Dl}^2}$$

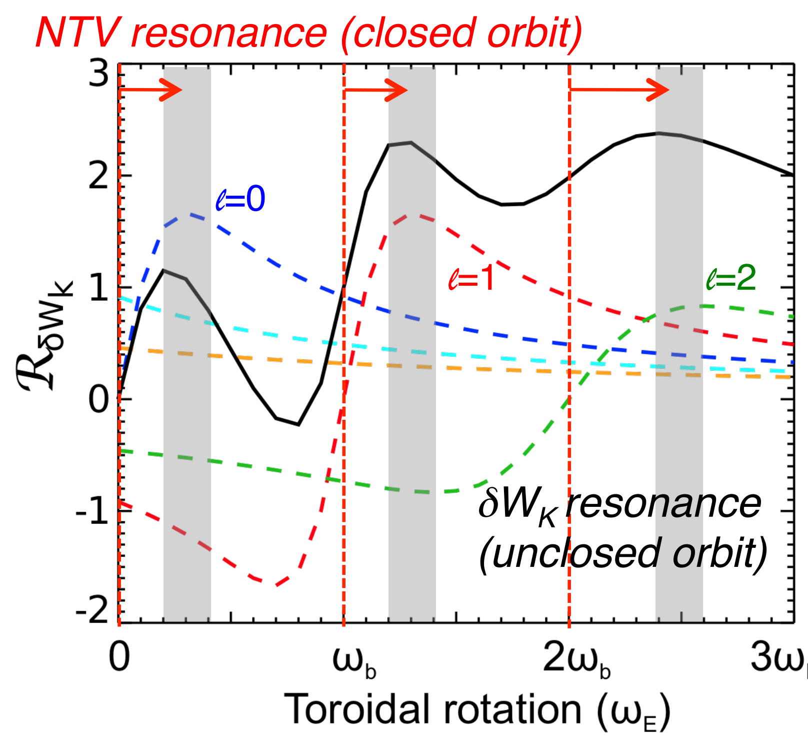




  - NTV resonance by bounce-harmonic resonance ExB rotation
  - Kinetic potential energy resonance shifted from bounce-harmonic resonances

Analytic integration of resonant operator in the kinetic theory is difficult, due to complicated bounce orbit

- Integration along bounce orbit *may not be well-defined*, unless orbits are closed
  - Bounce orbits are closed when NTV resonances occur ($n\omega_E=\ell\omega_b$)
    - Dominant contribution from resonant particles
    - Agreement between theory and particle code
  - Resonance frequencies for δW_K are shifted
    - Bounce integration can be poor due to unclosed bounce orbit
  - Semi-analytic integration of kinetic energy could be inaccurate and overestimated
    - Bounce orbit phase can be actually mixed, thus transport can be smaller
    - Closed orbit assumption can be broken

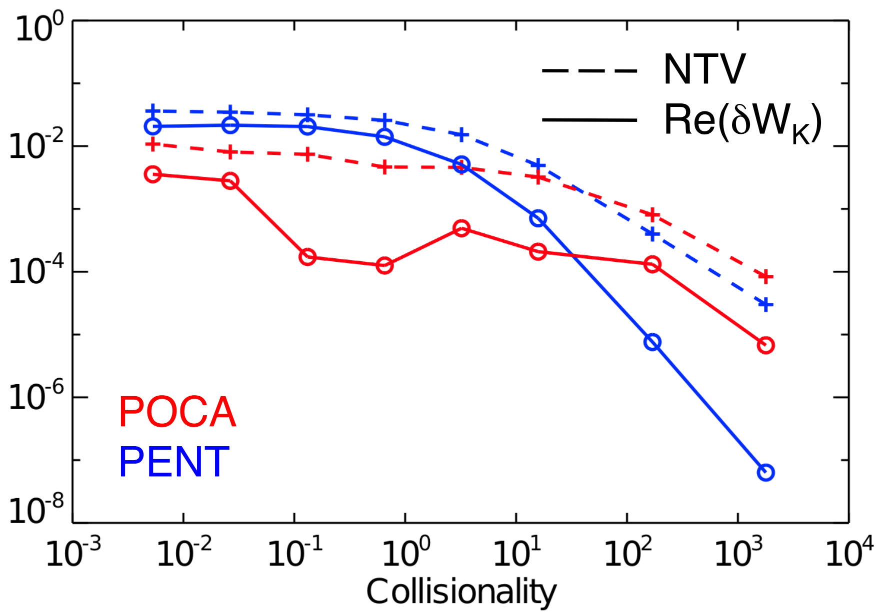


- Particle simulation can be another useful way for kinetic stability analysis
  - Semi-analytic calculations have been well-benchmarked (MISK, MARSK, PENT)
  - Particle simulation could give new physics in the kinetic stability analysis by tracking realistic orbit trajectories
  - Should extend the previous benchmarking work between MARS-K and HAGIS

[Y. Liu, POP (2008)]

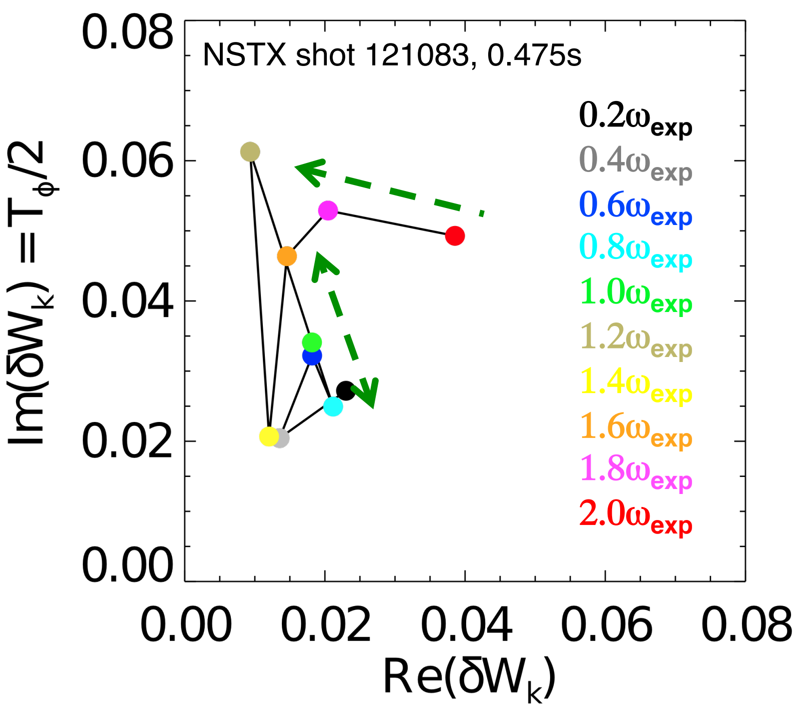
Preliminary Test in Solov'ev Equilibrium Compared to PENT

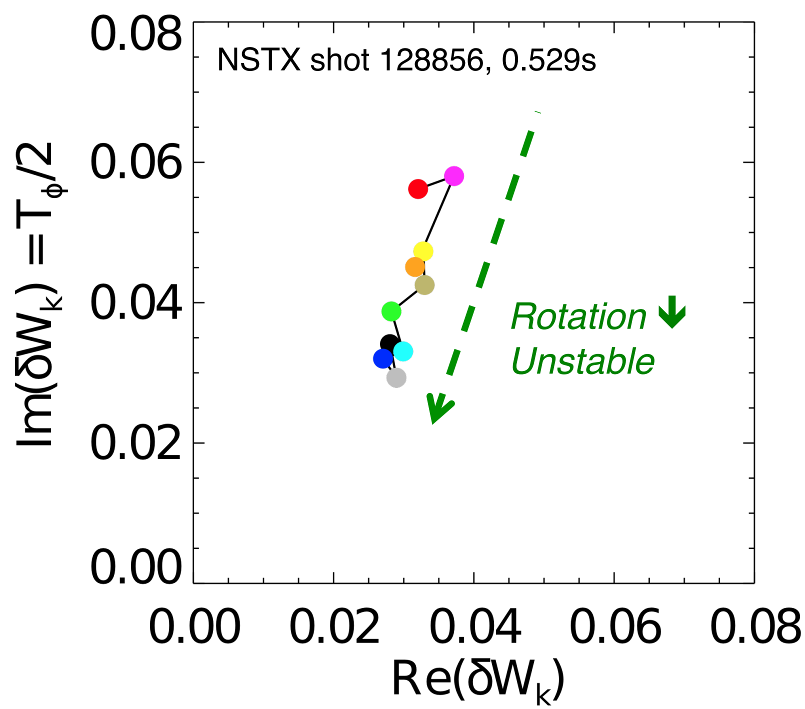
- Particle simulation largely agrees with semi-analytic calculation in NTV, but disagrees in kinetic potential energy
  - Solov'ev equilibrium with single rational surface
  - Consistent collisionality dependency and agreement in quantity for NTV **[N. Logan, Submitted]**
  - Disagreement in kinetic potential energy
    - Re(δW_K) by PENT: similar trend and comparable quantities to NTV in low collisionality
    - Re(δW_K) by POCA: less-similar and noisy trend, smaller than NTV by 1~2 orders of magnitude



Preliminary Test of Particle Simulation for δW_k: Resistive Wall Mode in NSTX (n=1)

- Test marginally stable n=1 resistive wall mode discharges in NSTX
  - Revisit excellent work by Columbia U using MISK **[J. Berkery, PRL (2010)]**
  - Re(δW_K) by POCA smaller than semi-analytic prediction by MISK
    - NTV torque can be greater than δW_K by an order of magnitude
    - Stability may the same, but path to the (un)stable condition can be different
  - This is a preliminary result; careful checks must be followed
    - Benchmark with MISK, MARS-K, PENT in the simplified condition





Summary and Future Work

- Particle orbit motion is a essential element to explain the toroidal momentum transport in the perturbed tokamaks
  - Distortion of banana orbits, interaction with collision and precession
- Guiding-center particle simulation is a useful tool for NTV study
  - Verify and validate theories, analyze and predict experiments, investigate new physics
- Particle simulation can contribute to kinetic stability analysis
  - Accurate bounce orbit description with finite orbit width effects may improve a prediction of kinetic potential energy
  - Rigorous benchmarking study with semi-analytic stability codes is required before experimental application
    - Start with nonresonant field in the simplified configuration
    - Compare and assess effects by collision, rational surfaces, rotation, fast ions

Back-up

Rotational resonances can enhance radial particle transport by suppressing random phase-mixing

- Semi-analytic NTV theory represents the fundamental NTV physics

$$\left\langle \hat{e}_\phi \cdot \nabla \cdot \vec{\Pi} \right\rangle_l = \frac{\varepsilon^{-1/2} P}{\sqrt{2}\pi^{3/2} R_0} \int_0^1 d\kappa^2 \delta_{\omega,l}^2 \int_0^\infty dx \underline{R_{1l}} \left[u^\varphi + 2.0\sigma \left| \frac{1}{e}\frac{dT}{d\chi} \right| \right]$$

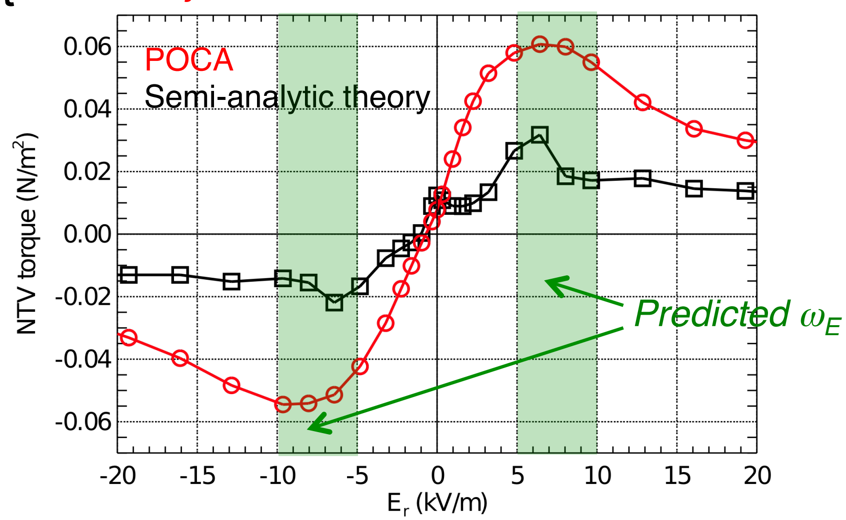
Resonance

  – Resonance frequency condition by particle's bouncing class ℓ

$$R_{yl} = \frac{1}{2} \frac{n^2 \left(1 + (l/2)^2\right) \frac{\nu}{2\varepsilon} x \left(x - \frac{5}{2}\right)^y e^{-x}}{\underbrace{\left(l\omega_{l,bounce} - n\omega_{E\times B} - n\omega_{\nabla B}\right)}_{=0}^2 + \left(\left(1 + (l/2)^2\right)\frac{\nu}{2\varepsilon}\right)^2 x^{-3}}$$

- NTV enhanced by the ExB rotation resonant to ℓ=1 bounce-harmonic resonance
  – Important driving mechanism of NTV in the fast rotating plasmas
  – Appear even in slow rotating plasma due to Maxwellian energy distribution
  – Appear every flux surface due to multi-harmonic magnetic perturbations

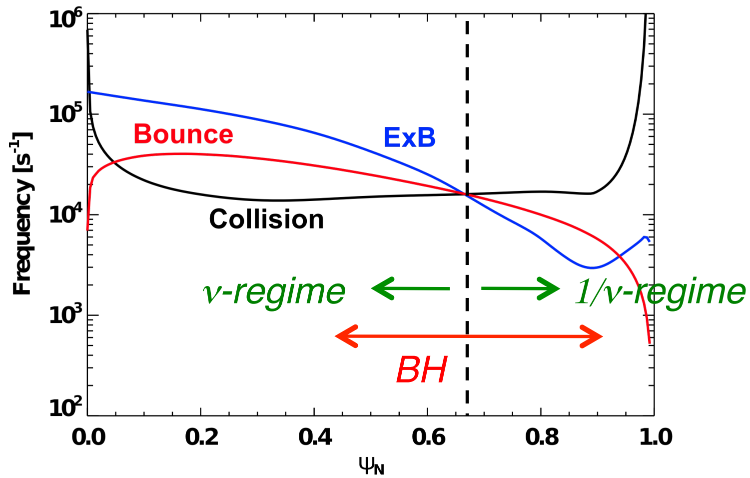
NTV by ℓ=1 bounce-harmonic resonances

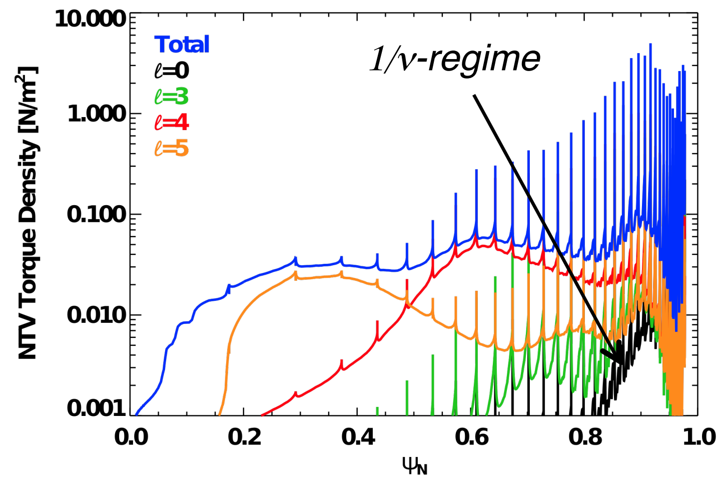


[K. Kim, PRL (2013)]

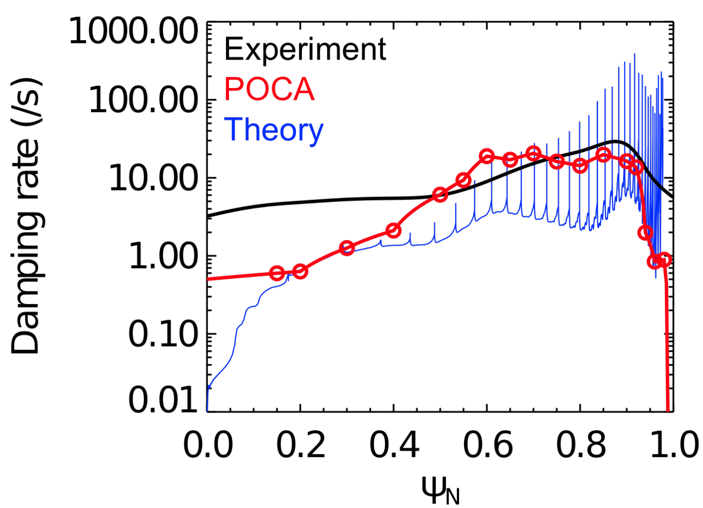
NTV prediction can be improved with particle simulation

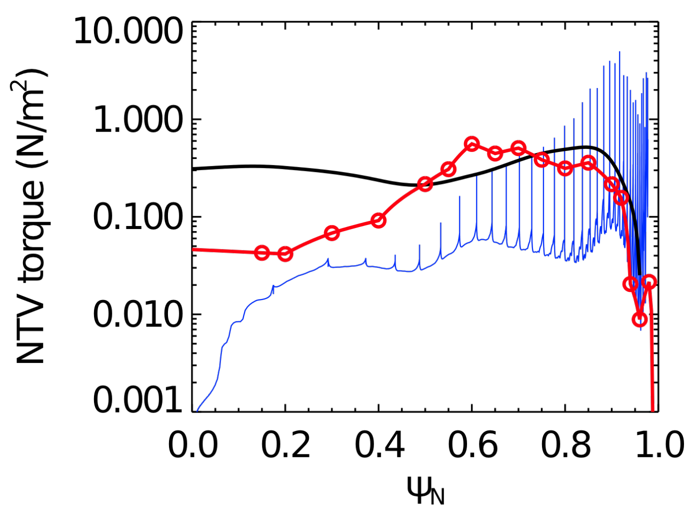
- Regime analysis indicates complicated overlapping of regimes in NSTX





- Bounce-harmonic resonances can dominate NTV transport

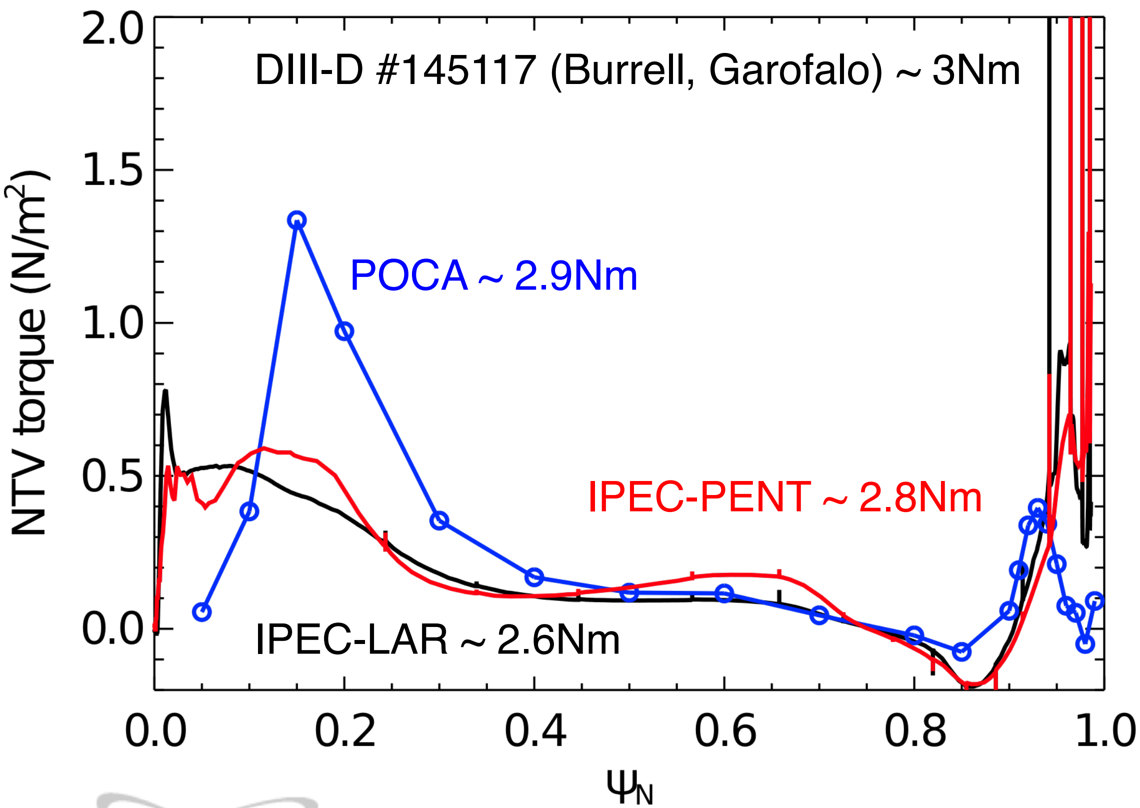


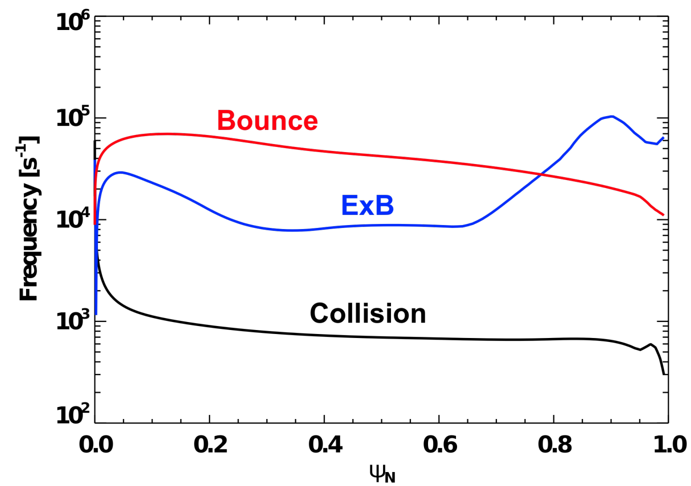


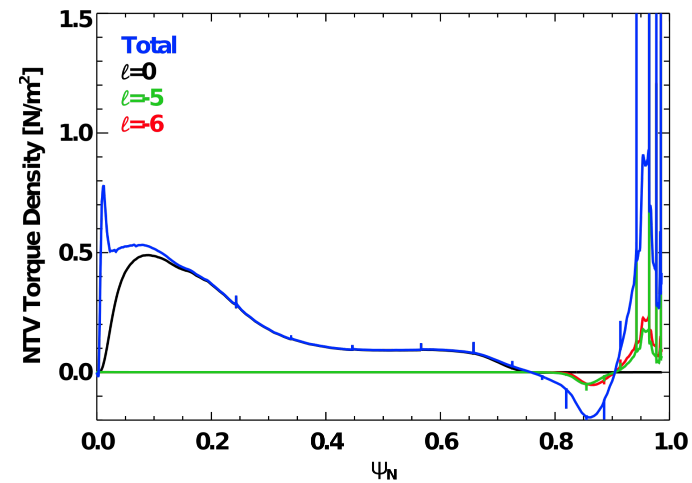
Exp. ~3.5Nm
POCA ~2.7Nm
Theory ~0.5Nm

Consistent NTV profile and total NTV was obtained in DIII-D n=3 magnetic braking

- Application of POCA to DIII-D n=3 magnetic braking (in QH mode experiments) using IPEC δB gives a consistent NTV







Various braking profiles are predicted in NSTX/NSTX-U

- POCA predicts variability of braking profiles by Midplane and NCC
  - Midplane coils can drive broad (n=2) or peaked (n=3) NTV profiles
  - (Full and/or partial) NCC can provide various braking by n=1 ~ 6, depending on phases
  - Polynomial degree for fitting δB should be carefully selected in high-n (=4, 6) cases for better radial resolution in NTV calculations (broad edge braking?)