When is $\vec{q} = q_{\|}\hat{b}$ Valid?

Robert J. Goldston

Princeton Plasma Physics Laboratory

$\vec{q} = q_{\parallel}\hat{b}$?

- **It is convenient to assume that SOL heat flux can be approximated by** $\vec{q} \approx q_{\parallel}\hat{b}$

- **Then in steady-state regions without volumetric heating** $q_{\parallel} \propto B$

$$\vec{\nabla} \cdot \vec{q} = \vec{\nabla} \cdot q_{\parallel} \frac{\vec{B}}{B} = \frac{q_{\parallel}}{B} \vec{\nabla} \cdot \vec{B} + \vec{B} \cdot \vec{\nabla} \frac{q_{\parallel}}{B} = \vec{B} \cdot \vec{\nabla} \frac{q_{\parallel}}{B} = 0$$

- ***However*** **heat only gets onto the SOL field lines *via*** $\vec{q}_{\perp}$ **.**

- **And** $\vec{q}_{\perp}$ ***produces*** **the SOL width!**

The 2-Point Model is Right for a Surprising Reason - I

- In Ref. [1] a nonlinear eigenmode solution was found for the heat equation with arbitrary power-law temperature dependences.

- A similar solution was found later in Ref. [2].

- Nonlinear heat equation

$$\frac{\partial}{\partial r}\chi_{\perp 0}T^{\alpha}\frac{\partial}{\partial r}T+\frac{\partial}{\partial z}\chi_{\parallel 0}T^{\beta}\frac{\partial}{\partial z}T=0$$

- Nonlinear eigenmode solution

$$T(r,z)=T_0 k_0^{1/(\alpha+1)}\left[\left(\frac{\chi_{\parallel 0}T_0^{\beta}/(\beta+1)}{\chi_{\perp 0}T_0^{\alpha}/(\alpha+1)}\right)^{1/2}\frac{d_0 r}{L_{Div}}\right]h_0^{1/(\beta+1)}\left(\frac{d_0}{L_{Div}}z\right)$$

- Analogous to n=0 eigenmode of linear heat equation

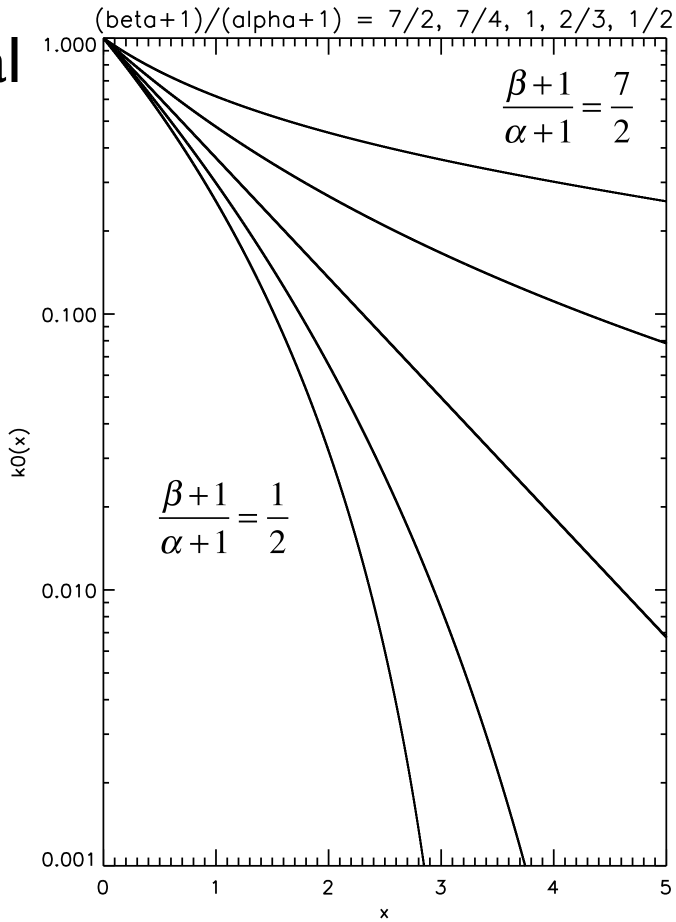
$$T(r,z)=T_0\exp\left(\left(\frac{\chi_{\parallel}}{\chi_{\perp}}\right)^{1/2}\frac{(\pi/2)r}{L_{Div}}\right)\cos\left(\frac{\pi z}{2L_{Div}}\right)$$

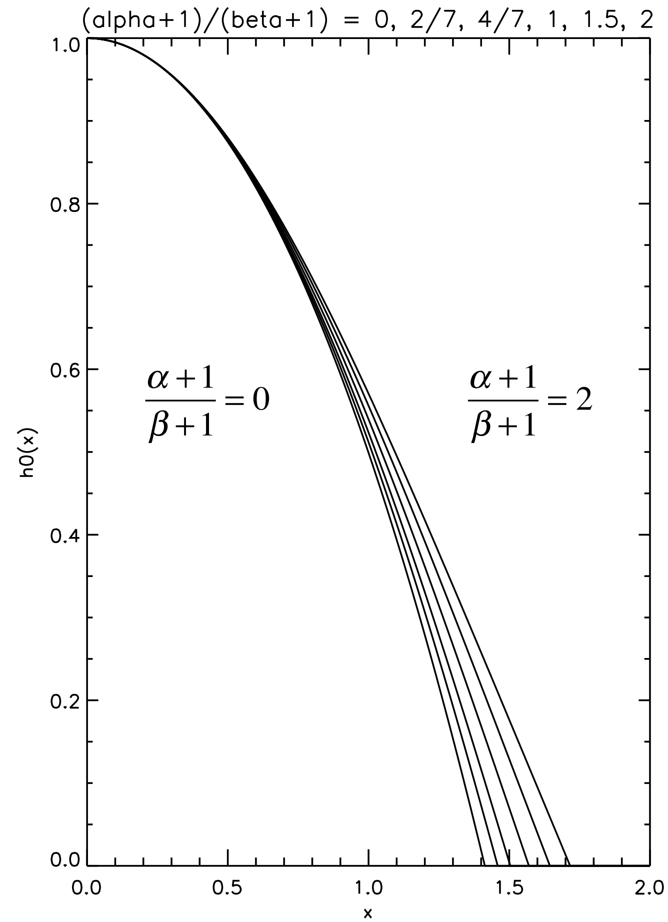
[1] S.-I. Itoh, M. Yagi, K. Itoh, Plasma Phys. Control. Fusion 38 (1996) 155

[2] R.J. Goldston, Physics of Plasmas 17 (2010) 012503

The 2-Point Model is Right for a Surprising Reason - II

Exponential - like along r





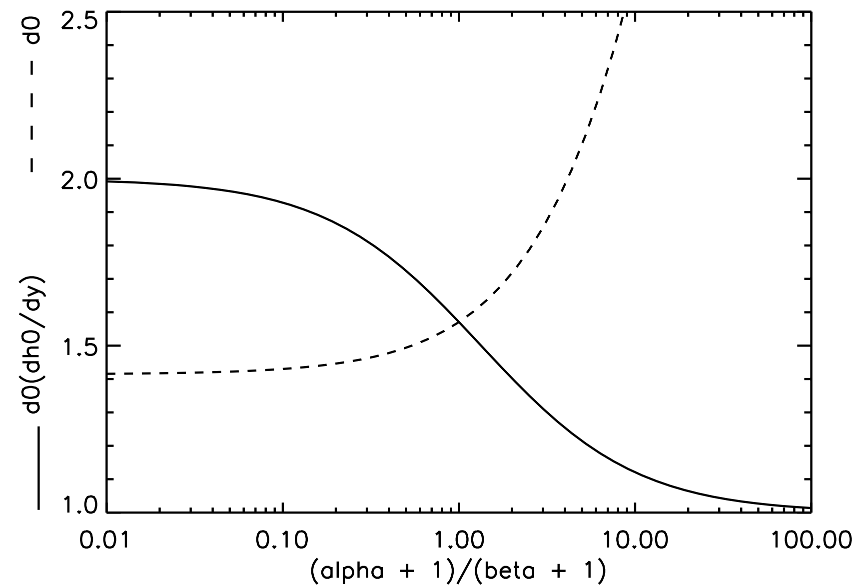
Cosine - like along z

d_0 = root of h_0

- **In Ref [2] it was shown that the eigenmode solution's heat flux at the divertor plate is proportional to $T^{\alpha+1}$ at the midplane, *e.g.*, $T^{7/2}$ for Spitzer $\chi_{\|}$, due to the specific balance between parallel and cross-field transport in the separation-of-variables solution.**

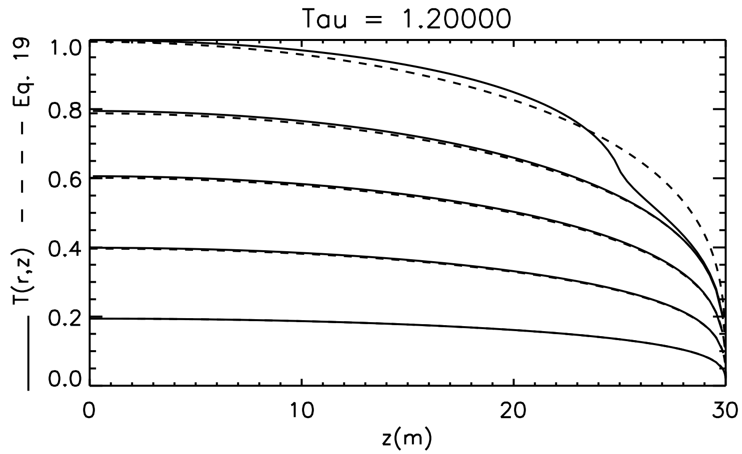
$$\frac{q_{\|,div}}{T^{\alpha+1}/(\alpha+1)}$$ **Depends on α & β**

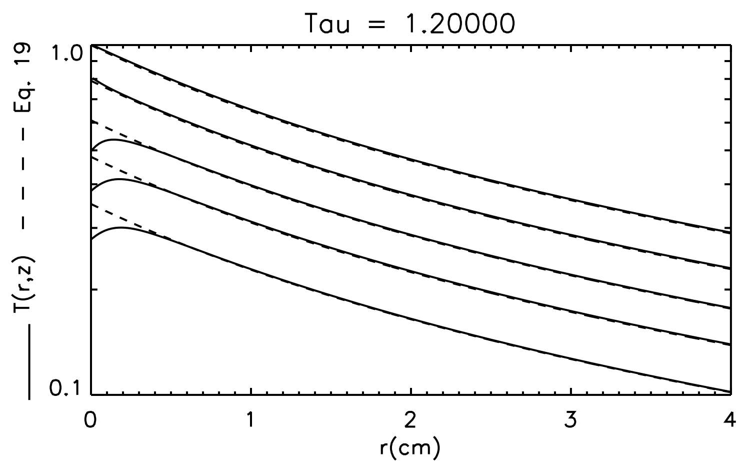
$(\alpha+1)/(\beta+1)$	$d_0\,(dh_0/dx)\|_{x=do}$	Comment
0	2.0	Heat source independent of z.
2/7	1.821	Spitzer parallel, constant perpendicular.
4/7	1.698	Spitzer parallel, Bohm perpendicular.
1	1.571 = π/2	$n = 0$ eigenmode of linear problem, and $\alpha = \beta$.
1.5	1.472	
2	1.402	
Infinity	1.0	Heat source at $z = 0$. Simple two-point model.

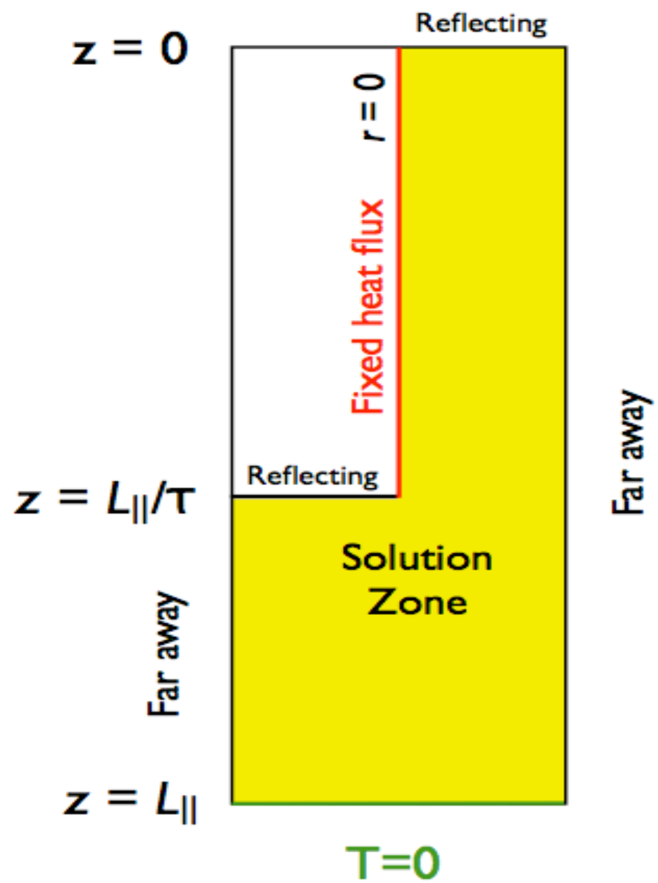


- **The proportionality between $q_{\|}L_{Div}/\chi_{\|0}$ and midplane $T^{\alpha+1}/(\alpha+1)$, *e.g.*, $(2/7)T^{7/2}$, is given by $d_0(dh_0/dx)$, where d_0 is the root of h_0.**
- **The well-known results for constant heat source along *z*, and for point heat source at *z* = 0 are reproduced.**
- **One should not, however, expect that the nonlinear eigenmode solution should hold across the full SOL. It is analogous to the *n* = 0 eigenmode of the linear heat equation.**

Numerical Results Differ from Eigenmode Near Separatrix



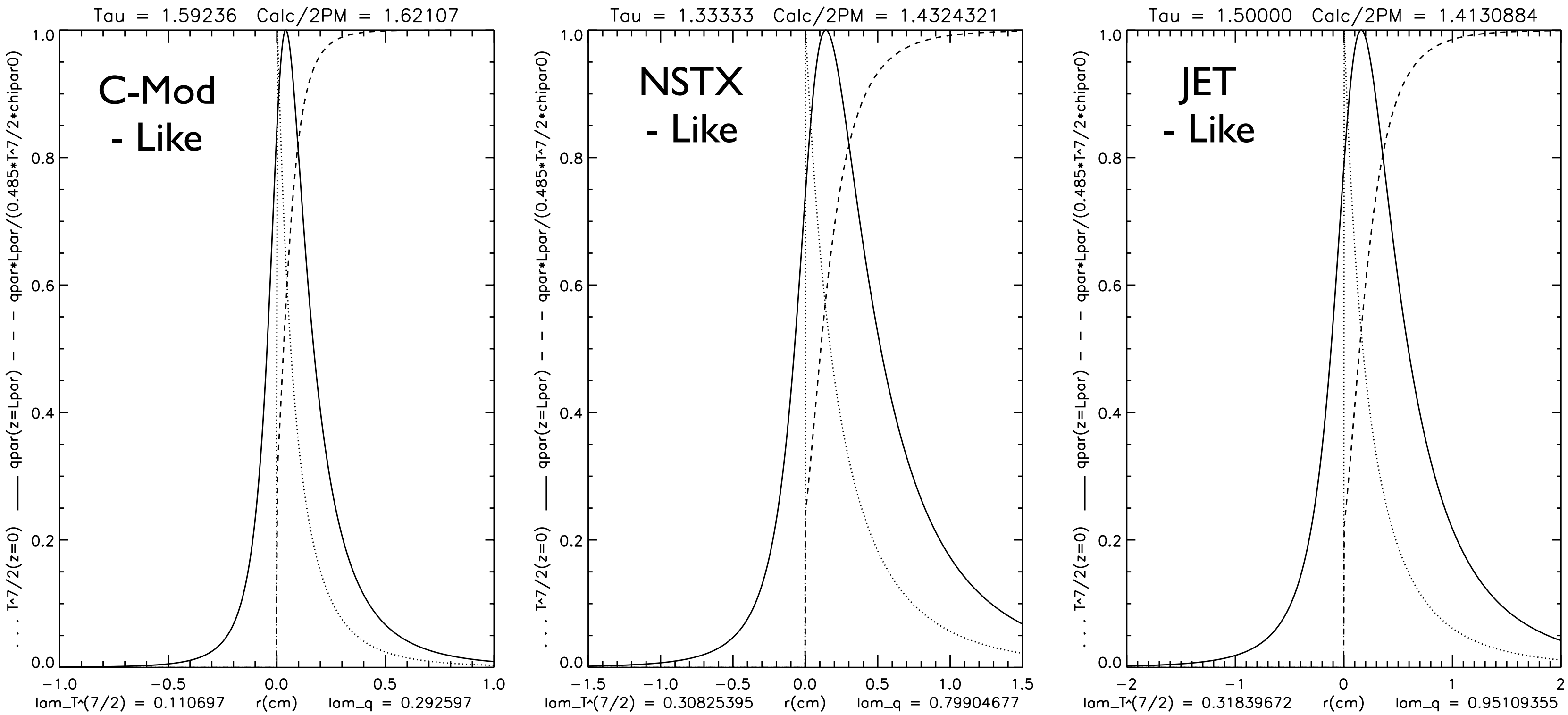




- **Agreement with the eigenmode is remarkably good away from the separatrix.**

- **Constant heat flux across the separatrix is not consistent with the eigenmode solution, so results differ in that region.**

2-Point Model Predicts Divertor Heat Flux Profile Poorly



- **These calculations assume Spitzer-like $\chi_{\|}$ and Bohm-like $\chi_{\perp}$. Heat flux across separatrix has cos$(\pi z/2L_x)$ dependence.**
- **Ratio ~ 1/4 to eigenmode proportionality near separatrix, due to influence of boundary condition. Excellent agreement away from separatrix.**
- **Integral/peak of upstream $T_e^{7/2}$ ~3x narrower than divertor heat flux.**

***Caveat Emptor*: This is a purely diffusive model, *e.g.*, no convection, no sheath resistance to heat flow, no density variation.**

2PM is Good for Estimating *T* at Midplane Separatrix

- **2PM with cosine heat source, $L_{Div}/L_x = \tau$**

$$\frac{T_0^{7/2}}{q_{||,Div}} = \frac{7}{2}\frac{L_{Div}}{\chi_0}\left(1 - \frac{1-2/\pi}{\tau}\right)$$

- **Integrate $q_{||}$ across divertor plate**

- **Integrate midplane $T^{7/2}$ for 2PM prediction of total heat flux to divertor plate**

	$q_{||,Div,tot}$ / 2PM	$q_{||,Div,tot}$ / cos-τ-2PM	Error in T_{max}
C-Mod-like	1.621	1.251	6.6%
NSTX-like	1.432	1.042	1.2%
JET-like	1.413	1.071	2.0%

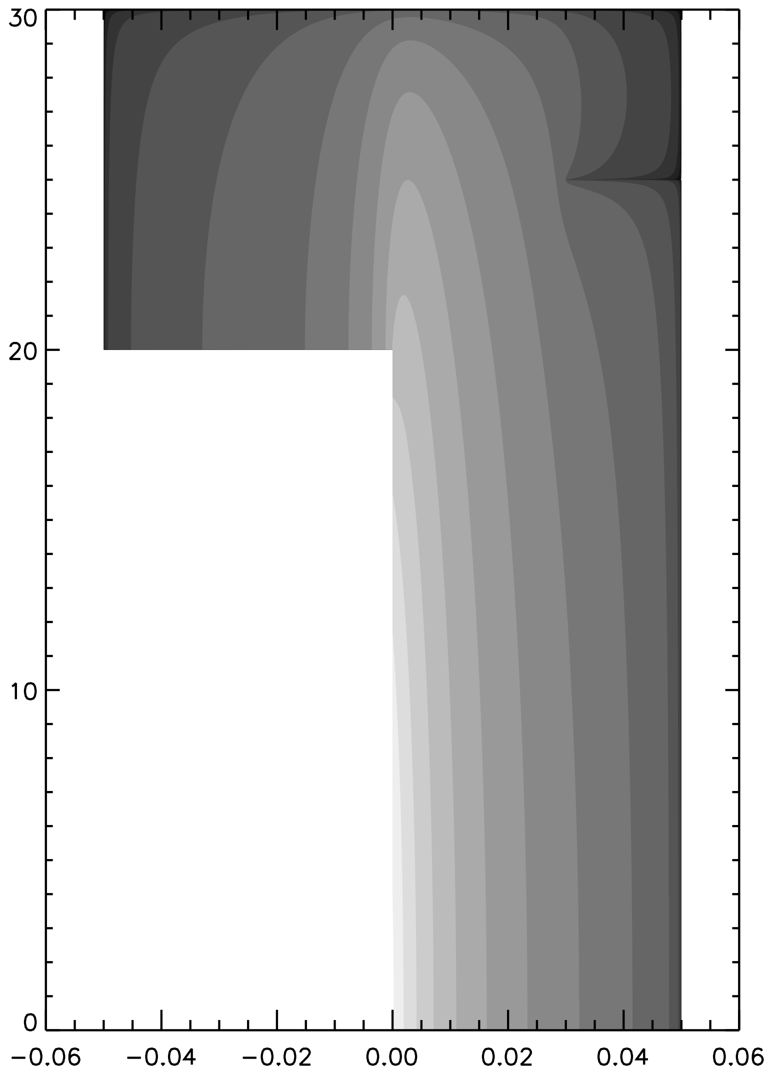
Version of $\vec{q} = q_{\|}\hat{b}$ for SOL Scrapers

- **Conventional wisdom for estimating power flow to scrapers, shaped limiters, *etc*., in diffusive SOL:**
  - Calculate heat to divertor in "shadow" of scrapers

    – but without scrapers present
  - Assume this heat is available to divertor + upstream scrapers
  - Total heat to each scraper ∝ 1/(# of scrapers)
    - Peak heat flux = "shadow" heat flux / $\#^{0.5}$
    - Heat flux width = width at divertor / $\#^{0.5}$
  - Assume all heat flux is parallel to *B*
- **Not valid if diffusion is important**
  - Heat flows to cold surfaces, even across *B.*
  - *The analogy to electrostatics is informative.*
    - *For example sharp edges have infinite electric field ⇒ heat flux*

High-Resolution Numerical Results Support Electrostatics Analogy - I

- **Knife-edge scraper in numerical model**



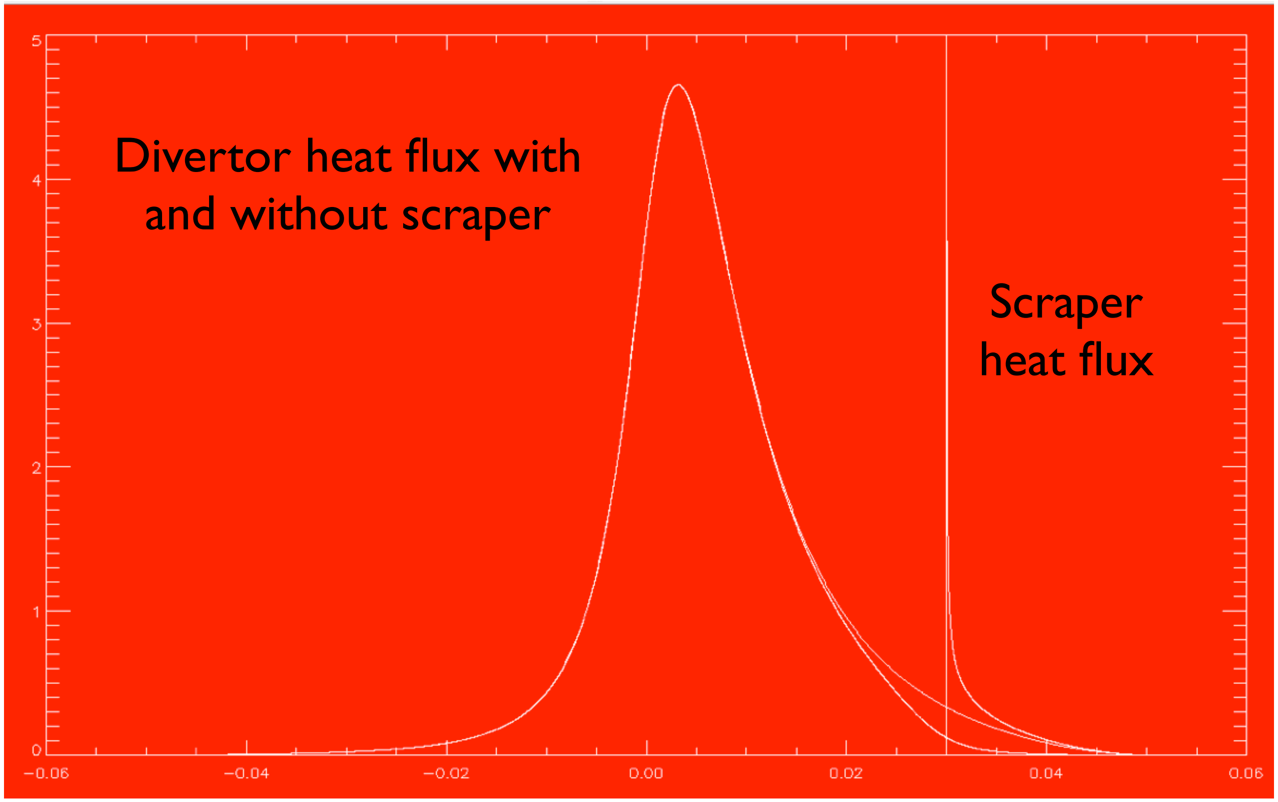
- Scraper is infinitesimally wide: *between computational zones*
- Jacobi iteration
- Alternating direction implicit
- Time step exponentially decays to Courant condition
- Better than 1% power balance
- Up to 8000 radial zones x 4000 zones along *B*

⇒ down to 12.5μ x 7.5mm

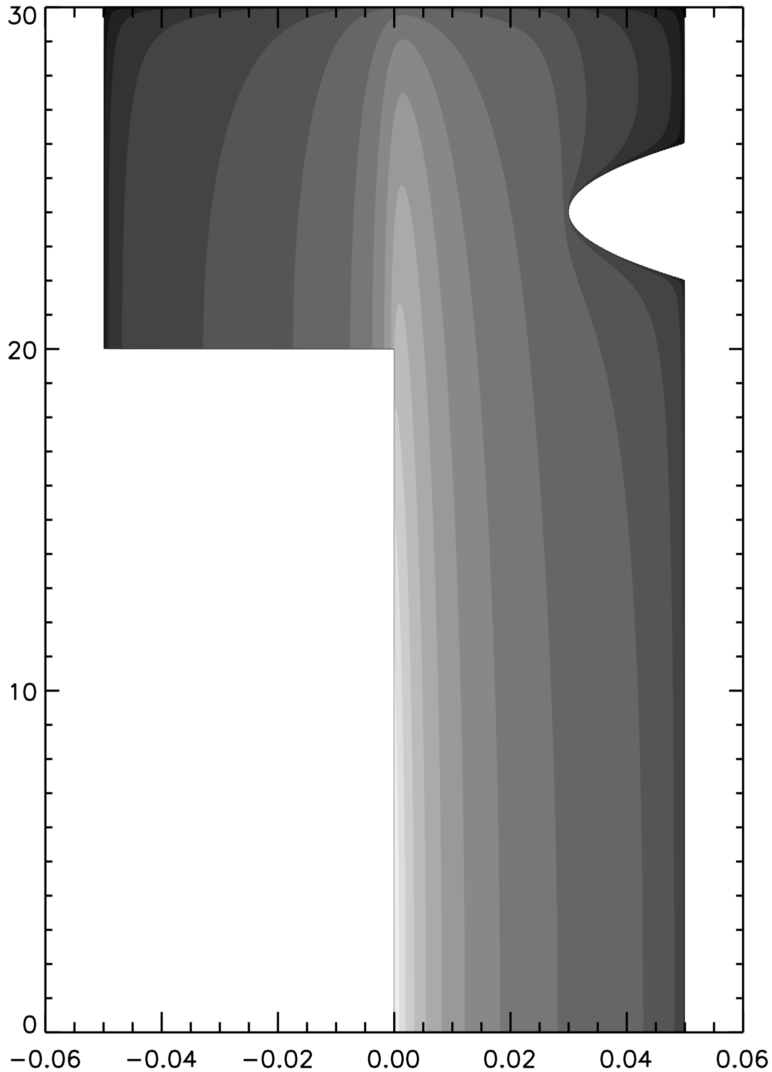
High-Resolution Numerical Results Support Electrostatics Analogy - II

- **Local heat flux at thin scraper diverges like analytic electrostatics result, $1/\delta r^{0.5}$, even in this highly nonlinear and highly anisotropic case**

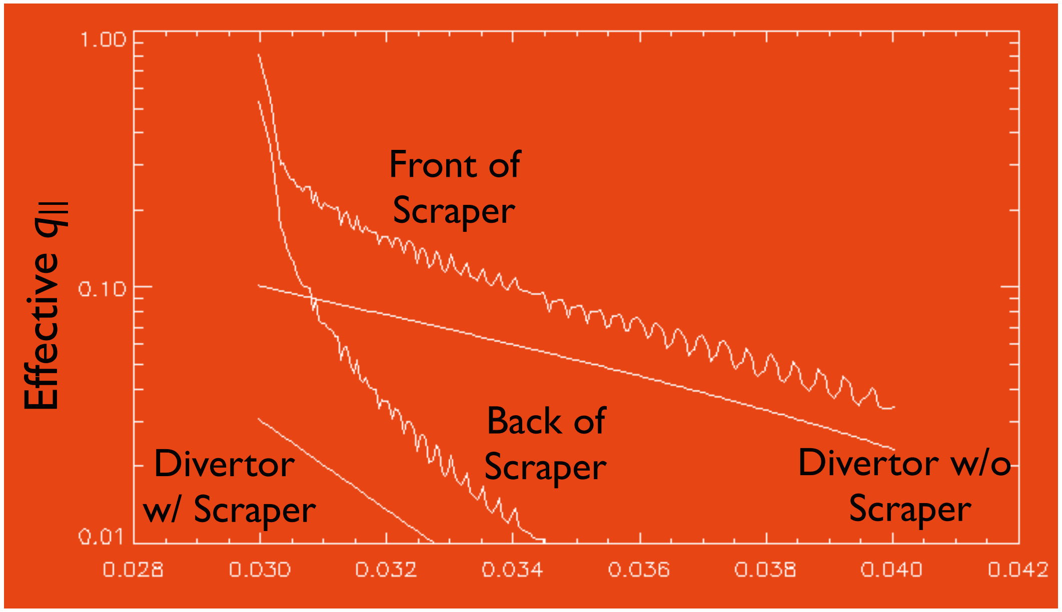
- **Greatly exceeds heat flux to divertor plate at same location.**



~ Realistic Scraper Shape: Heat Flux at Front is Still Large



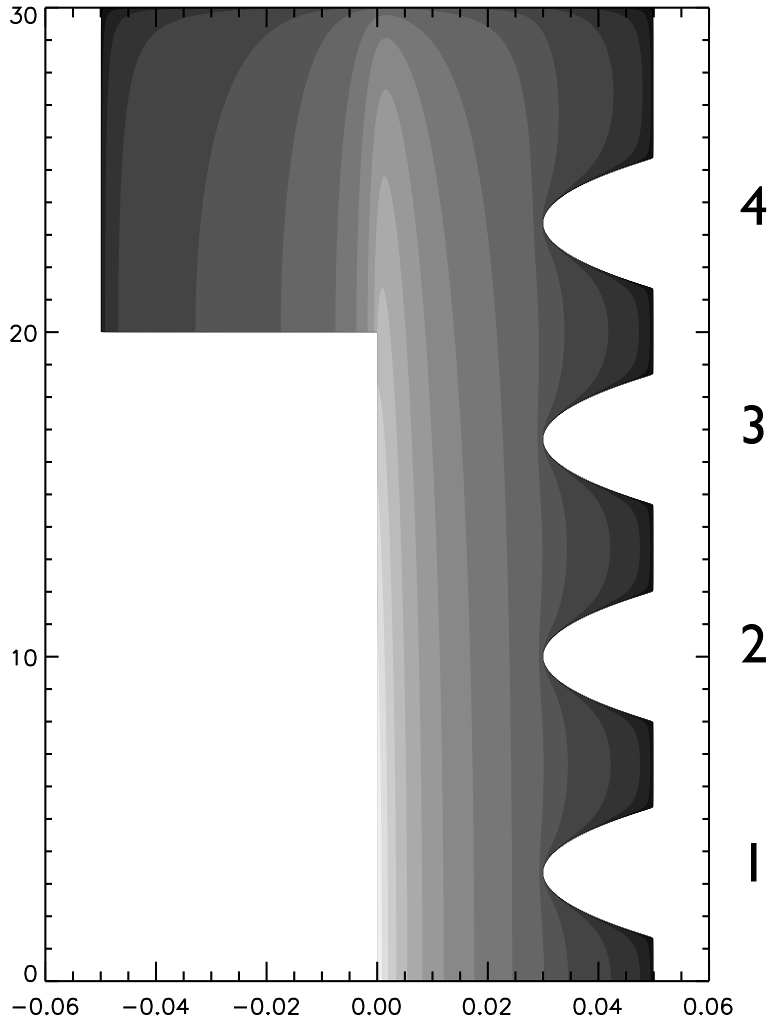
- **Scraper shape parabolic, 4m x 2cm**
- ***Cross field flux dominates nearest plasma, reminiscent of Stangeby funnel effect [3]. Included in 2, 3-D SOL models??***
- **Heat flux to scraper = 1.85%**
- **Heat flux to divertor behind scraper, w/o the scraper, = 0.742%**



[3] P.C. Stangeby, C.S. Pitcher, and J.D. Elder, Nucl. Fusion 32 (1992) 2079

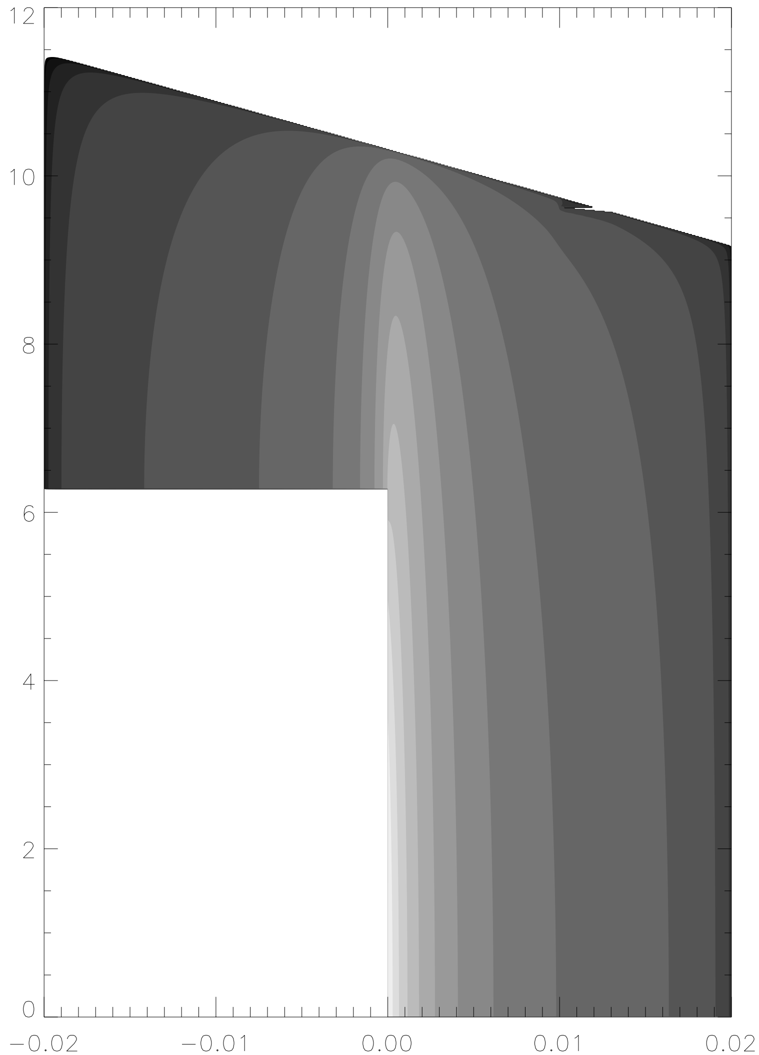
Multiple Scrapers Absorb Much More Power than in Conventional Model

- **Four 4m x 2cm parabolic scrapers absorb 5x more heat flux than divertor "behind them" w/o scrapers**



	% of influx
Divertor behind, *without scrapers*	0.742
Scraper 1, front	0.588
Scraper 1, back	0.556
Scraper 2, front	0.557
Scraper 2, back	0.509
Scraper 3, front	0.485
Scraper 3, back	0.415
Scraper 4, front	0.358
Scraper 4, back	0.267
Total to scrapers	3.75
Total to four 1m x 2cm scrapers	3.6

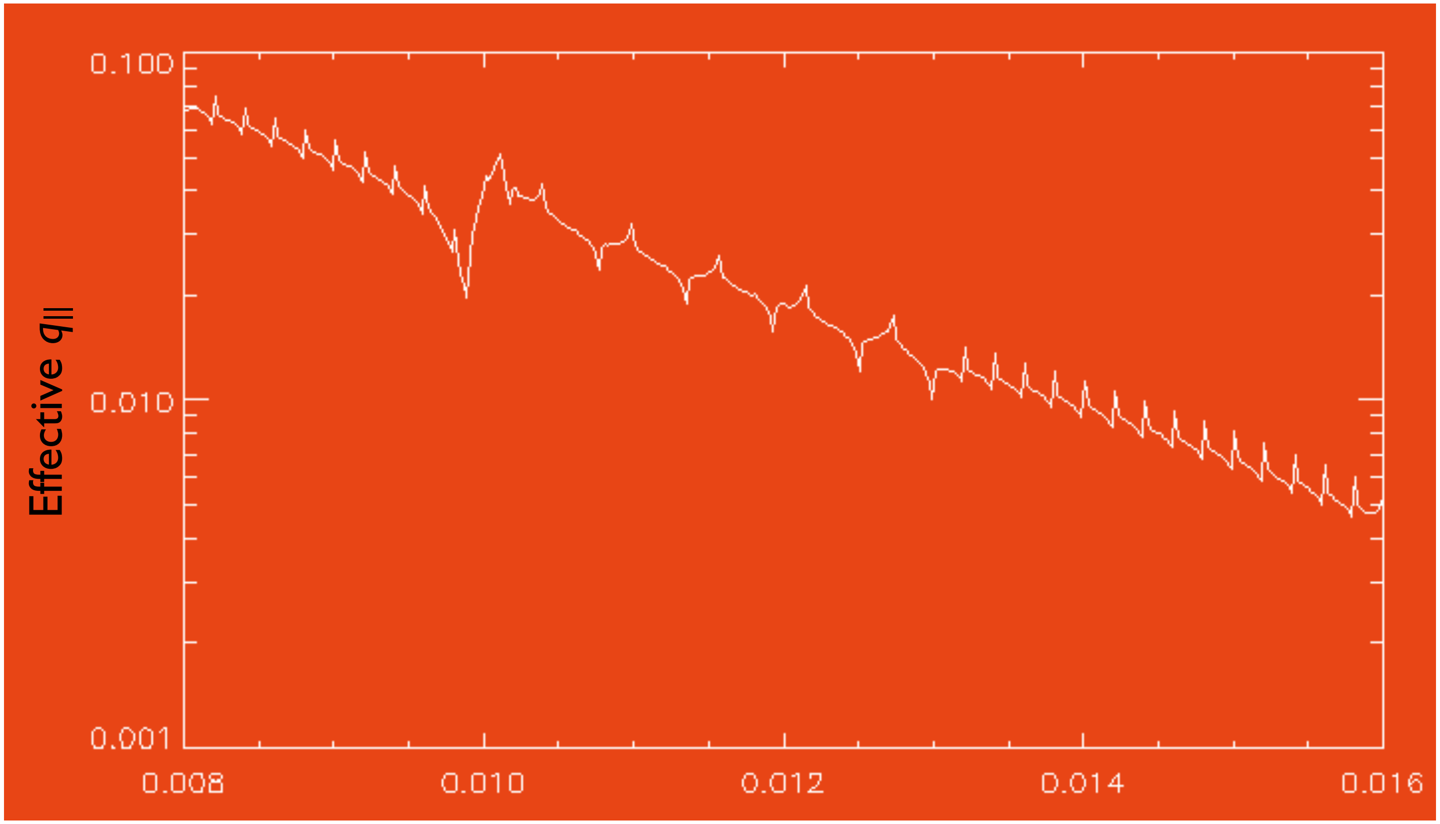
Can We Model the C-Mod Scraper?



Note vastly distorted z vs. r scales.

- **Not really.**
- **Scrapers in this code are axisymmetric.**
- **However if we use**
  - C-Mod vertical divertor target geometry
    - T=0 boundary at ~1° to separatrix
  - C-Mod scraper height (2mm) and length (6cm) along B
  - C-Mod length along B from mid-plane to X-point, scraper, target
- **We can see if this could be an interesting effect.**

Preliminary C-Mod Results Interesting



- **Noise due to staircase zoning smoothed here.**
- **Scraper nose tip at 0.01m from separatrix.**
- **$q_{||}$ is not disturbed near bridge of nose ~ 0.013m**
- **Looks like ~ 1.3 enhancement near nose "tip", for this case.**

$\vec{q} = q_{\|}\hat{b}$ is Tricky

- **Poor** for mapping from $T^{7/2}$ in midplane to heat flux profile at divertor.
- **Good** for finding T_{max} at midplane.
- **Awful** for calculating heat flux to front face of scrapers / shaped first walls.
- **Awful** for calculating heat flux to multiple scrapers.
- ***Caveat Emptor:*** ***This analysis is based only on solving the nonlinear anisotropic heat equation with T = 0 boundary conditions.***