NSTX-U

Supported by U.S. DEPARTMENT OF ENERGY | Office of Science

Development of a new reduced model for fast ion transport in TRANSP

M. Podestà

M. Gorelenkova, R. B. White,

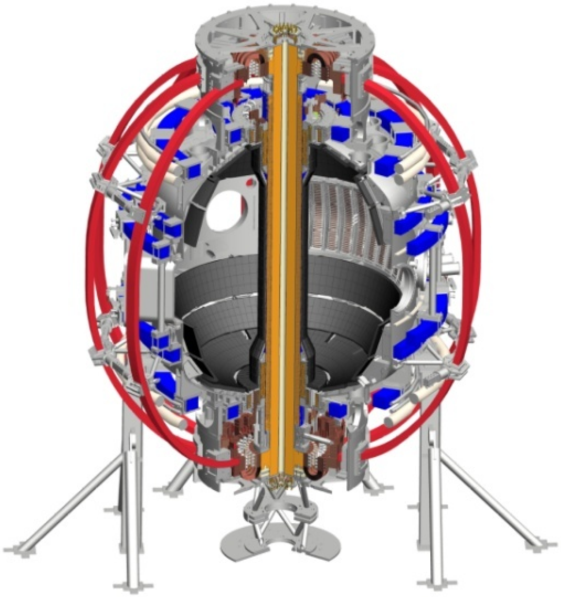
NSTX-U Energetic Particles Topical Science Group and NSTX-U Team

US TTF 2014

San Antonio, TX

April 23rd, 2014

Coll of Wm & Mary
Columbia U
CompX
General Atomics
FIU
INL
Johns Hopkins U
LANL
LLNL
Lodestar
MIT
Lehigh U
Nova Photonics
Old Dominion
ORNL
PPPL
Princeton U
Purdue U
SNL
Think Tank, Inc.
UC Davis
UC Irvine
UCLA
UCSD
U Colorado
U Illinois
U Maryland
U Rochester
U Tennessee
U Tulsa
U Washington
U Wisconsin
X Science LLC



Culham Sci Ctr
York U
Chubu U
Fukui U
Hiroshima U
Hyogo U
Kyoto U
Kyushu U
Kyushu Tokai U
NIFS
Niigata U
U Tokyo
JAEA
Inst for Nucl Res, Kiev
Ioffe Inst
TRINITI
Chonbuk Natl U
NFRI
KAIST
POSTECH
Seoul Natl U
ASIPP
CIEMAT
FOM Inst DIFFER
ENEA, Frascati
CEA, Cadarache
IPP, Jülich
IPP, Garching
ASCR, Czech Rep

Supported by DOE contract no. DE-AC02-09CH11466

New model introduces selectivity in phase space, generalizes *ad-hoc* (diffusive) models in TRANSP

- Info on phase space dynamics is of paramount importance for Verification&Validation of codes, theory-experiment comparison
- "Fluid" (integral) quantities do not provide enough information
  - Re-computed ("inverted") F_{nb} based on measured *integral* quantities (f.i. density, neutrons, E_r/rotation, NB-driven J_{nb}, ...) **is not unique**

> ***Need details on fast ion energy, pitch and their (consistent) evolution***

  - E.g. *AE bursts transiently (and **selectively**) modify phase space; effects propagate during slowing-down

The new model must be "simple enough" to be included in TRANSP/NUBEAM for routine use ⟶ ***reduced model***

*see M. Podestà et al., PPCF **56** (2014) 055003*

Outline

- New ‘kick model’: basic ideas
- Verification against full ORBIT simulations
- Example from NSTX case w/ TAE avalanche
- Summary

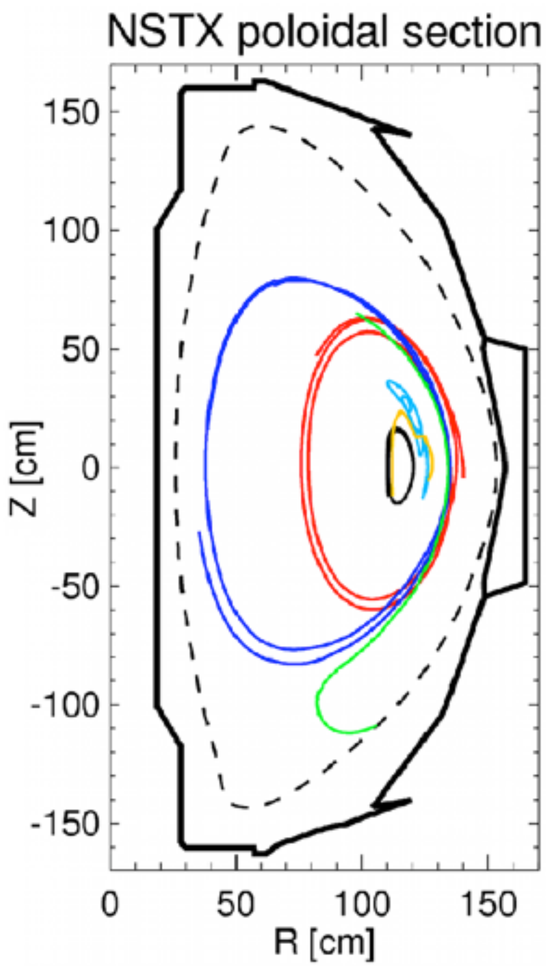
Outline

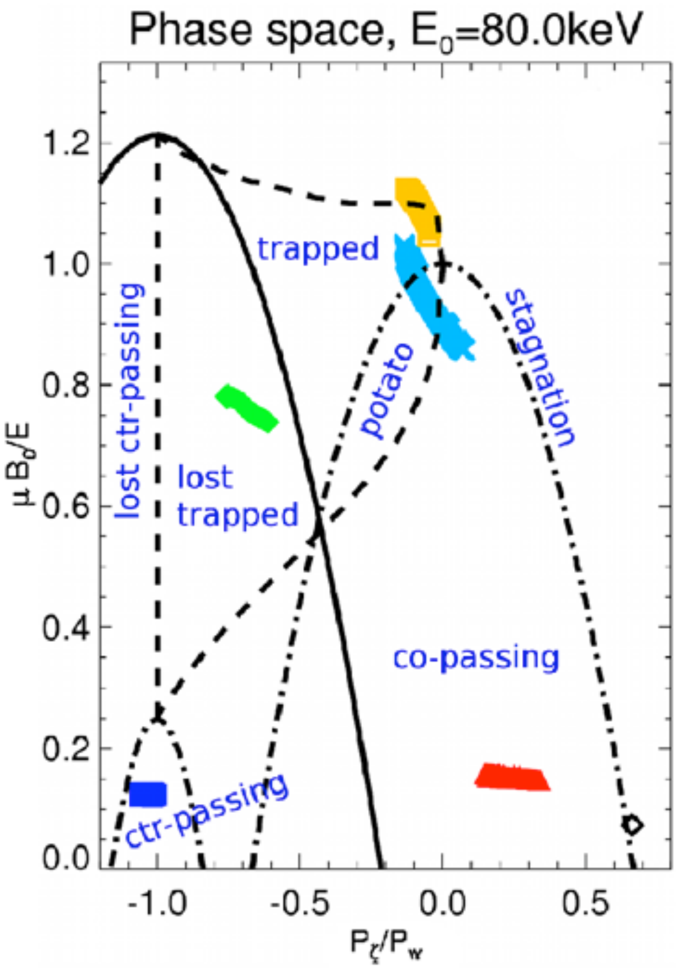
- New ‘kick model’: basic ideas
- Verification against full ORBIT simulations
- Example from NSTX case w/ TAE avalanche
- Summary

'Constants of motion' are the natural variables to describe resonant wave-particle interaction

Each orbit is fully characterized by:

- E, energy
- $P_\zeta \sim mRv_{par} - \Psi$, canonical ang. momentum
- $\mu \sim v_{perp}^2/(2B)$, magnetic moment





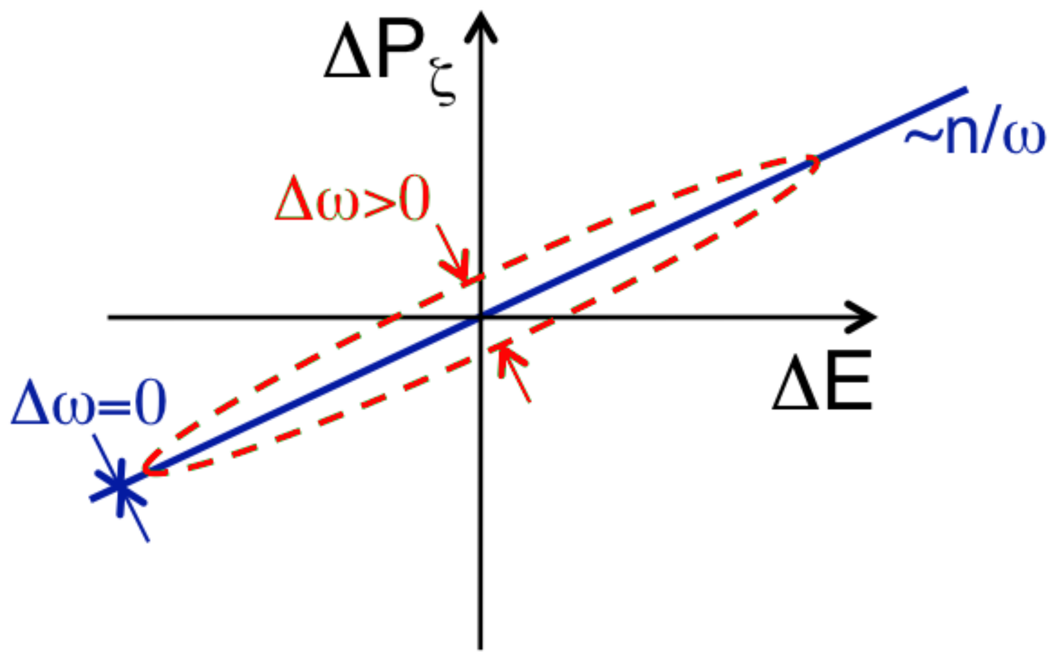
R. B. White, Theory of toroidally confined plasmas, Imperial College Press (2001)

Resonances introduce fundamental constraints on particle's trajectory in (E,P_ζ,μ)

From Hamiltonian formulation – single resonance:

$$\omega P_\zeta - nE = const. \Longrightarrow \Delta P_\zeta / \Delta E = n/\omega$$

$\omega=2\pi f$, mode frequency $\quad$ n, toroidal mode number



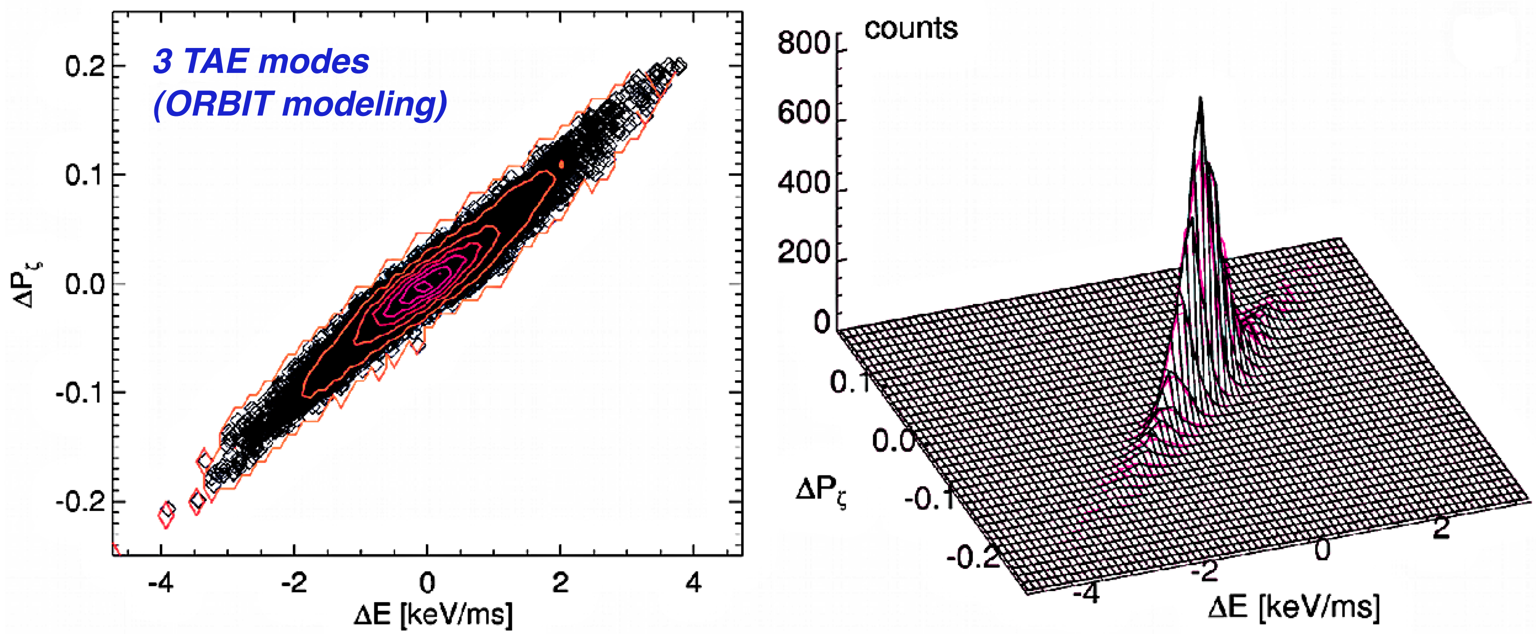
Multiple modes, resonances can lead to complicated $(\Delta E,\Delta P_\zeta)$ distributions

New 'kick model' is based on a *probability distribution function* for particle transport

For each "orbit" in (E,P_ζ,μ), describe steps in $\Delta E,\Delta P_\zeta$ by

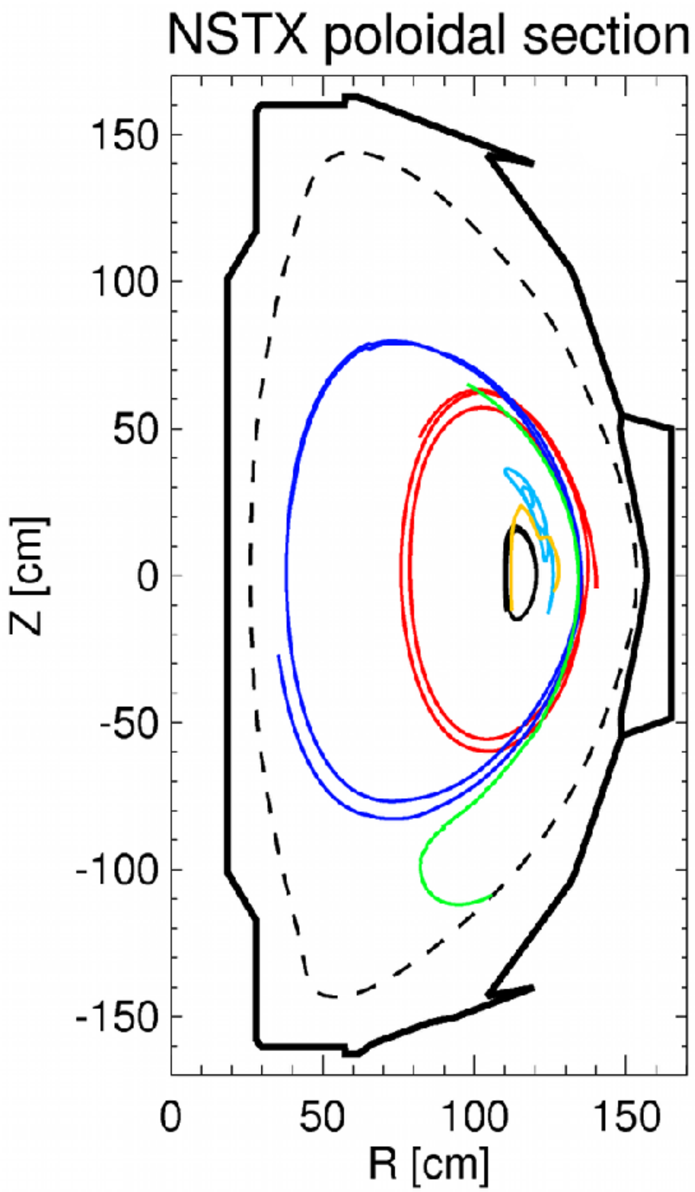
$$p(\Delta E, \Delta P_\zeta | P_\zeta, E, \mu, A)$$

which can include the effects of <u>multiple</u> modes, resonances: ***correlated random walk in E, P_ζ***



In practice, will use the full 5D matrix for $p(\Delta E,\Delta P_\zeta|P_\zeta,E,\mu)$ plus a time-dependent "mode amplitude scaling factor"

'Transport probability' $p(\Delta E,\Delta P_\zeta|P_\zeta,E,\mu)$ can be computed through numerical codes (e.g. ORBIT) or theory

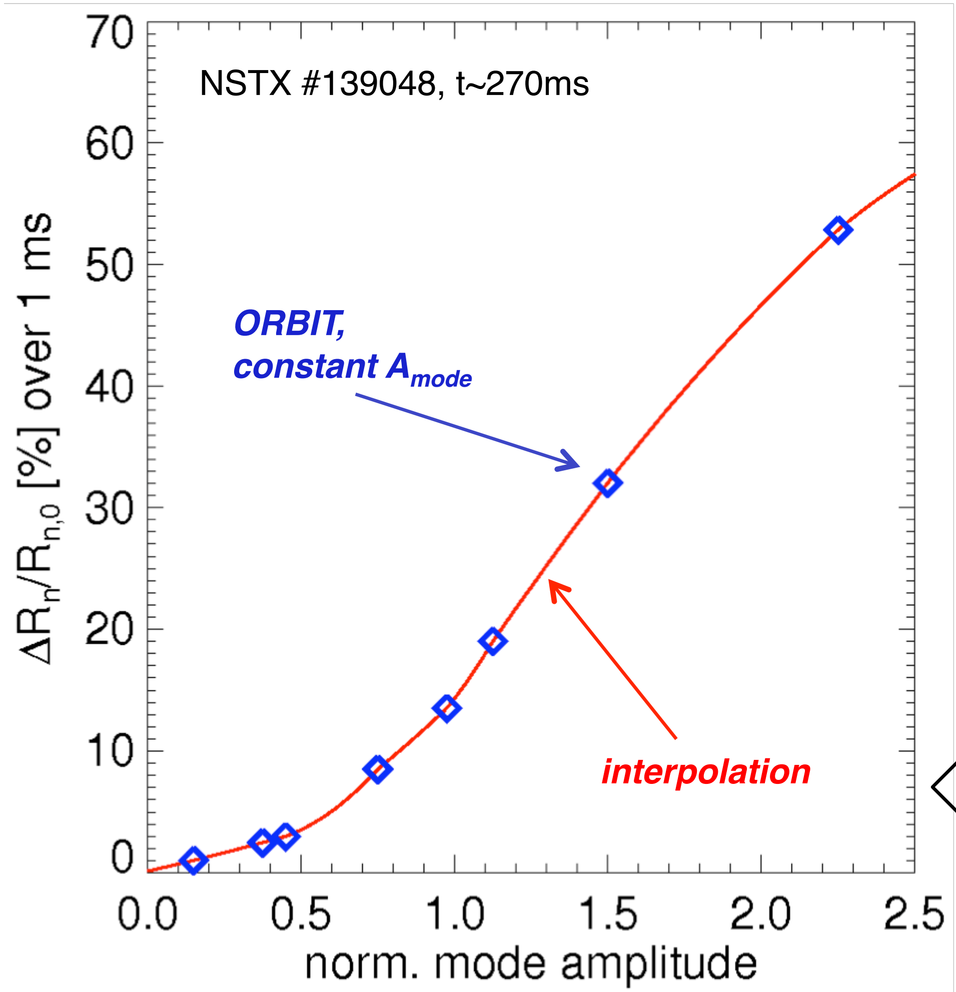


- Compute modes, e.g. through NOVA + measurements
- Run ORBIT with A_{mode}=1, constant
- Track (P_ζ,E,μ) in time for each particle, steps δt_{sim}~25-50μs
- Compute steps $\Delta E,\Delta P_\zeta,\Delta\mu$
- Re-bin over (P_ζ,E,μ) space

> *Obtain $p(\Delta E,\Delta P_\zeta|P_\zeta,E,\mu)$ for each bin in phase space*

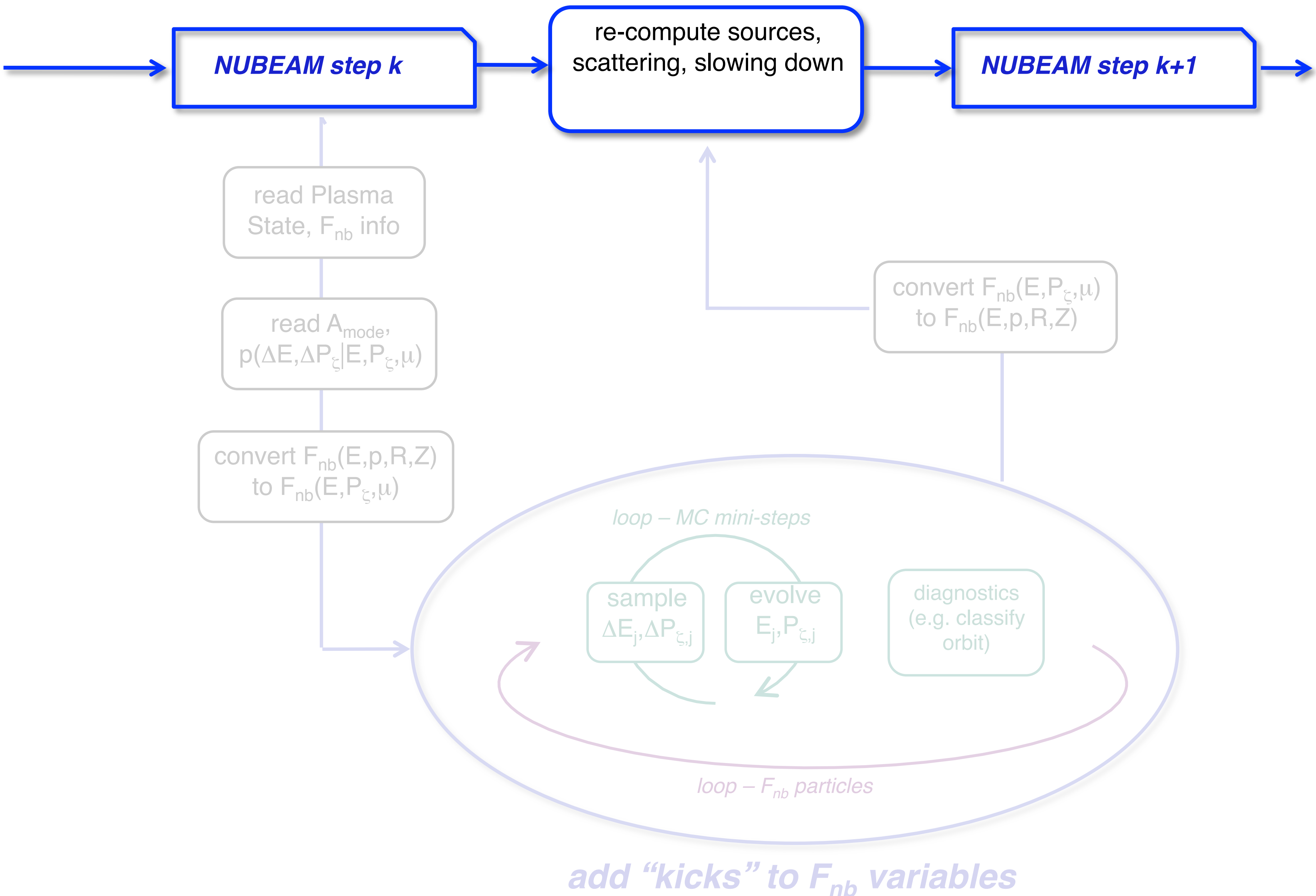
Scaling factor $A_{mode}(t)$ is obtained from measurements, or from other *observables* such as neutron rate + modeling

- Best option: use experimental data (e.g. reflectometers, ECE)
- If no mode data directly available, A_{mode} can be estimated based on other measured quantities

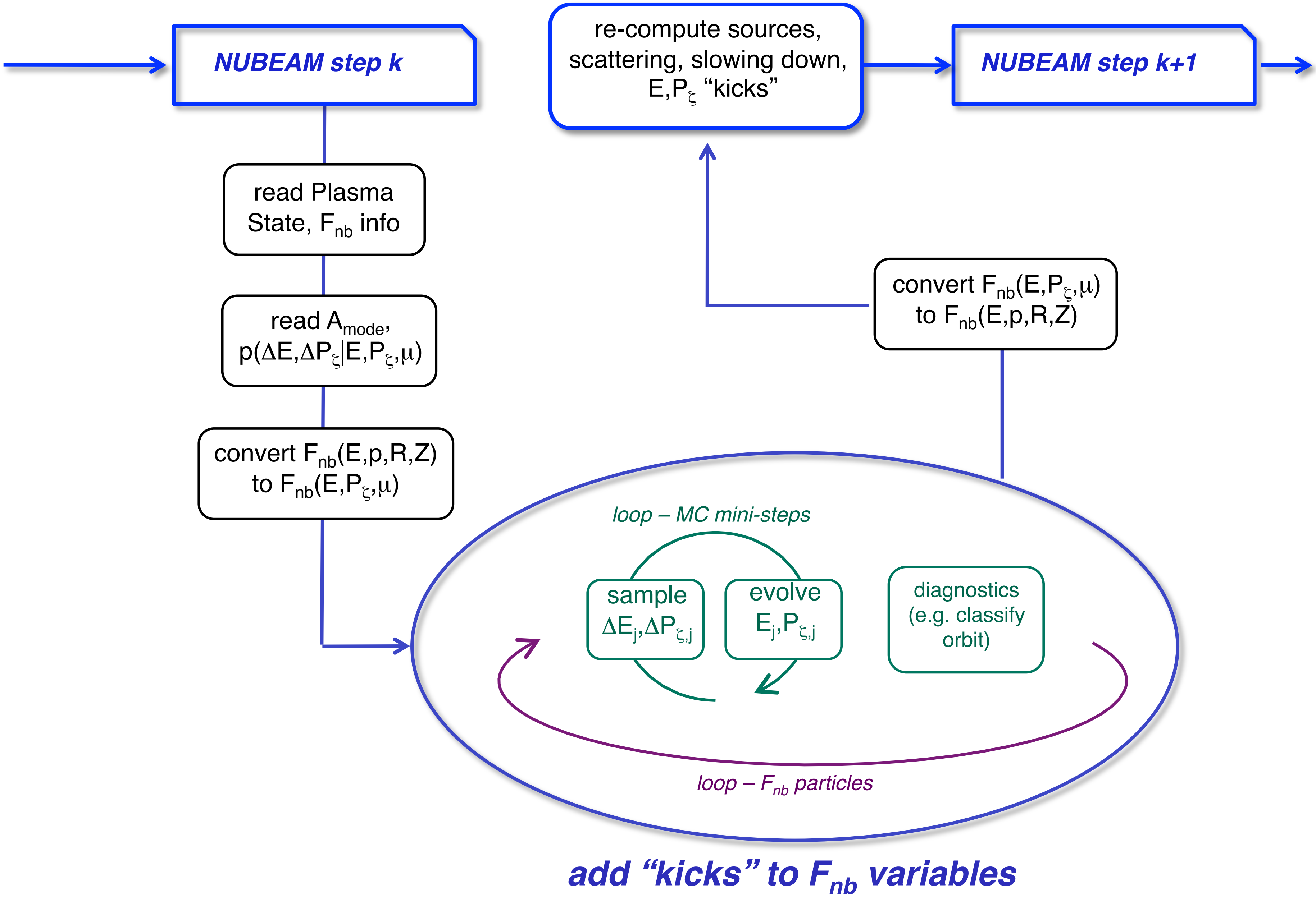


- **Example: use measured neutron rate**
  - Compute ideal modes through NOVA
  - Rescale relative amplitudes from NOVA according to reflectometers
  - Rescale total amplitude based on computed neutron drop from ORBIT
  - Scan mode amplitude w.r.t. experimental one, A_{mode}*=1*: get table
  - Build $A_{mode}(t)$ from neutrons vs. time, table look-up

Scheme to advance fast ion variables according to transport probability in NUBEAM module of TRANSP



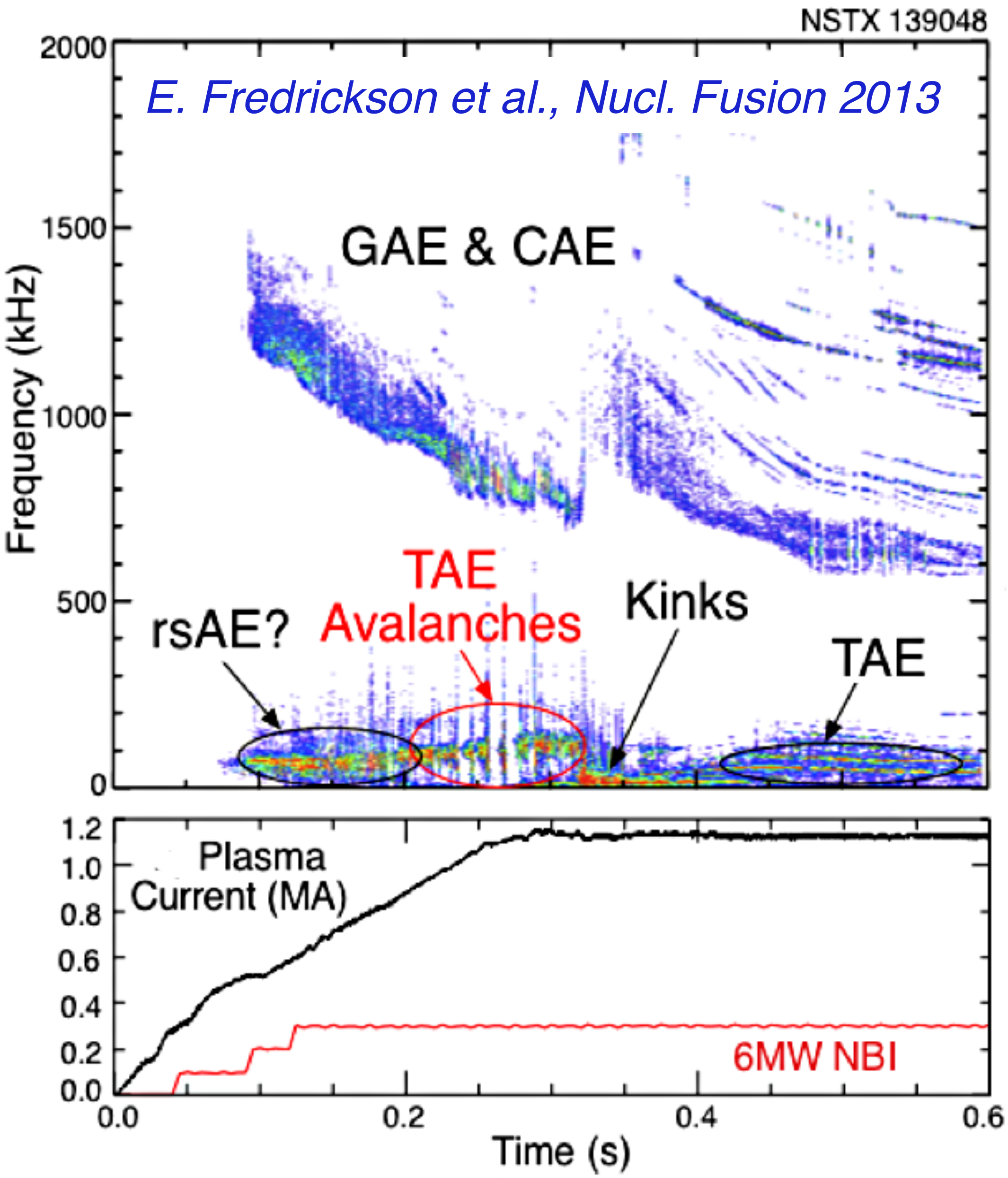
Scheme to advance fast ion variables according to transport probability in NUBEAM module of TRANSP

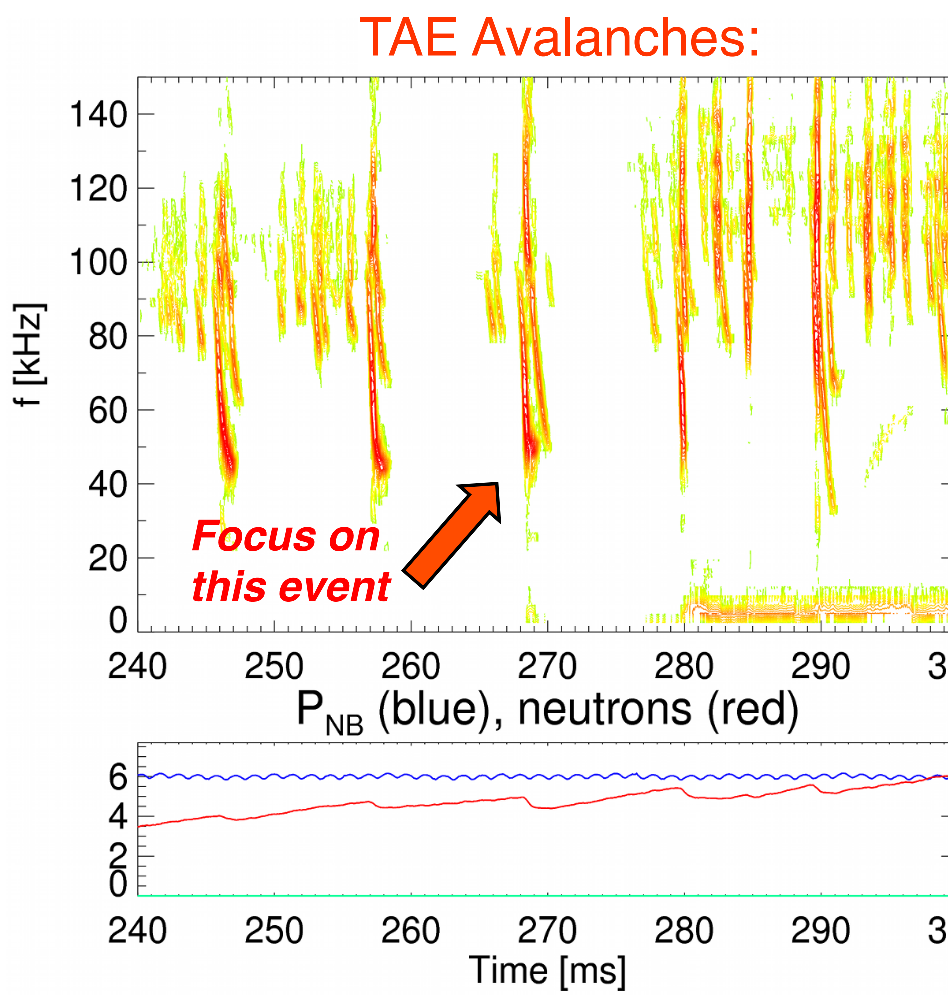


Outline

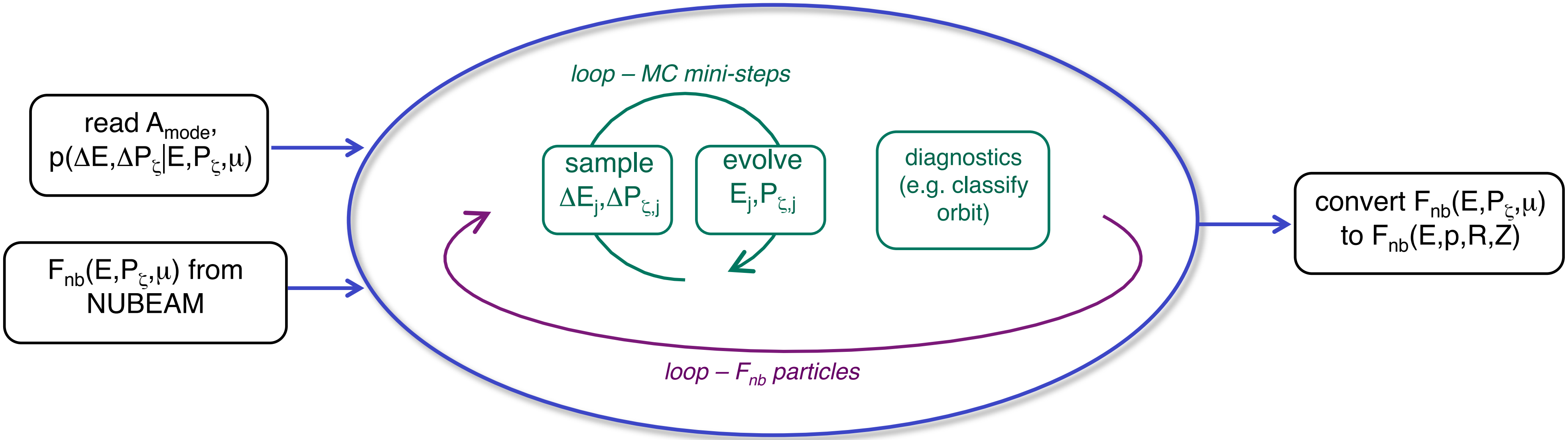
- New ‘kick model’: basic ideas
- Verification against full ORBIT simulations
- Example from NSTX case w/ TAE avalanche
- Summary

Experimental scenario for initial verification: NSTX H-mode plasma with bursts of TAE activity





Evolve F_{nb} from NUBEAM assuming fixed background, compare with ORBIT

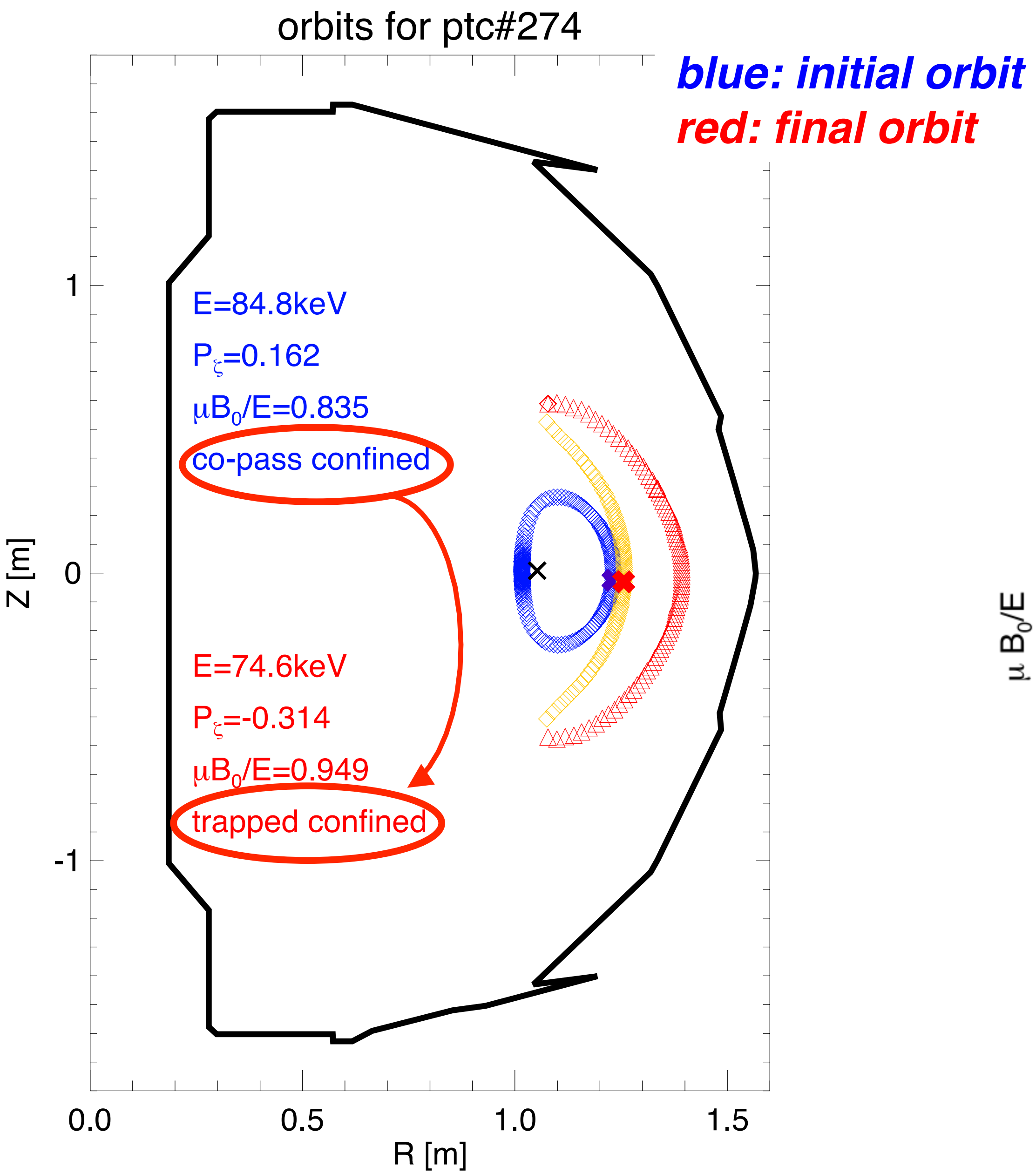


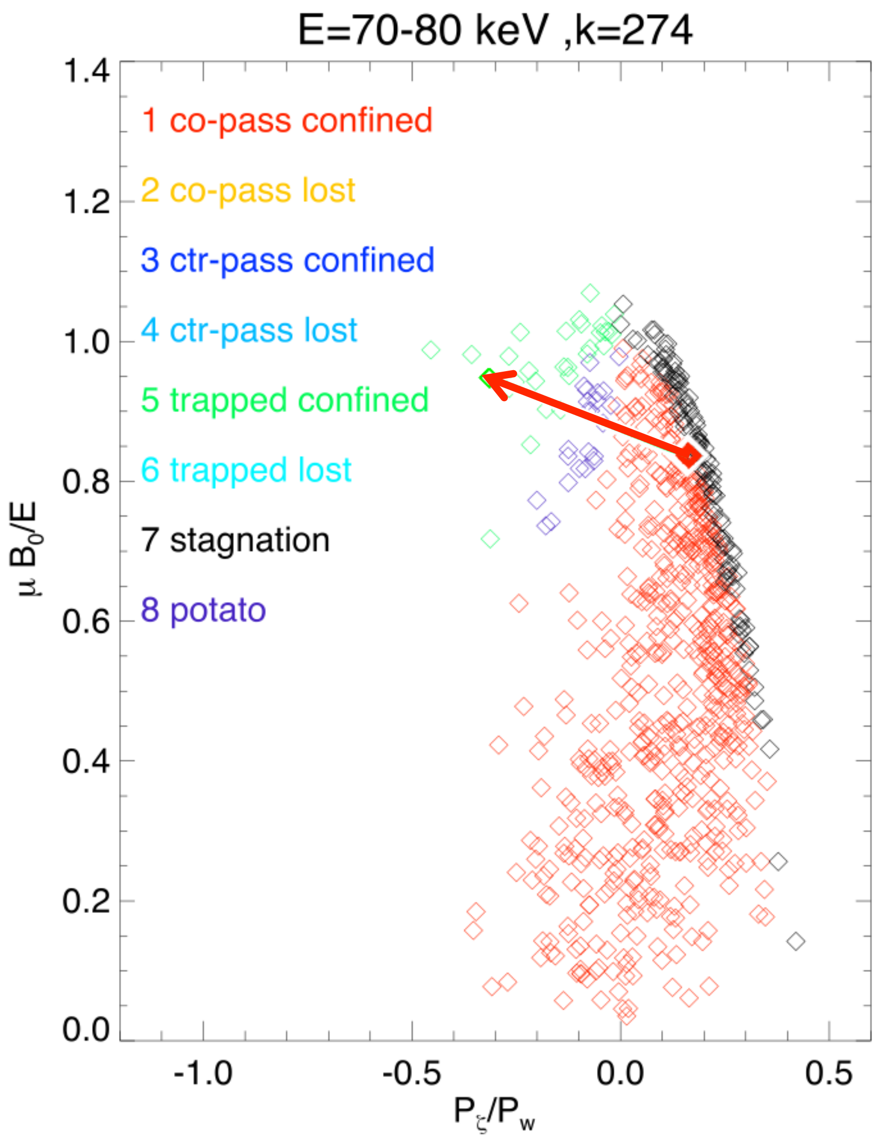
add "kicks" to F_{nb} variables

<u>*Verify agreement for*</u> :

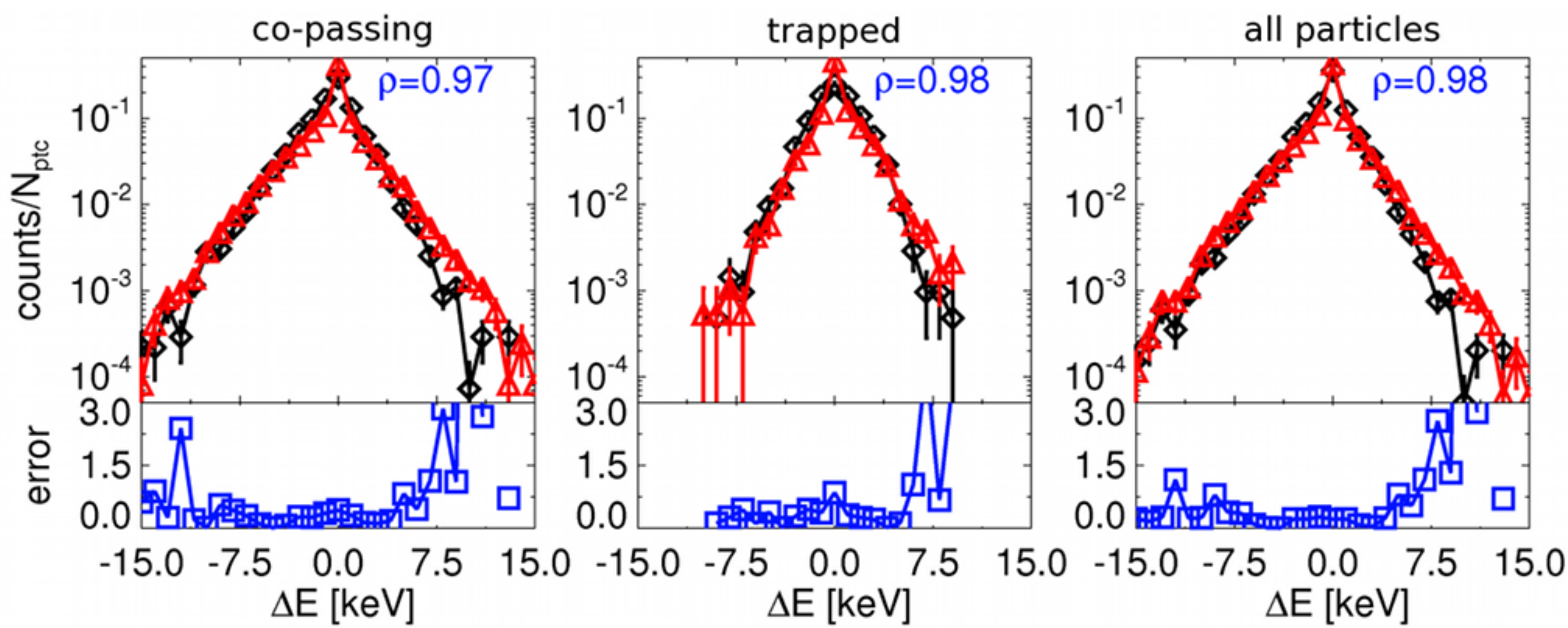
- Single particle orbits
- Whole ensemble of particles (preserve statistics)
- Global quantities: density profile

Change in orbit type observed for some particles: fast ions *are* kicked around in phase space by TAEs





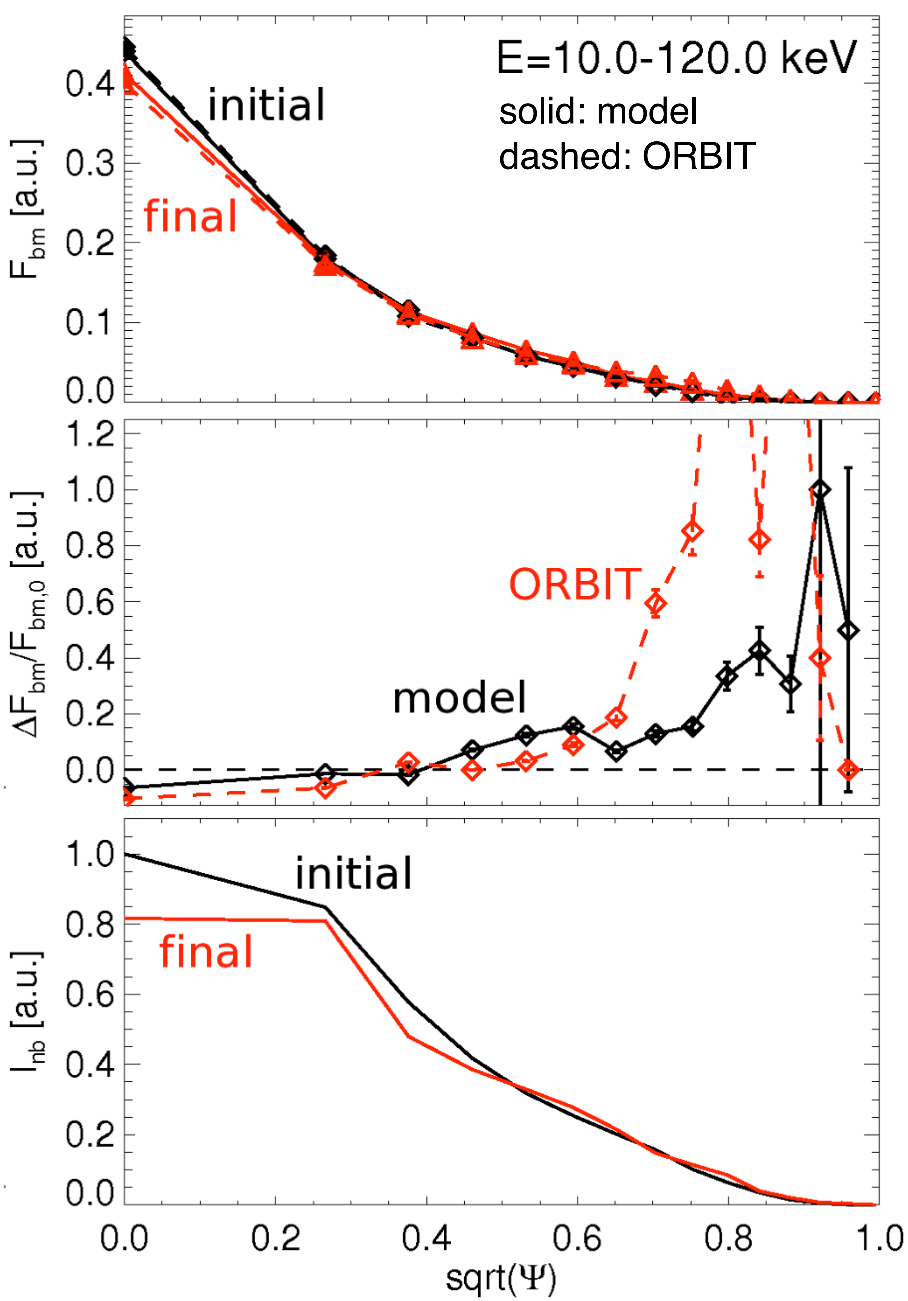
Good agreement with ORBIT is preserved when evolving F_{nb} over 5 ms, typical macro-step of NUBEAM



black: ORBIT ***red: Kick Model*** *ρ: correlation coefficient* *error: (ORBIT-model)/ORBIT*

- Similar results for P_ζ
- Satisfactory reconstruction over ~3 orders of magnitude for co-, trapped (, counter- not present in *this* input F_{nb}) particles

F_{nb} after 5ms shows fast ion redistribution, only modest losses; NB driven "current" also affected.

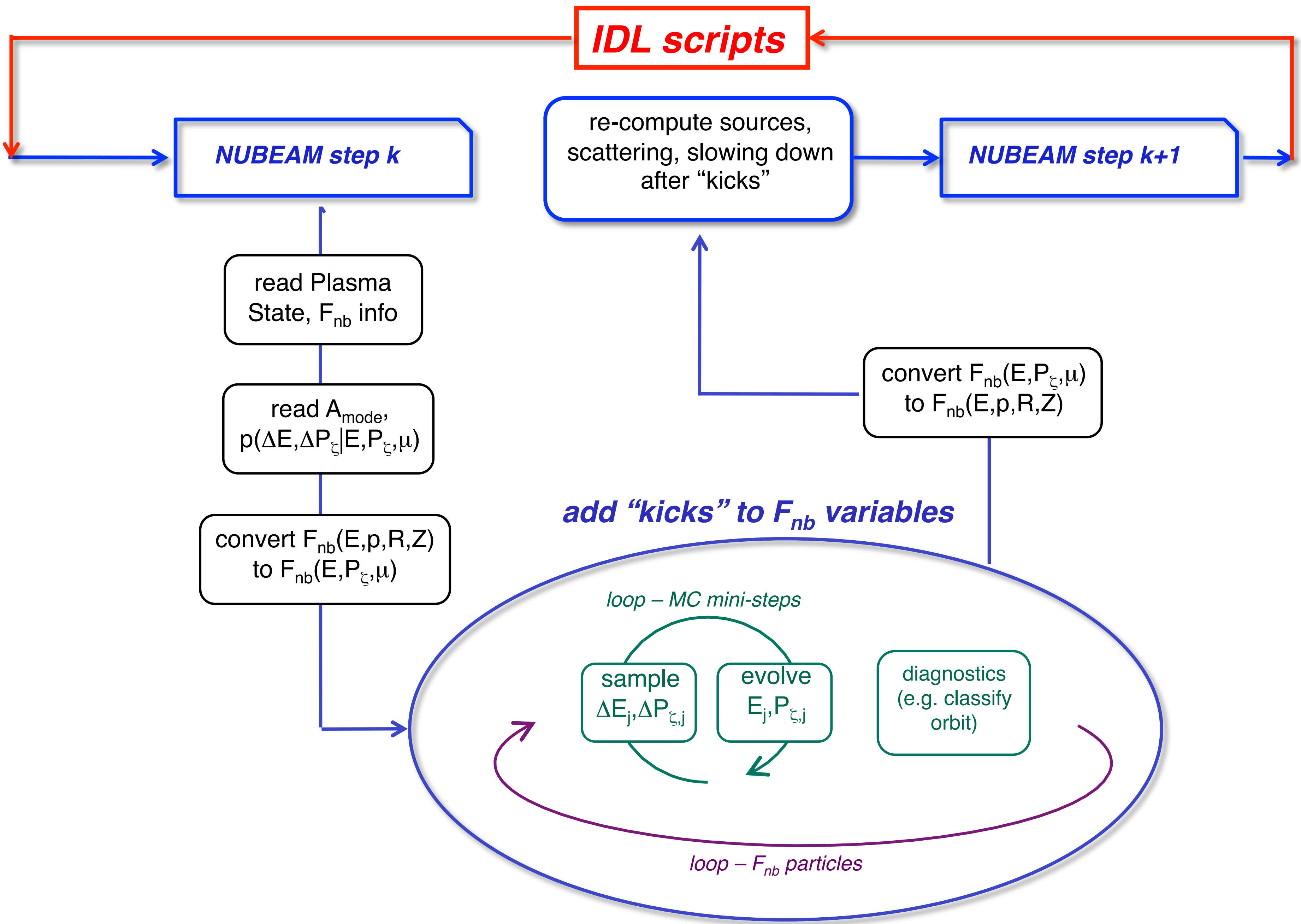


- Compare full ORBIT simulation with reconstructions from reduced model
- F_{nb} drops in the core, fast ions redistributed to larger radii
- Define rough proxy for NB-driven (parallel) current, $I_{nb} \sim F_{nb}$ p v
  - Larger (relative) variations for I_{nb} than for F_{nb} in the core
  - > **Need NUBEAM/TRANSP for more quantitative calculations of I_{nb}**

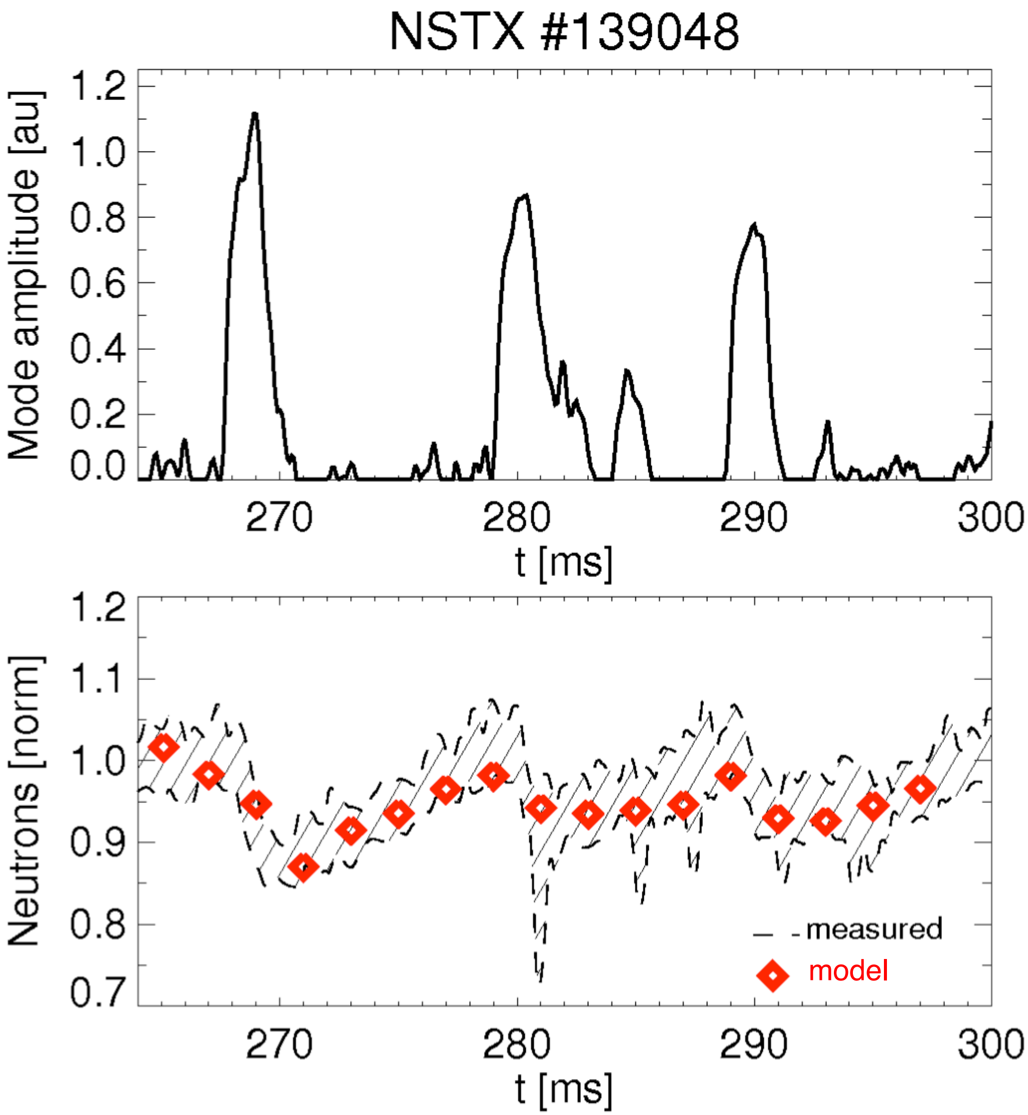
Outline

- New ‘kick model’: basic ideas
- Verification against full ORBIT simulations
- Example from NSTX case w/ TAE avalanche
- Summary

Next step: use stand-alone NUBEAM and IDL scripts to simulate F_{nb} evolution for >>5 ms



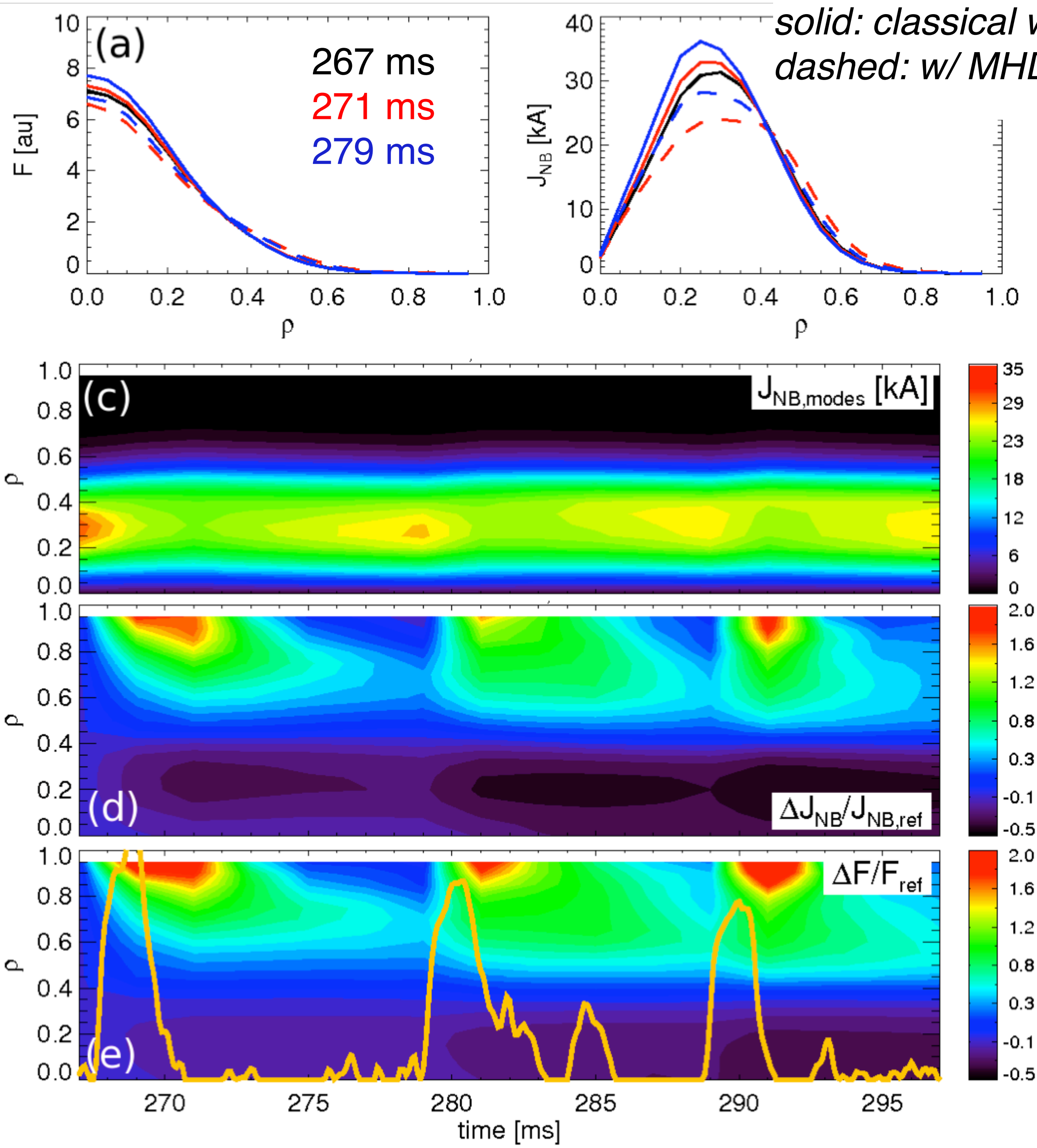
Reduced model reproduces neutron evolution for nominal $A_{mode}(t)$ from neutrons, Mirnovs



- Initial comparison of model predictions w/ experimental neutron rate
- Use stand-alone version of NUBEAM, iterate with reduced model (IDL scripts)
- Background plasma is fixed
  - Normalize exp. neutron rate to central ion density vs. time
  - Normalize all neutron rates at t=267 ms (before first burst)

Satisfactory agreement for $A_{mode}(t)$ from neutron rate, Mirnovs

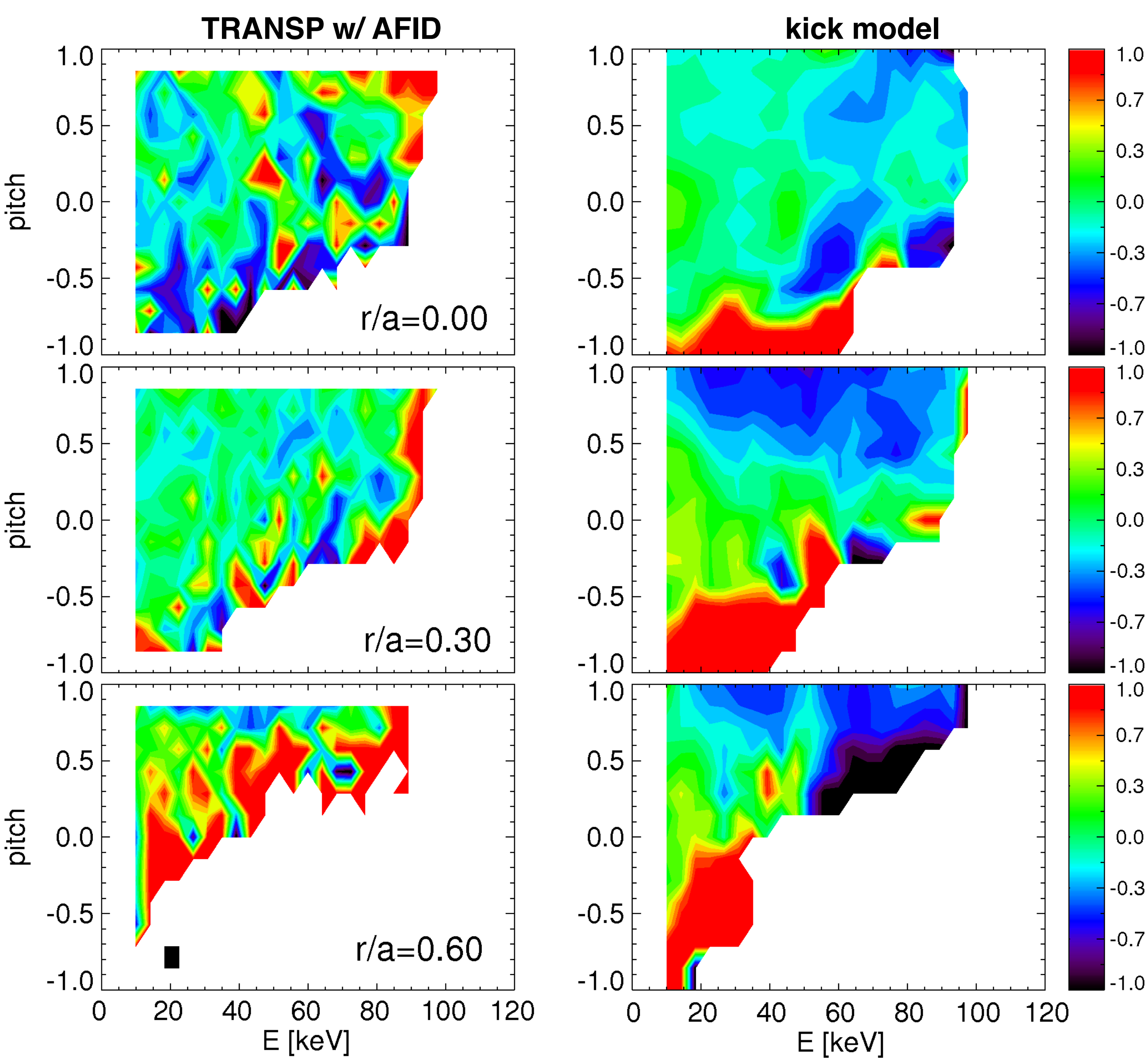
Reduced model + NUBEAM reproduces fast ion redistribution; NB-driven current strongly affected, too



- Initial tests show fast ion redistribution induced by each TAE avalanche
- Clear effects on NB-driven current, J_{nb}
  - Stronger effect than on F_{nb}
- AE effects persist on slowing down time scales
- Constant NB injection counteracts F_{nb}, J_{nb} depletion

Clear *resonance patterns* seen on F_{nb} modified through kick model – not seen in "standard" TRANSP

Relative variation $(F_{nb,1}-F_{nb,0})/F_{nb,0}$, t=265-275ms



- Resonances appear as localized in energy, pitch
- Simple diffusion not adequate to reproduce this
- Will enable more direct comparison with measurements sensitive to phase space modifications

Example: *weight function* for FIDA measurements [Heidbrink, RSI 2010]

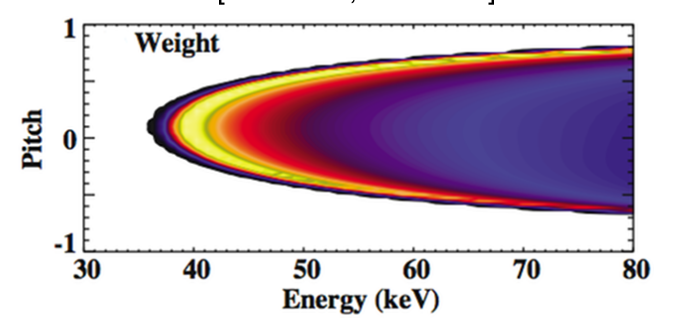
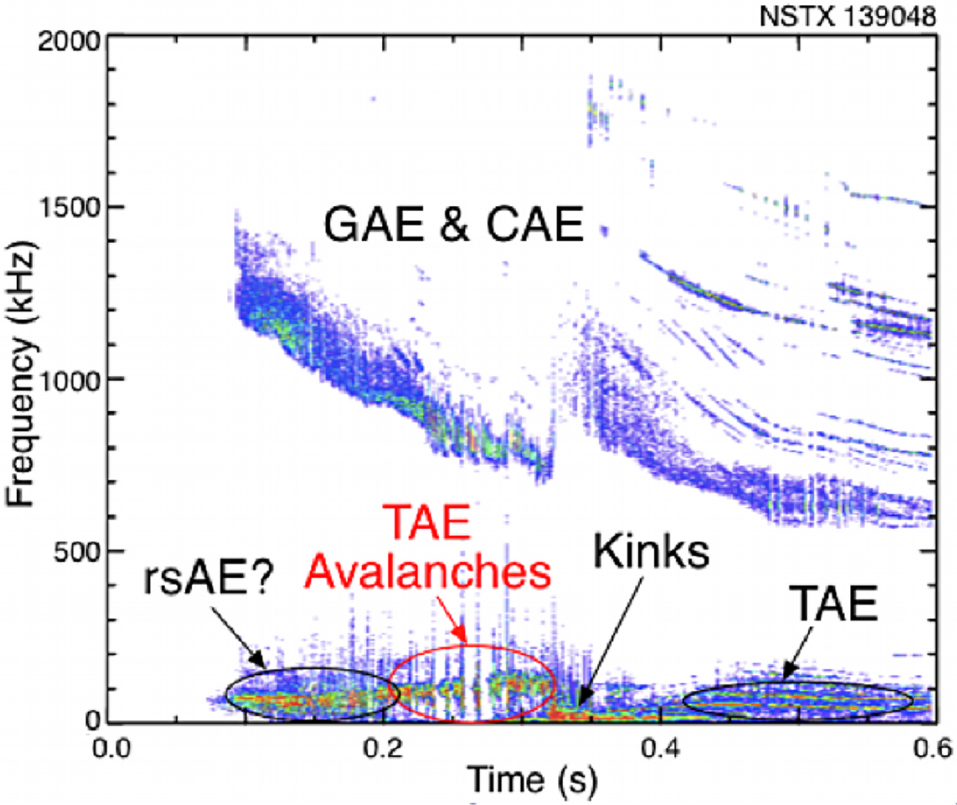
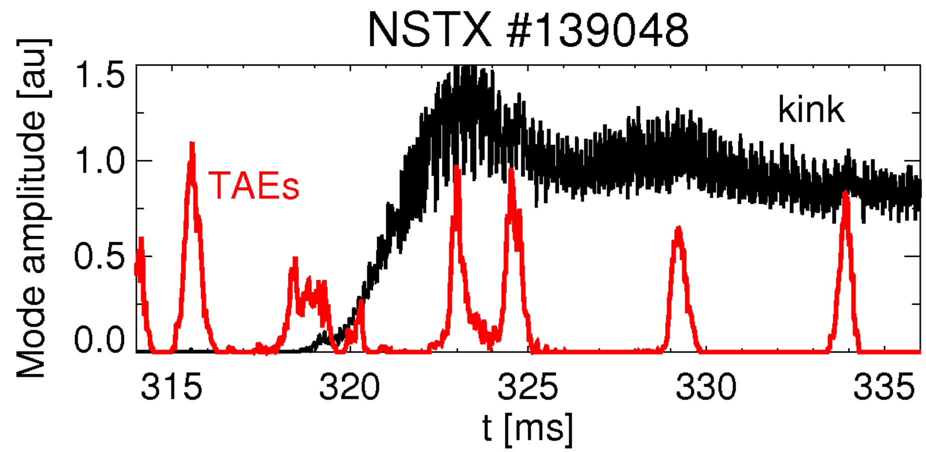


FIG. 6. (Color online) FIDA weight function W vs energy and pitch for a nearly vertical view in DIII-D. The contours are on a linear scale for a blueshifted wavelength corresponding to E_λ=40 keV. Total weight function

The model has been recently improved to account for multiple "classes" of modes in same discharge

Quite common scenario:





> Provides more realistic F_{nb} evolution when modes have comparable amplitude

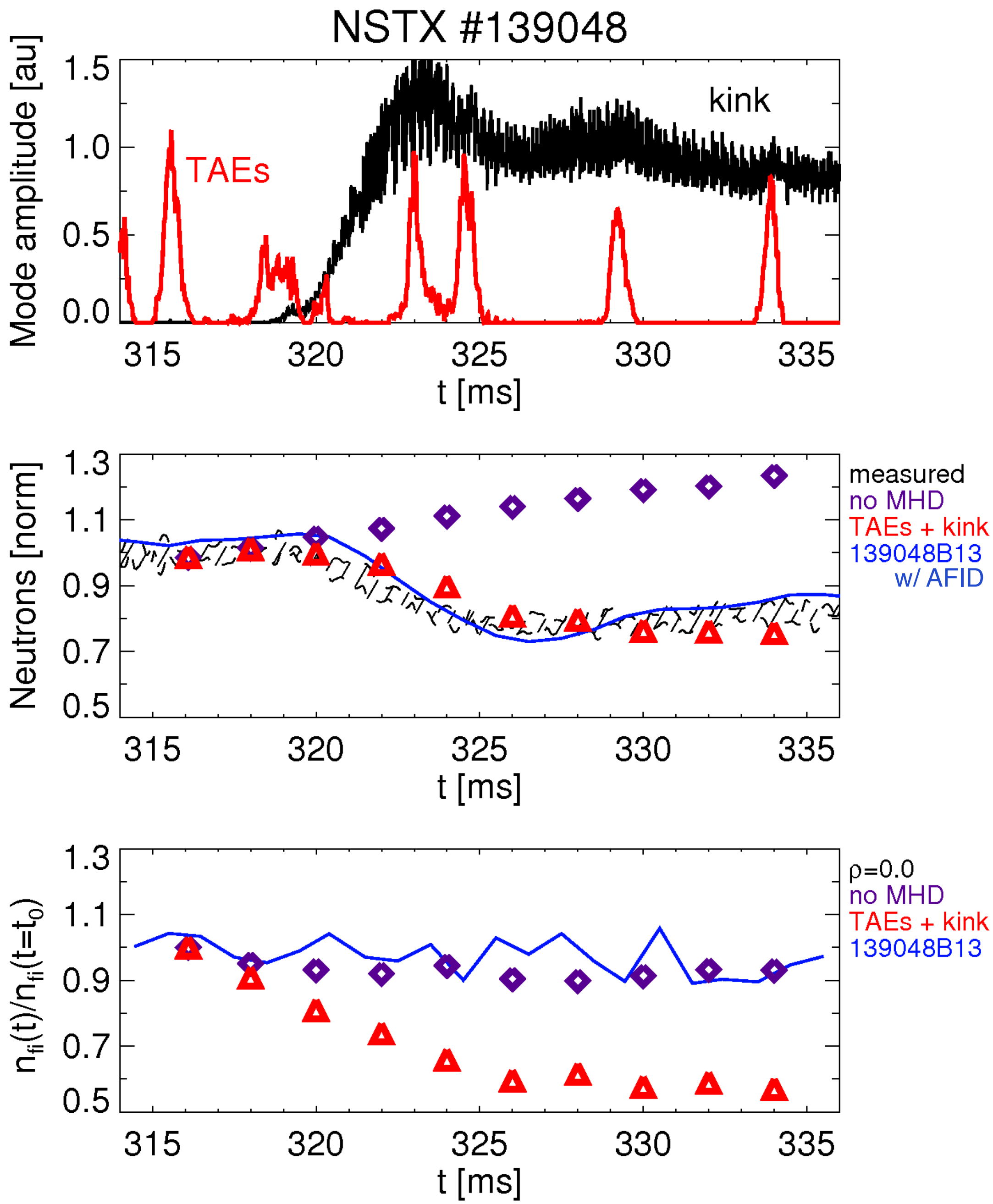
> Account for differences in equilibrium, modes at different stages of the discharge

> Each "class" is modeled by $p_k(\Delta E,\Delta P_\zeta)$, weighted by $A_{mode,k}$
  - Total kicks result from $\Sigma_k\ A_{mode,k}(t) \times p_k(\Delta E,\Delta P_\zeta)$

Preliminary example: NSTX scenarios with simultaneous TAEs and kink-like modes

TAEs + kink-like:

- Anomalous diffusivity (AFID) up to t=320ms to match neutrons
- Set AFID=0 after 320ms
- Keep profiles fixed
- Use $p_1(\Delta E, \Delta P_\zeta)$ for TAEs, same as for avalanches
- Use $p_2(\Delta E, \Delta P_\zeta)$ for kink-like modes
  - **Crude model w/ n=1,2,3**
  - *Just a test case!*
  - > Analysis in progress to obtain "real" kink structure



Outline

- New ‘kick model’: basic ideas
- Verification against full ORBIT simulations
- Example from NSTX case w/ TAE avalanche
- Summary

Summary

- Test algorithm for reduced fast ion transport model developed, being verified
  - Results compared with full ORBIT runs
  - Confirmed validity of approach for practical case

*more details: M. Podestà et al., PPCF **56** (2014) 055003*

- Successful preliminary tests for NSTX case with TAE modes (*avalanches*)
  - Shot#139048, t~265-320 ms: H-mode avalanches
  - Strong redistribution (energy, radius) of fast ions inferred
  - Modest losses, consistent with previous detailed modeling
  - NB-driven current J_{nb} is strongly perturbed

> **Implementation in NUBEAM under way**
> > **Extensive Verification & Validation planned (multi-machine, multi-mode)**
> > **Identify issues, possible improvements to the model**

Backup

Four models are presently implemented in TRANSP for fast ion diffusion/convection coefficients, D_b & v_b

All have diffusive/convective nature in radial coordinate; little/no phase space selectivity:

[from http://w3.pppl.gov/~pshare/help/transp.htm]

1) $D_b = k_{ADIFB} \times D_e$

k_{ADIFB} : *multiplier*
$D_e(x,t)$: electron particle diffusivity

2) $D_b = k_{ADIFB} \times D_e^{WP}$

$D_e^{WP}(x,t)$: electron particle diffusivity from Ware-pinch corrected flux

3) $\Gamma_{fi} = -D_b \nabla n_b + v_b n_b$

diffusion/convection model

4) $D_b(E,t,x) = \Sigma_k \, \alpha_k \, D_k$

$D_k(E,x,t)$: diffusivity for deeply trapped, barely trapped, co-barely passing, ...

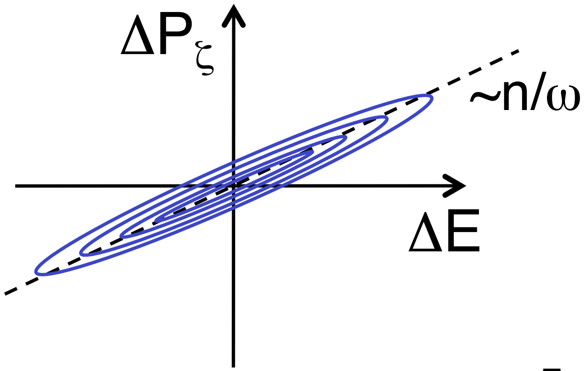
For low-frequency *AEs with $\omega << \omega_{ci}$ such as TAEs, magnetic moment μ is conserved (...but maybe it's not)

- *In this presentation, it is assumed that $\Delta\mu=0$*

- <u>However</u>: $\Delta\mu=0$ hypothesis can break down if
  - ρ_f ~ radial width of the modes
  - ρ_f ~ scale-length of equilibrium profiles
  - ➩ *Both conditions are likely to be met in spherical tokamaks (e.g. NSTX)*

➩ **Proposed model can be generalized to cases where μ is *not* conserved**
  - Also important for ω_{ci}-range instabilities: GAE/CAEs

Example for single-resonance case: analytical *probability distribution function*



For each *bin* in (E,Pζ,μ), steps in ΔE,ΔPζ are approximated by a bivariate $p(\Delta E, \Delta P_\zeta | P_\zeta, E, \mu, A)$

$$p = p_0\, e^{-\frac{1}{2(1-\rho)}\left[\frac{(\Delta E-\Delta E_0)^2}{\sigma_E^2}+\frac{(\Delta P_\zeta-\Delta P_{\zeta 0})^2}{\sigma_{P_\zeta}^2}-2\rho\frac{(\Delta E-\Delta E_0)(\Delta P_\zeta-\Delta P_{\zeta 0})}{\sigma_E\,\sigma_{P_\zeta}}\right]}$$

$$p_0 = \frac{1}{2\pi\,\sigma_E\,\sigma_{P_\zeta}\sqrt{1-\rho^2}}$$ *normalization*

$$\rho = \frac{<(\Delta E-\Delta E_0)(\Delta P_\zeta - \Delta P_\zeta 0)>}{\sigma_E\,\sigma_{P_\zeta}}$$ *correlation parameter,*

such that $\Delta P_\zeta(\Delta E) = \Delta P_{\zeta 0} + sign(\rho)\times\frac{\sigma_{P_\zeta}}{\sigma_E}(\Delta E - \Delta E_0)$

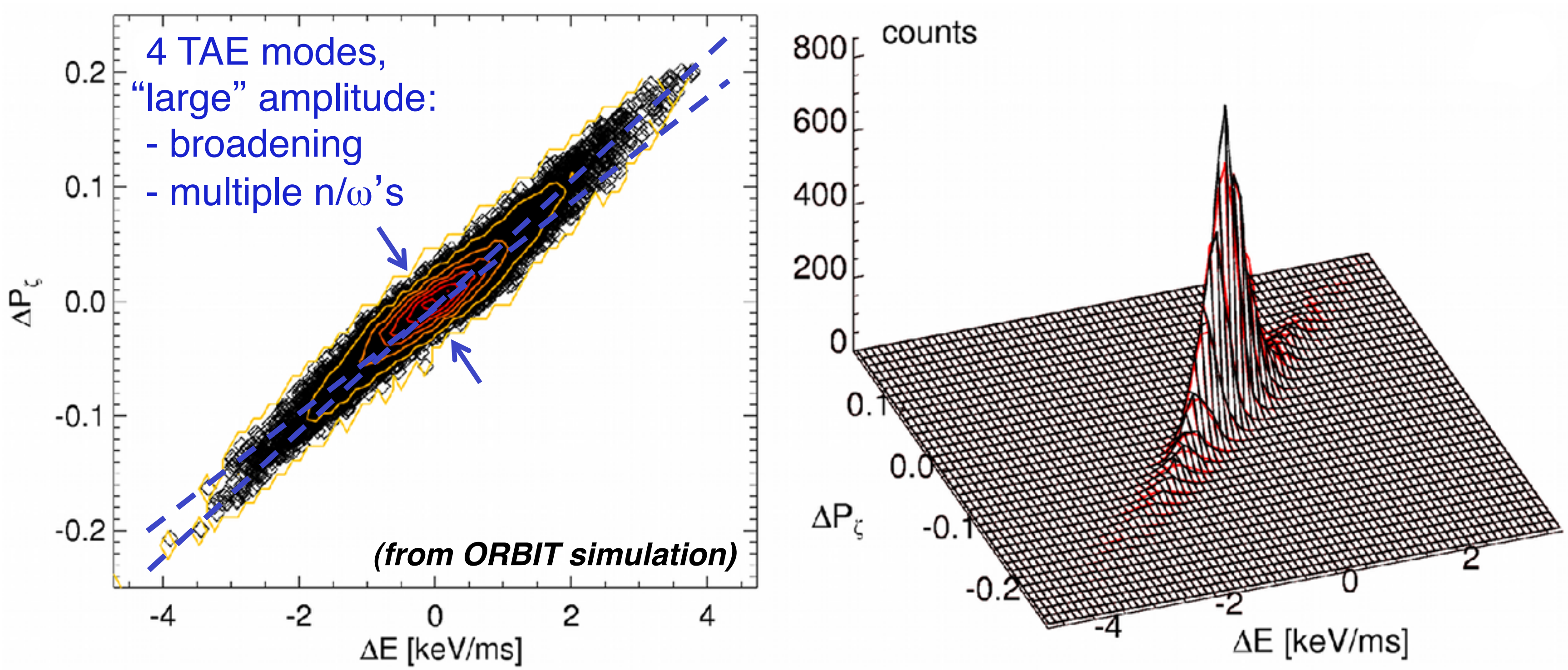
...and all parameters depend, in principle, on the mode amplitude A=A(t)

Single (isolated) resonances introduce fundamental constraints on particle's trajectory in (E,P_ζ,μ)

- From Hamiltonian formulation:

$$\omega P_\zeta - nE = const. \Longrightarrow \Delta P_\zeta / \Delta E = n/\omega$$

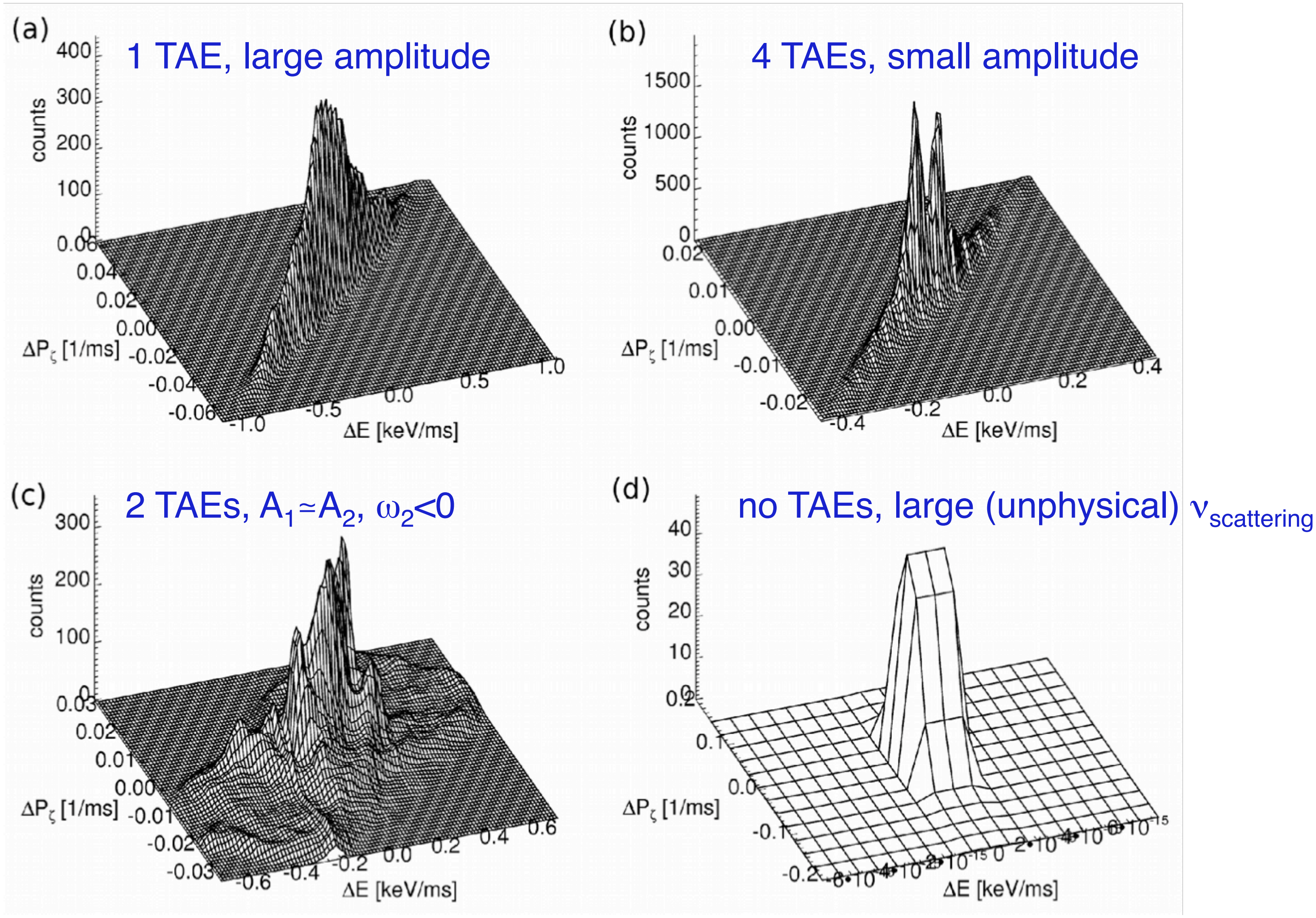
$\omega=2\pi f$, mode frequency $\qquad$ n, toroidal mode number



Presence of multiple modes/resonances distorts the 'ideal' (linear) relationship

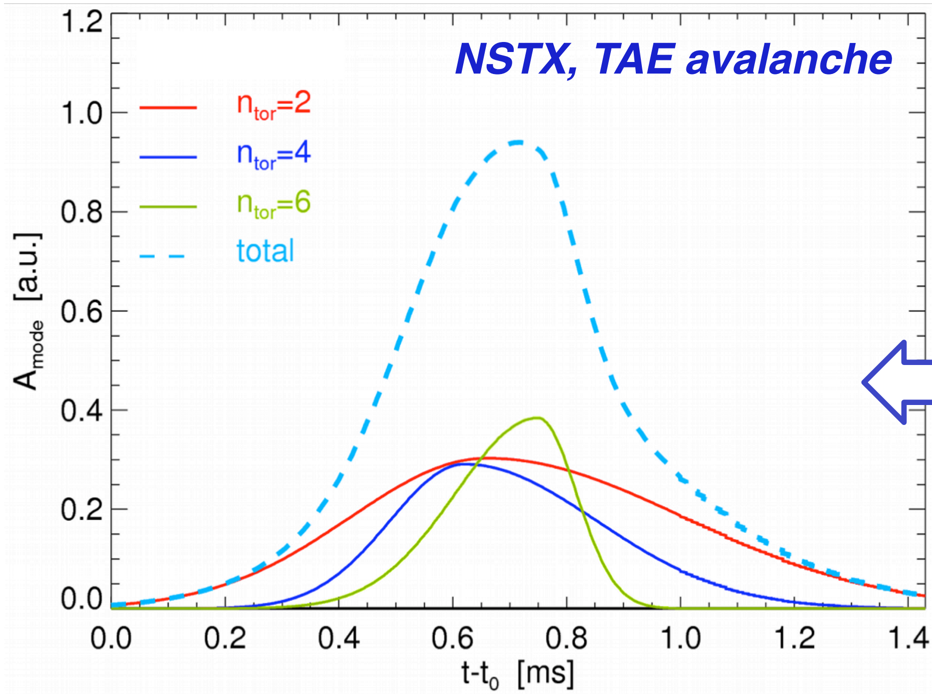
An analytical formulation for $p(\Delta E, \Delta P_\zeta | P_\zeta, E, \mu)$ *could* be developed – but it would be quite unpractical

E.g., use: $p(\Delta E, \Delta P_\zeta | P_\zeta, E, \mu) = \Sigma_{i=1}^{N} w_i \, p_i(\Delta E, \Delta P_\zeta | P_\zeta, E, \mu)$



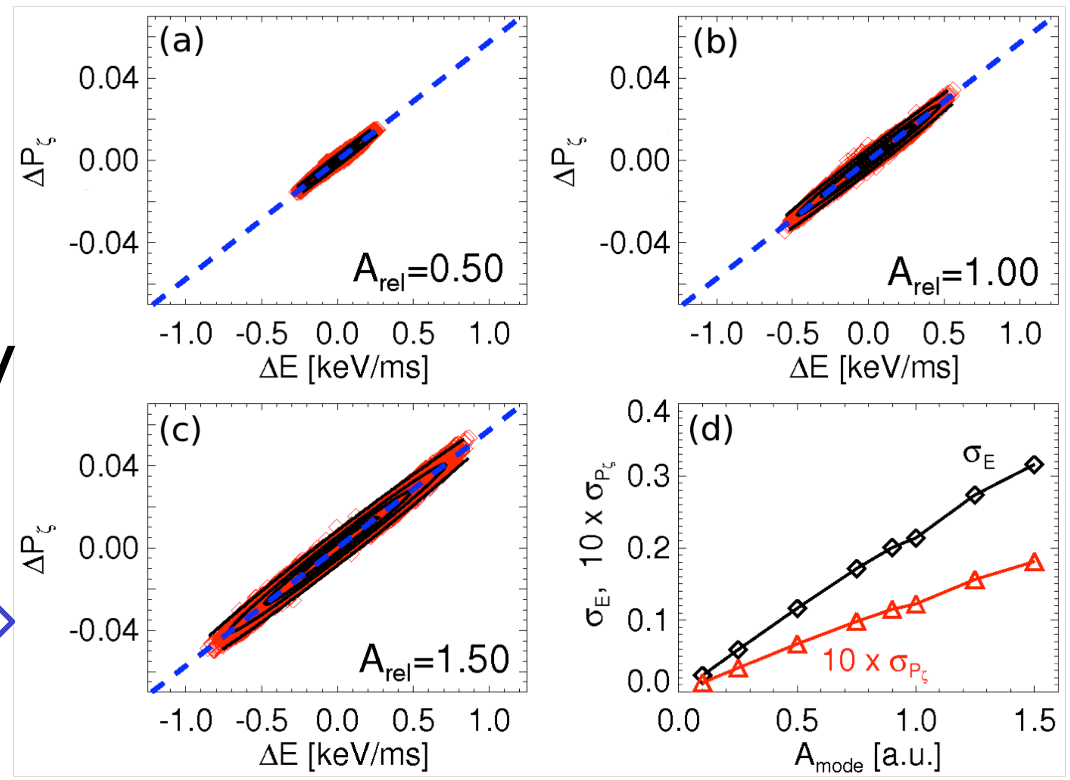
In practice, will use the full 5D matrix for $p(\Delta E, \Delta P_\zeta | P_\zeta, E, \mu)$

Mode amplitude can evolve on time-scales shorter than typical TRANSP/NUBEAM steps of ~5-10 ms



F_{nb} evolution must be computed as a sequence of sub-steps

- Duration δt_{step} sufficiently shorter than time-scale of mode evolution
- Examples here have δt_{step}~25-50 μs

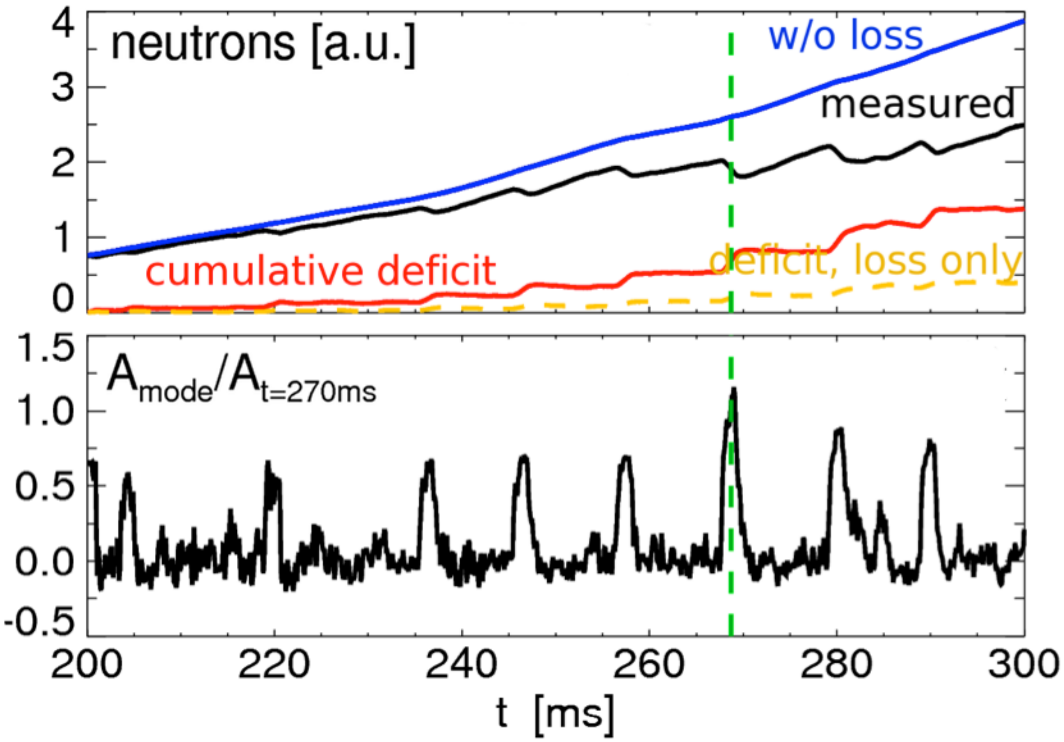


Energy and P_ζ steps assumed to scale linearly with mode amplitude

- Roughly consistent with ORBIT simulations

Example: $A_{mode}(t)$ computed from measured neutron rate or Mirnov coils' signal

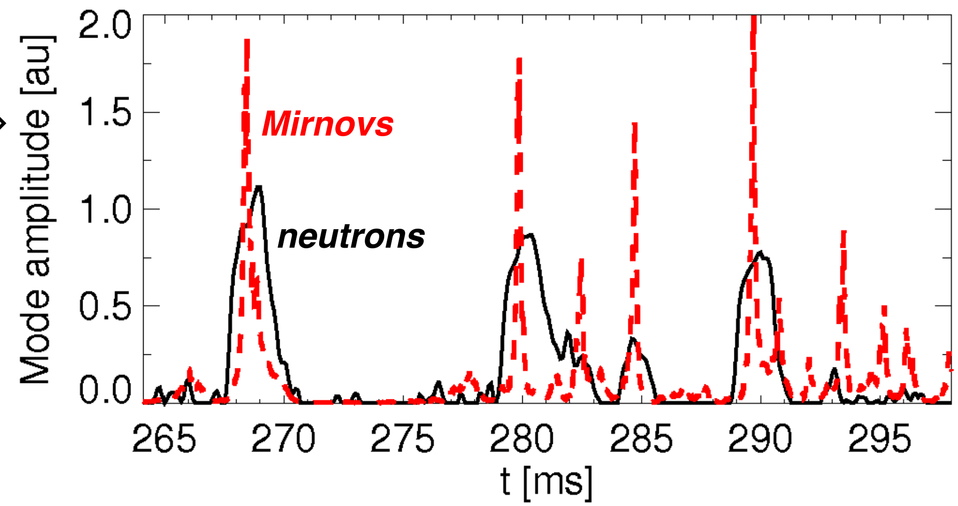
Get *A(t)* from measured neutrons+table look-up:



– Compute fractional R_n drops vs. time

– Use table R_n vs A_{mode} to find corresponding (normalized) mode amplitude

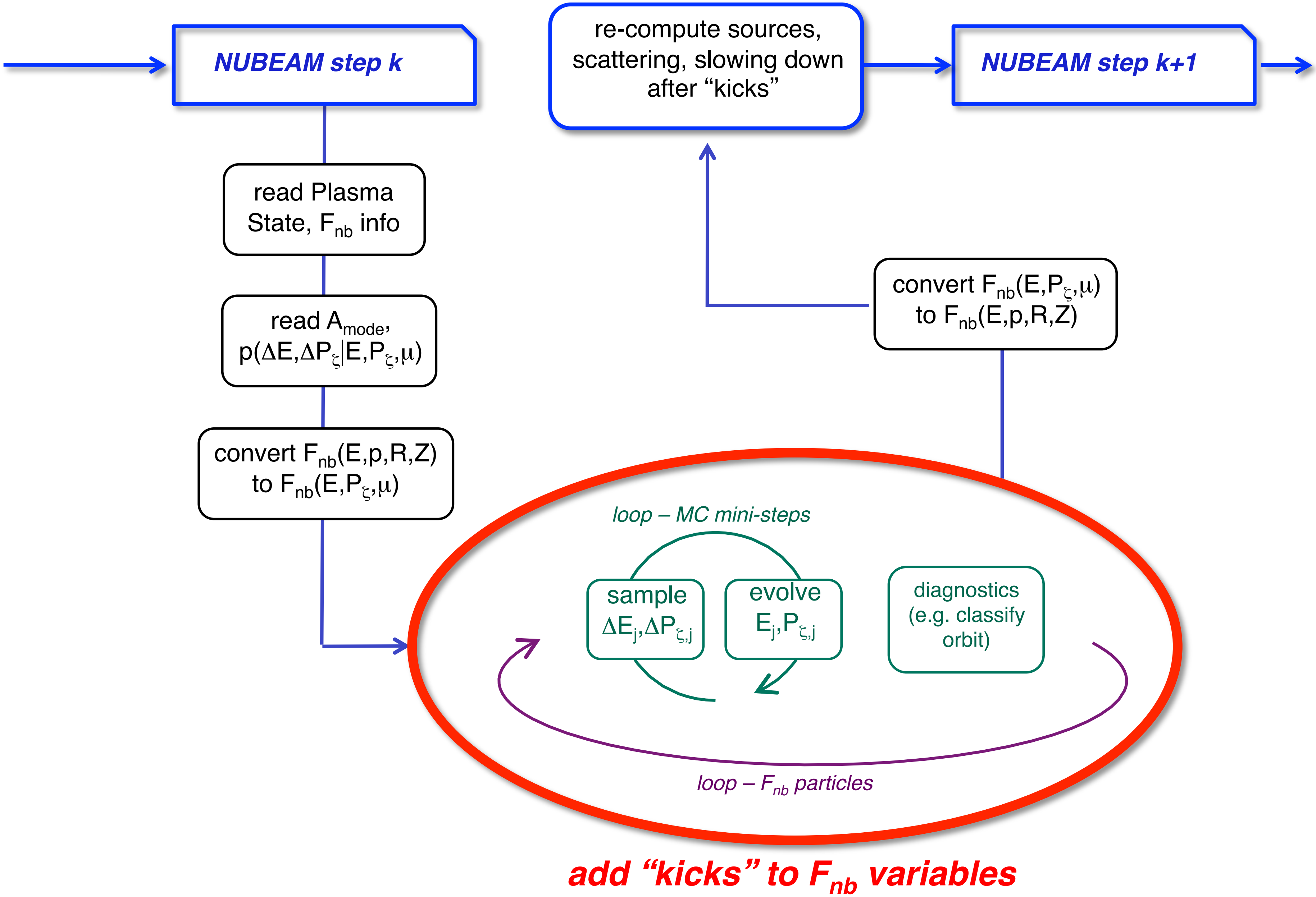
–Comparison w/ $A_{mode}(t)$ from Mirnov coils



–Do different waveforms lead to differences in fast ion evolution?

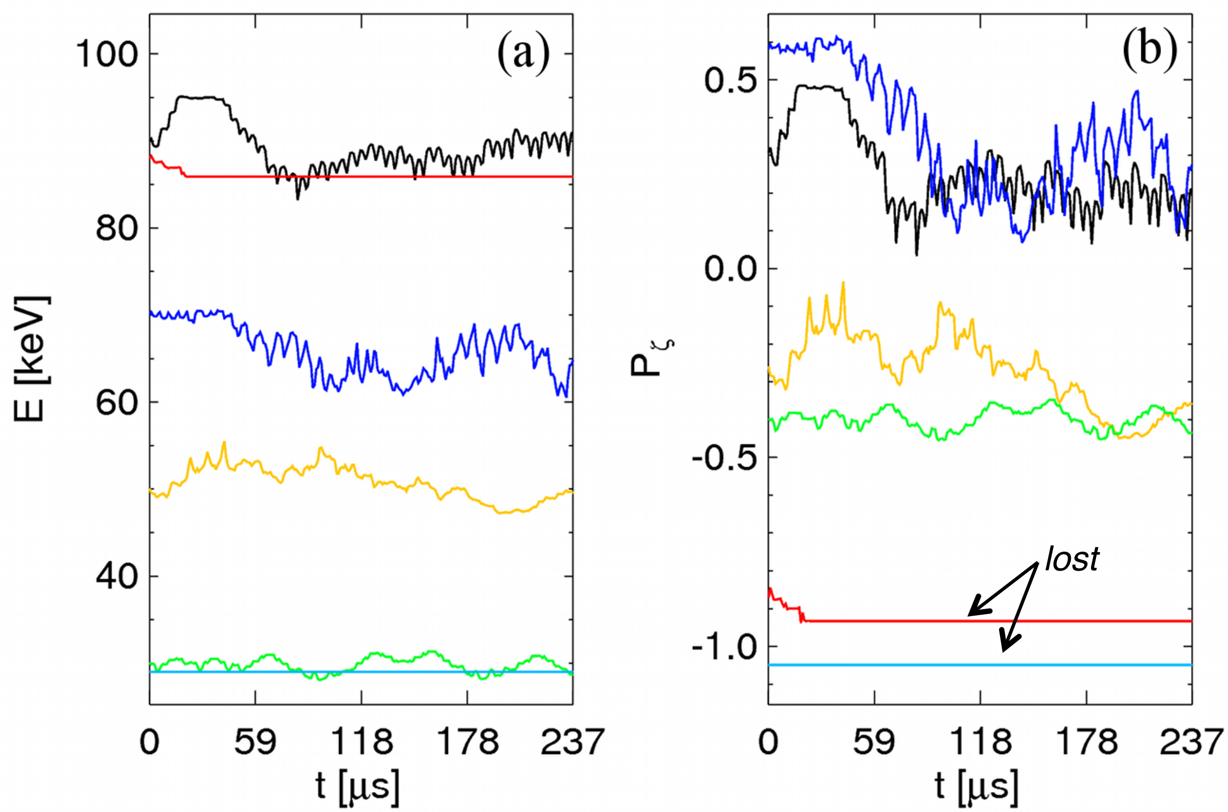
Not on relevant time scales > 1ms

Scheme to advance fast ion variables according to transport probability in NUBEAM module of TRANSP



Scheme to evolve F_{nb} takes into account (in a semi-empirical way) constraints of resonant interaction

- Particle's motion is characterized by different time-scales:
  - Fast oscillation in wave field – *neglected*
  - 'Jumps' $\Delta E, \Delta P_\zeta$ around instantaneous energy, P_ζ
  - Slow (secular) drift from initial energy, P_ζ



Putting all together

At each 'macroscopic' NUBEAM step:

Loop over particles

I. Re-normalize bins (P_ζ,E,μ) based on q-profile, fields, ...

II. Identify 'bin' in (P_ζ,E,μ) for current 'particle' (i.e. orbit)

III. Extract steps σ_E,$\sigma_{P\zeta}$ (σ_μ) from multivariate p(ΔE,ΔP_ζ,$\Delta\mu$)

IV. Compute sign $S_{r,i}$ from p(ΔE,ΔP_ζ): positive or negative kicks

V. Rescale steps based on A_{modes}(t)

VI. Advance E, P_ζ (μ): $\left\{ \begin{array}{l} \overline{\Delta E}_i = S_{r,i} \times A_{mode}(t=\bar{t}) \times \sigma_{E,i} \\ \Longrightarrow \; E_i = E_i + \overline{\Delta E}_i \end{array} \right.$

VII. Advance particle's trajectory in phase space for next step

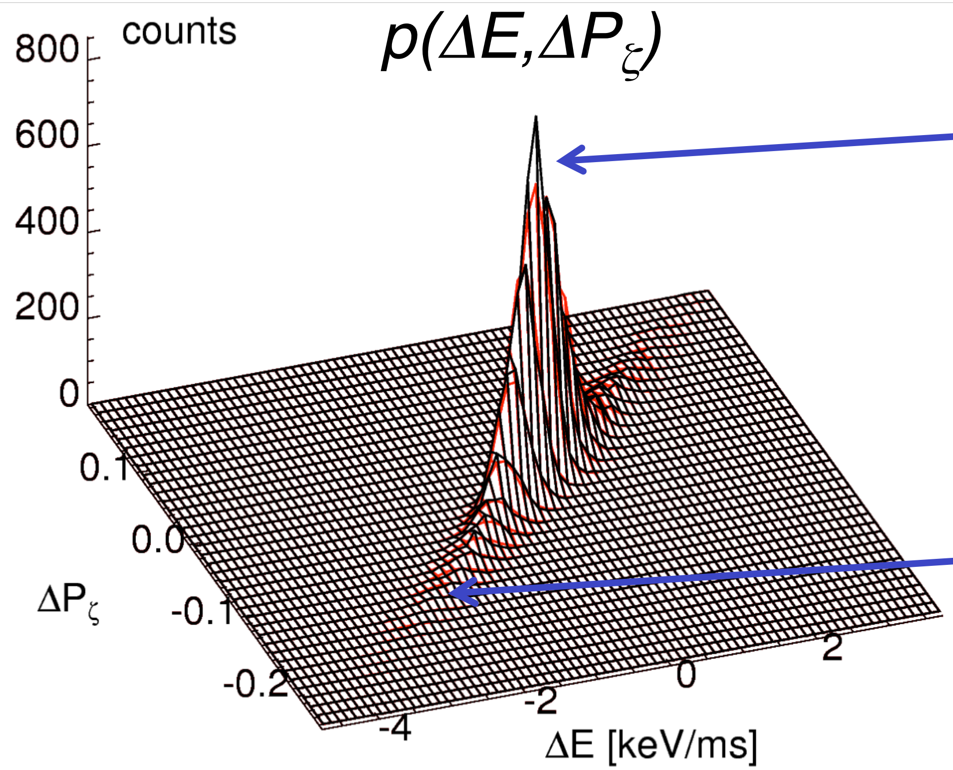
VIII. Compute slowing down, scattering (NUBEAM)

where steps II-VII are divided in sub-steps for each particle

Required input, e.g. through 'Ufiles':

scaling factor ("mode amplitude") A_{mode}, probability p(ΔE,ΔP_ζ)

Discrete bins in (P_ζ,E,μ) can contain both *resonant* and *non-resonant* particles



- 'Non-resonant' particles have small fluctuations around initial (E, P_ζ)
- 'Resonant' particles can experience large ΔE, ΔP_ζ variations

- To keep track of particle's class:
  - Sample steps σ_E, $\sigma_{P\zeta}$ at first step only
  - This mimic the "correlated random walk" experienced by the particles
  - Exception: particle move to a different bin -> re-sample

$p(\Delta E, \Delta P_\zeta | P_\zeta, E, \mu)$ can be skewed to positive/negative $\Delta E, \Delta P_\zeta$, causing overall "drift" of $F_{nb}(P_\zeta, E, \mu)$

- Introduce 'random sign' for *i*–th step in MC procedure, $S_{r,i}$
- For each particle (e.g. pair of correlated steps σ_E, $\sigma_{P\zeta}$), calculate $S_{r,i}$ from probability of positive vs. negative steps

  - From $p(\Delta E, \Delta P_\zeta)$ compute

$$p_+ \doteq p(\sigma_{E,i}, \sigma_{P_\zeta,i}) \quad ; \quad p_- \doteq p(-\sigma_{E,i}, -\sigma_{P_\zeta,i})$$

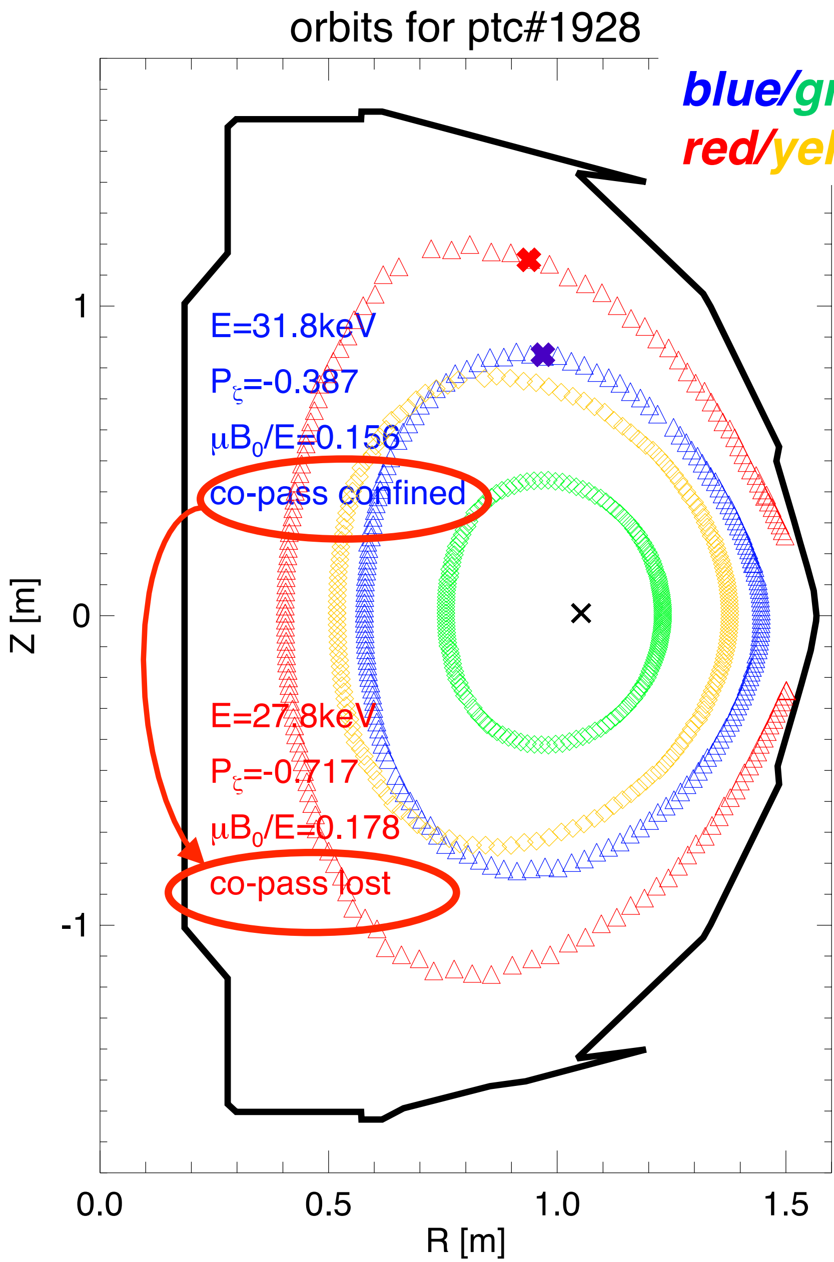
  - Then define f_{sign}:

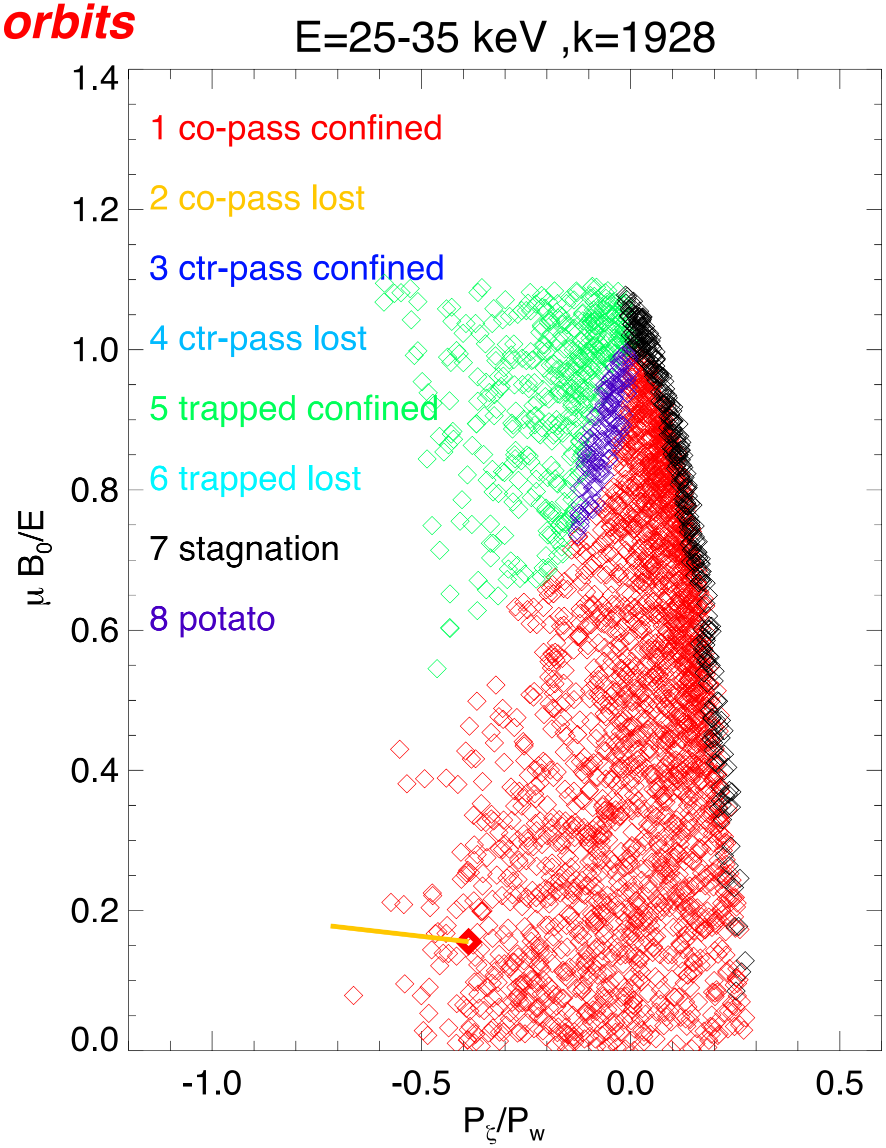
$$f_{sign} = \frac{p_+}{p_+ + p_-}$$

  - Finally, use $0 < f_{sign} < 1$ to bias random extraction of $S_{r,i} = +1, -1$

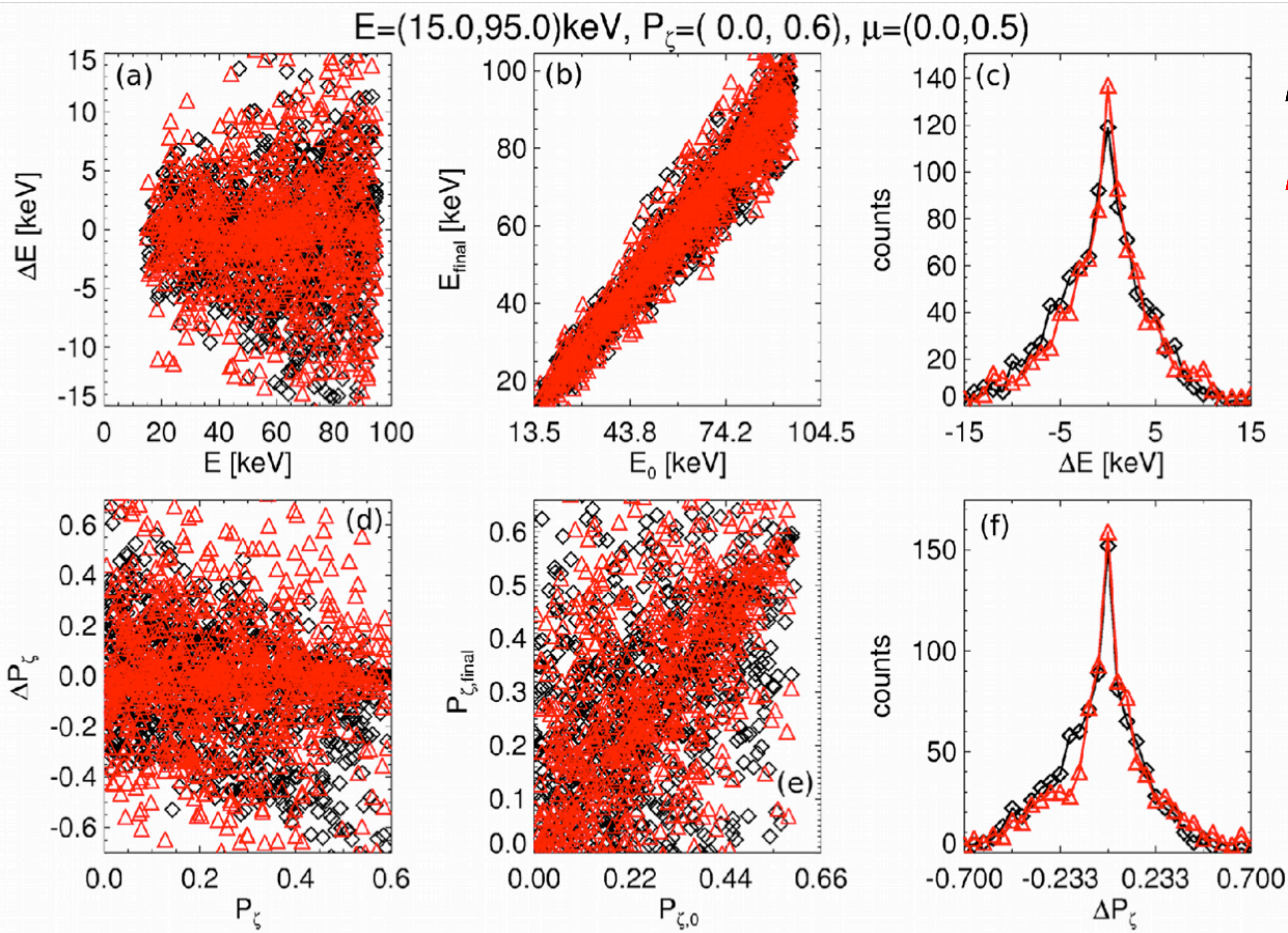
Only a few particles (~0.1%) are actually lost for the cases examined here; redistribution dominates

blue/green: initial orbits
red/yellow: final orbits





Example: evolving F_{nb} over 270 μs in 5 sub-steps

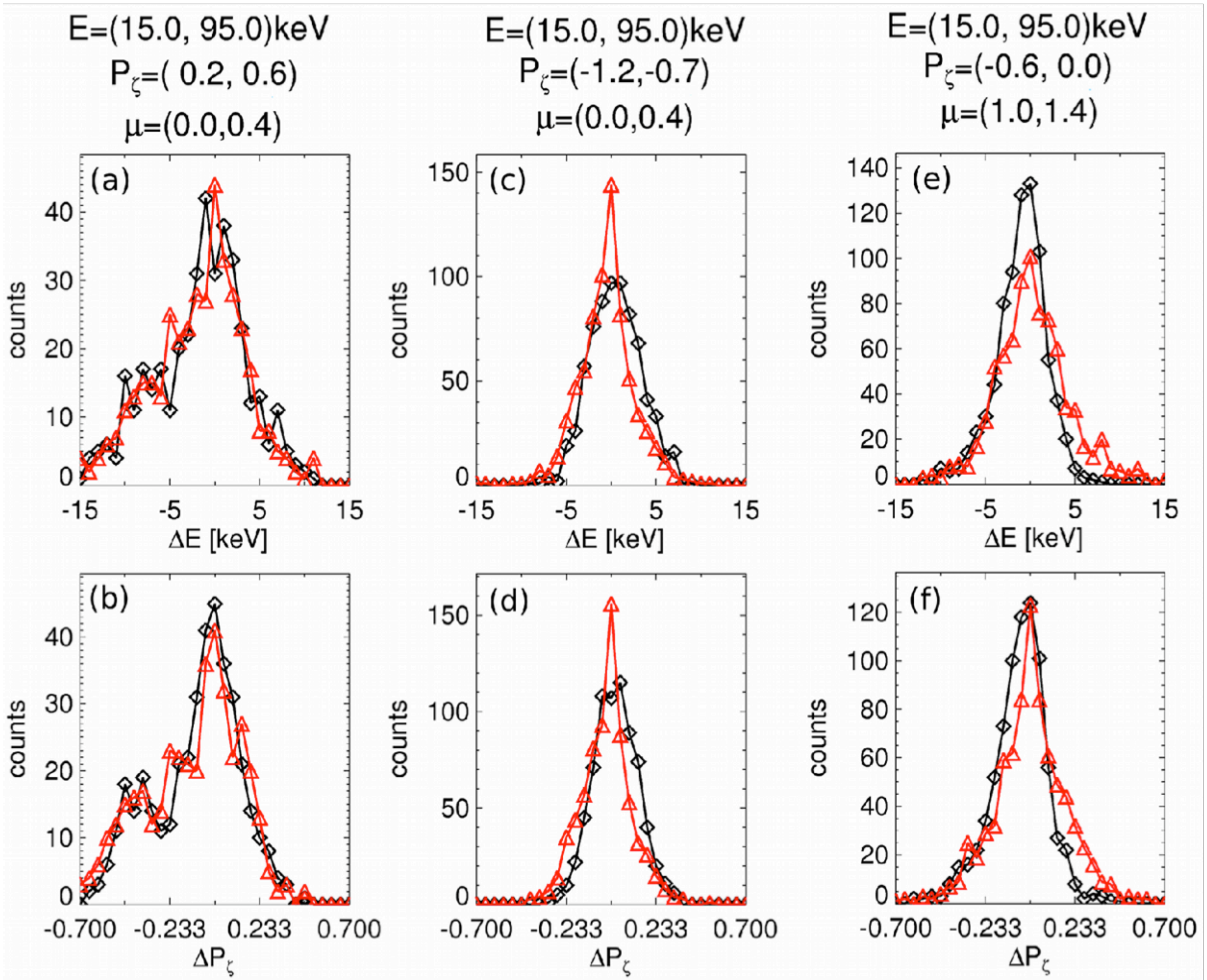


black: ORBIT

red: Model

Reconstruction works for different classes: co-, counter-, trapped

co-passing | *counter-passing* | *trapped*

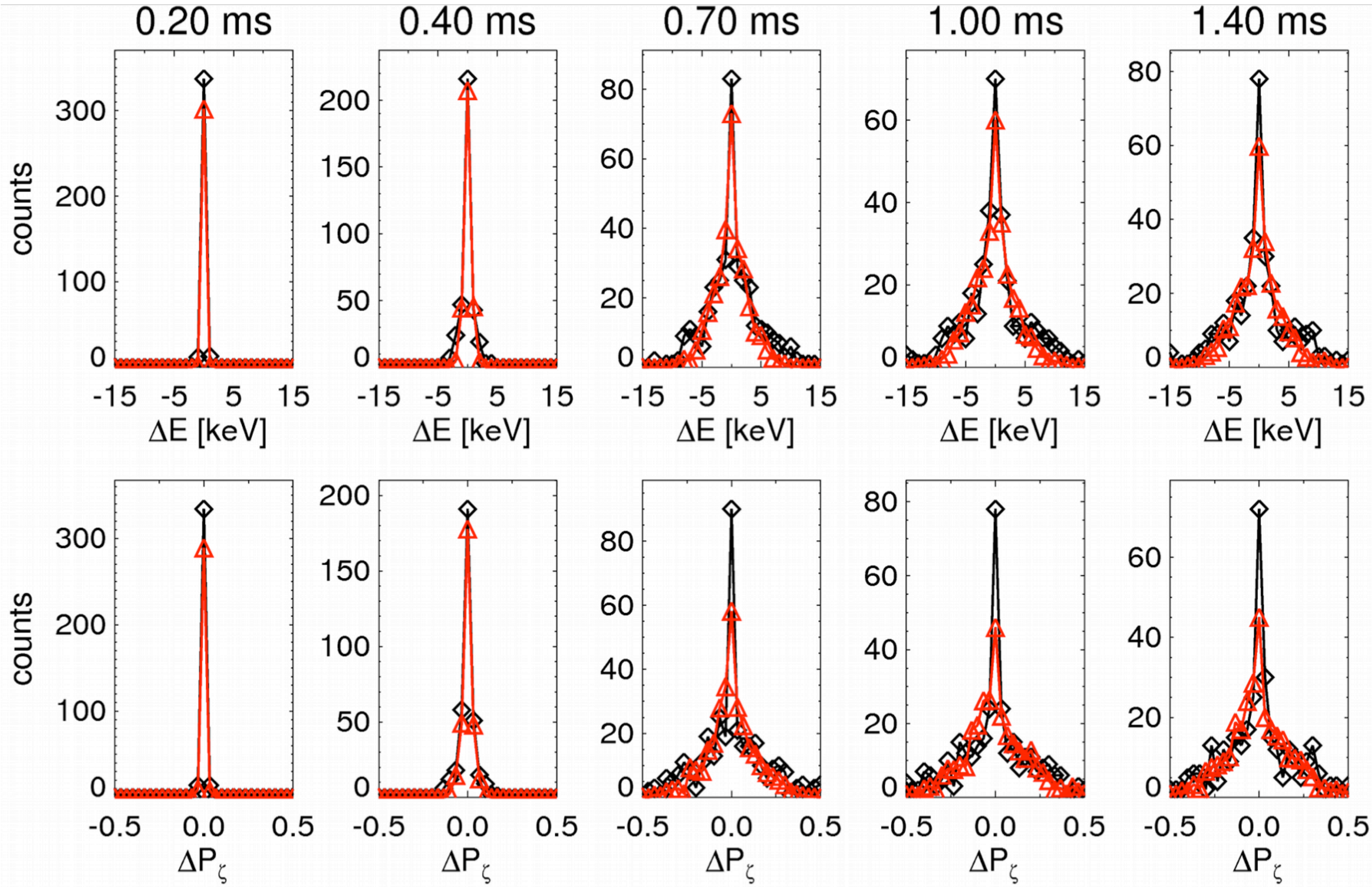


black: ORBIT

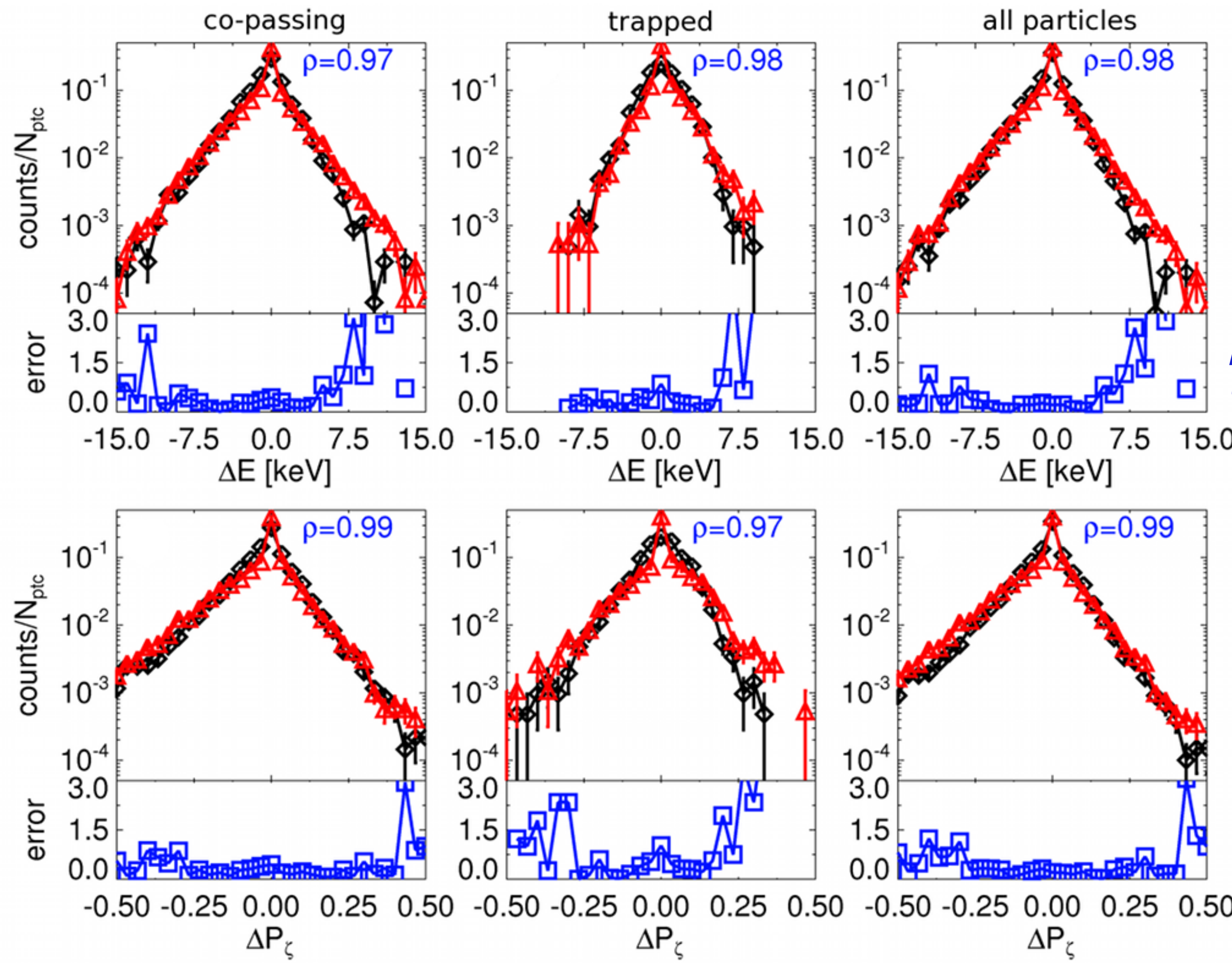
red: Model

Reconstruction works at different sub-steps

black: ORBIT ***red: Model***



Good agreement with ORBIT is preserved when evolving F_{nb} over 5 ms, typical macro-step of NUBEAM



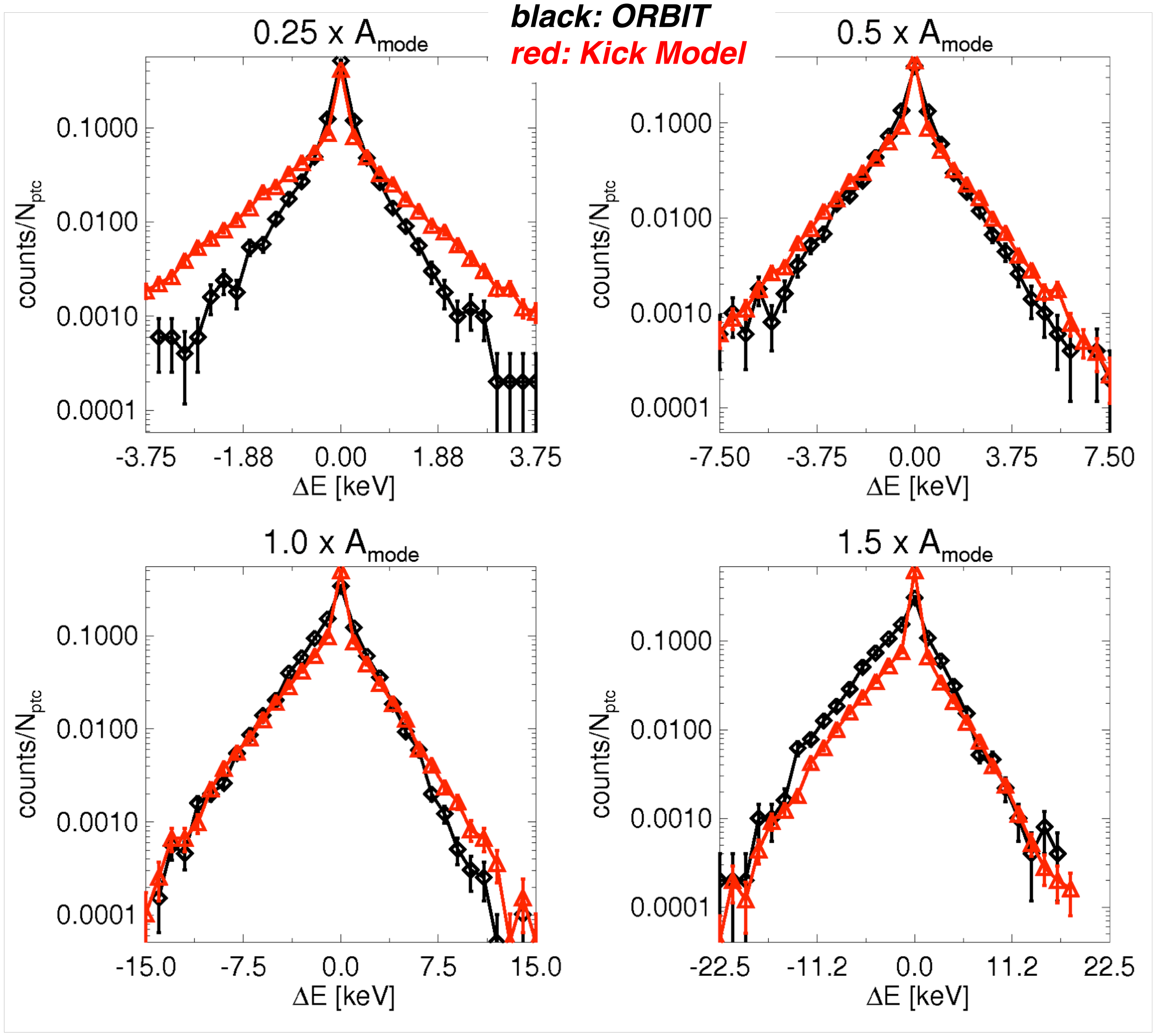
black: ORBIT

red: Kick Model

ρ: correlation coefficient
error: (ORBIT-model)/ORBIT

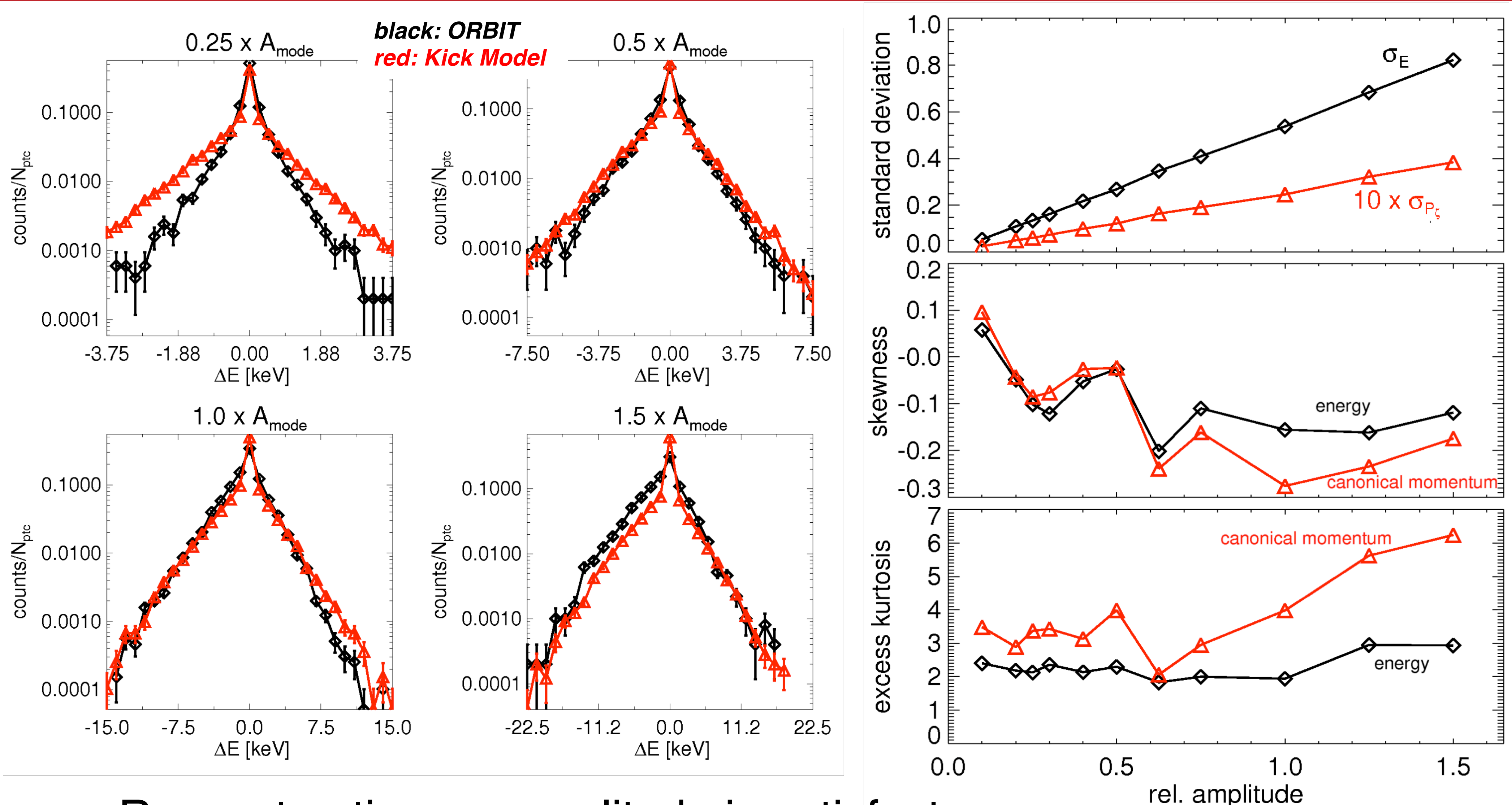
Satisfactory reconstruction over 3 orders of magnitude for co-, trapped (, counter- not present in *this* input F_{nb}) particles

Tests assuming different mode amplitudes are satisfactory, though not perfect…



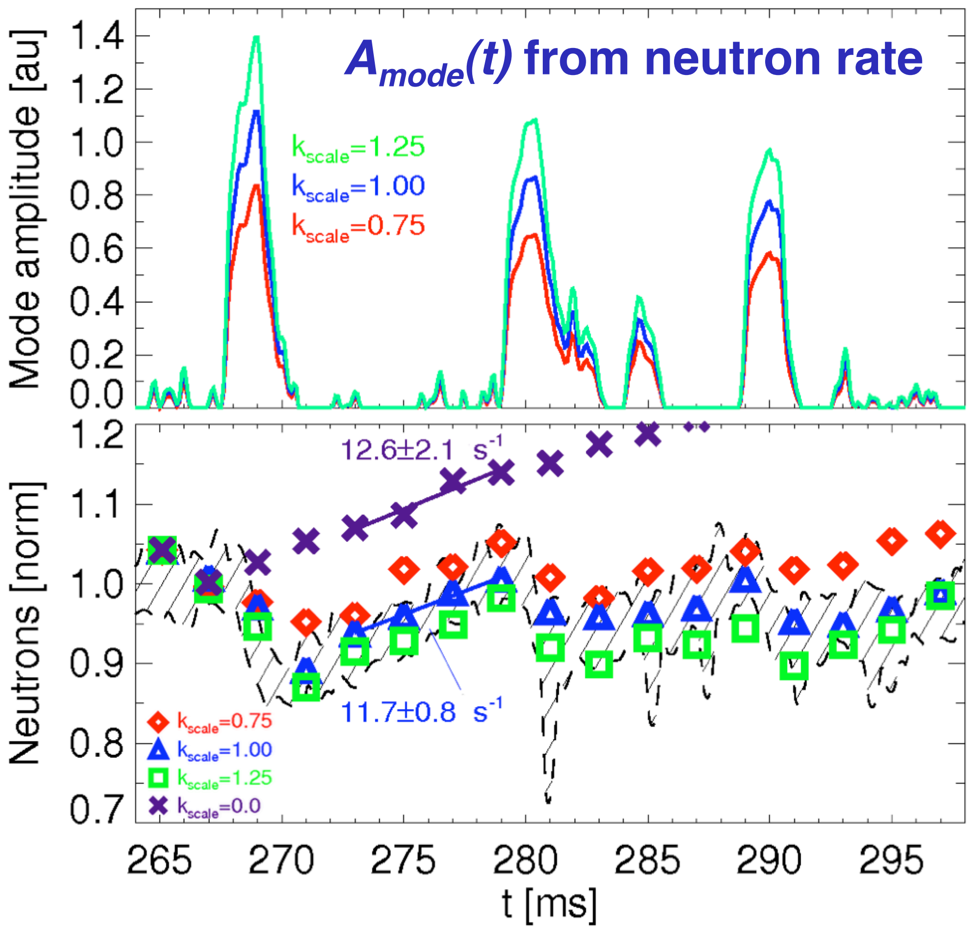
- Reconstruction vs. amplitude is satisfactory

Tests assuming different mode amplitudes are satisfactory, though not perfect…



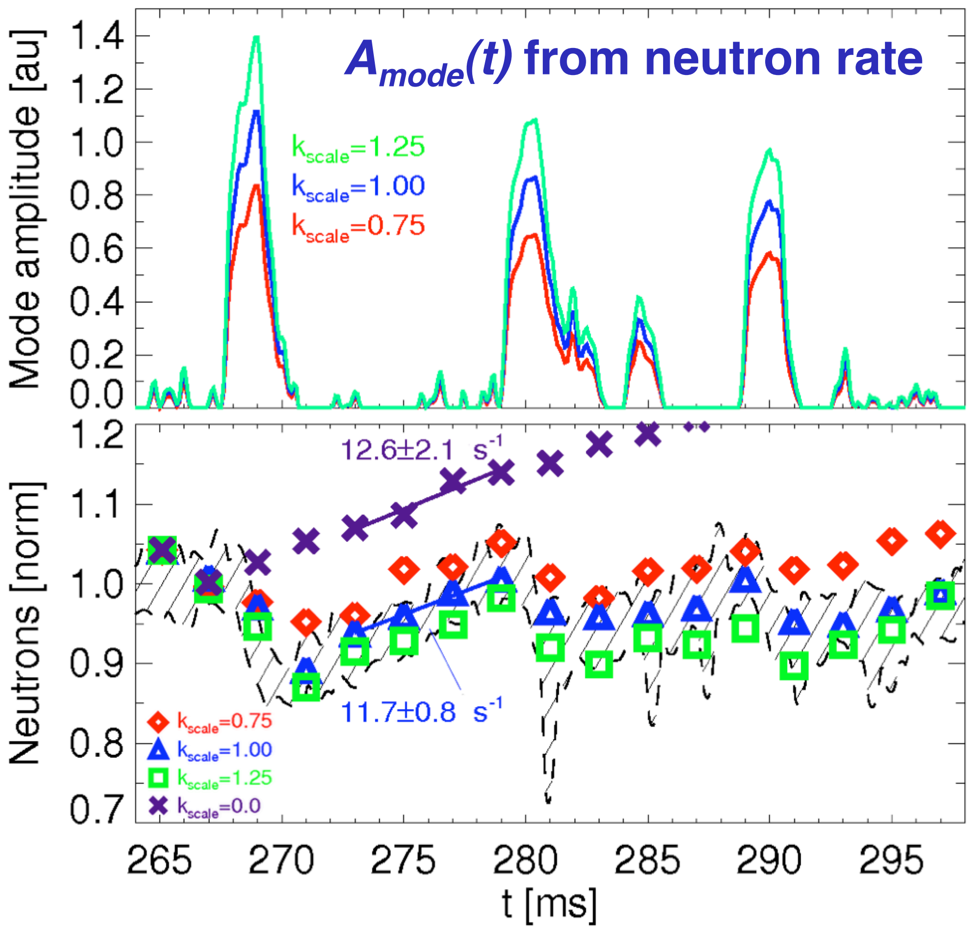
- Reconstruction vs. amplitude is satisfactory
- **Differences ascribed to simplifications in p(ΔE,ΔP_ζ) scaling with A_{mode}: shape *does* change**

Results are sensitive to input mode amplitude; time steps must be chosen to satisfy “statistical” approach

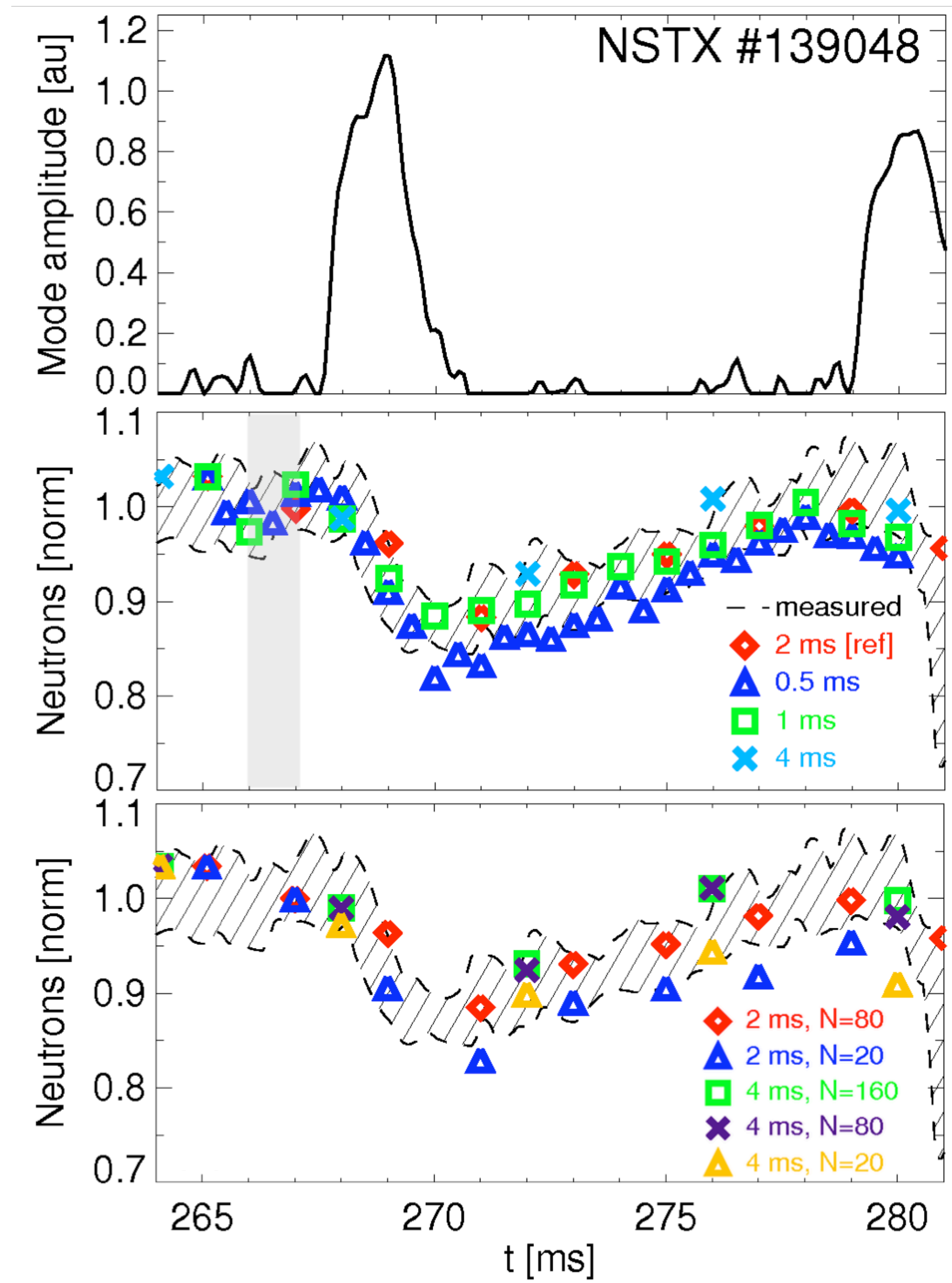


- Typical time scales:
  - Slowing down: 15-30ms
  - Collisions: 2-5ms
  - AEs: 0.1-2ms

Results are sensitive to input mode amplitude; time steps must be chosen to satisfy "statistical" approach

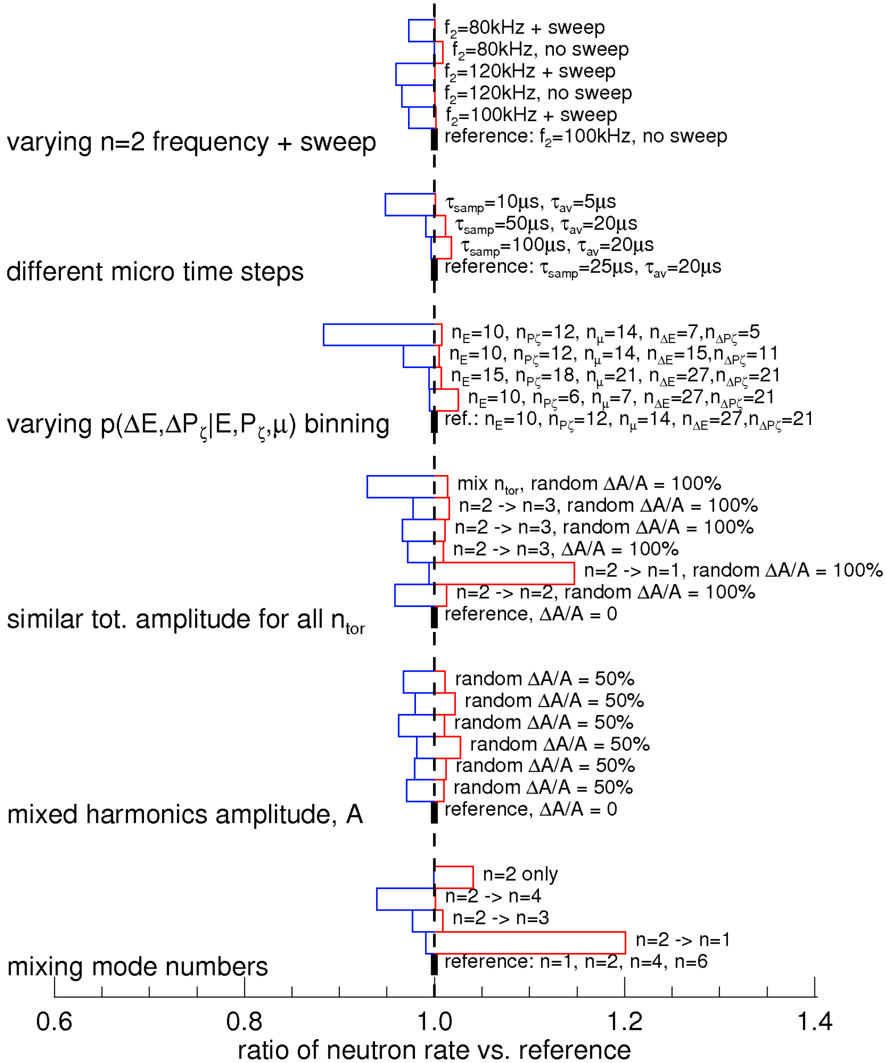


- Typical time scales:
  - Slowing down: 15-30ms
  - Collisions: 2-5ms
  - AEs: 0.1-2ms
- N: number of micro-steps for MC particle evolution, duration 25μs

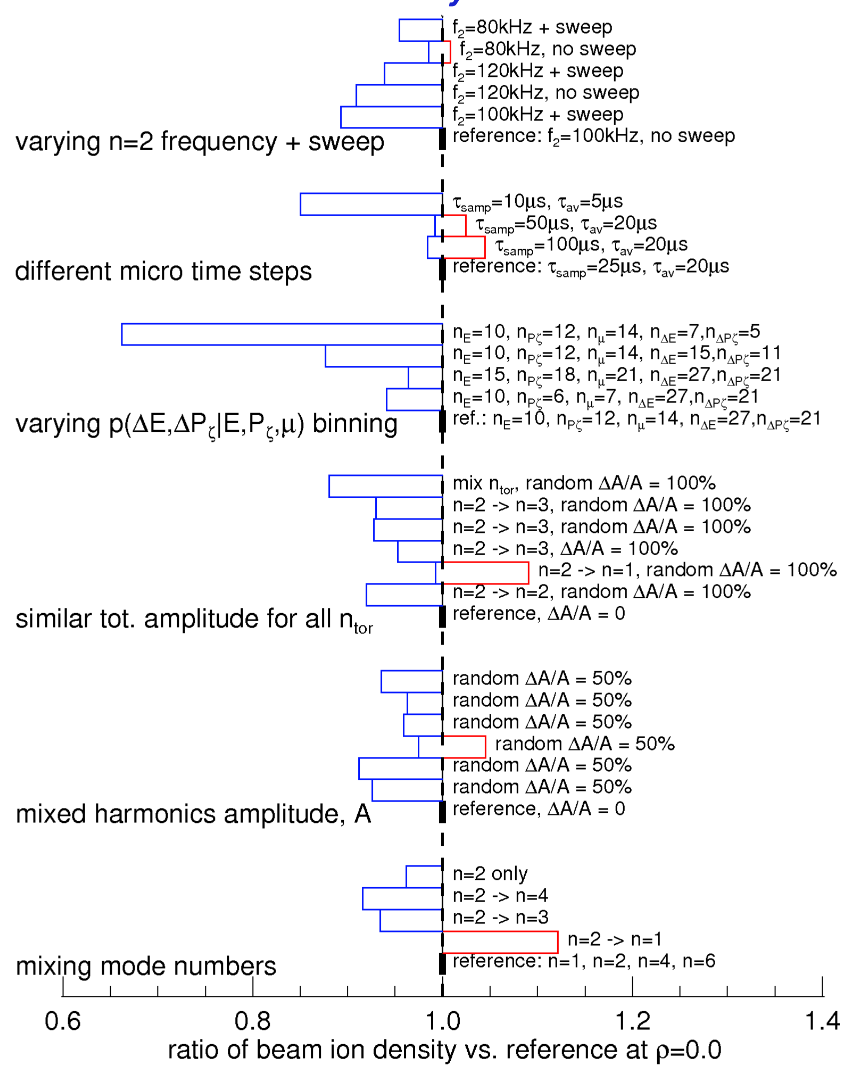


Sensitivity study suggests fast ion density is good candidate for thorough model-experiment comparison

neutron rate / reference



fast ion density / reference



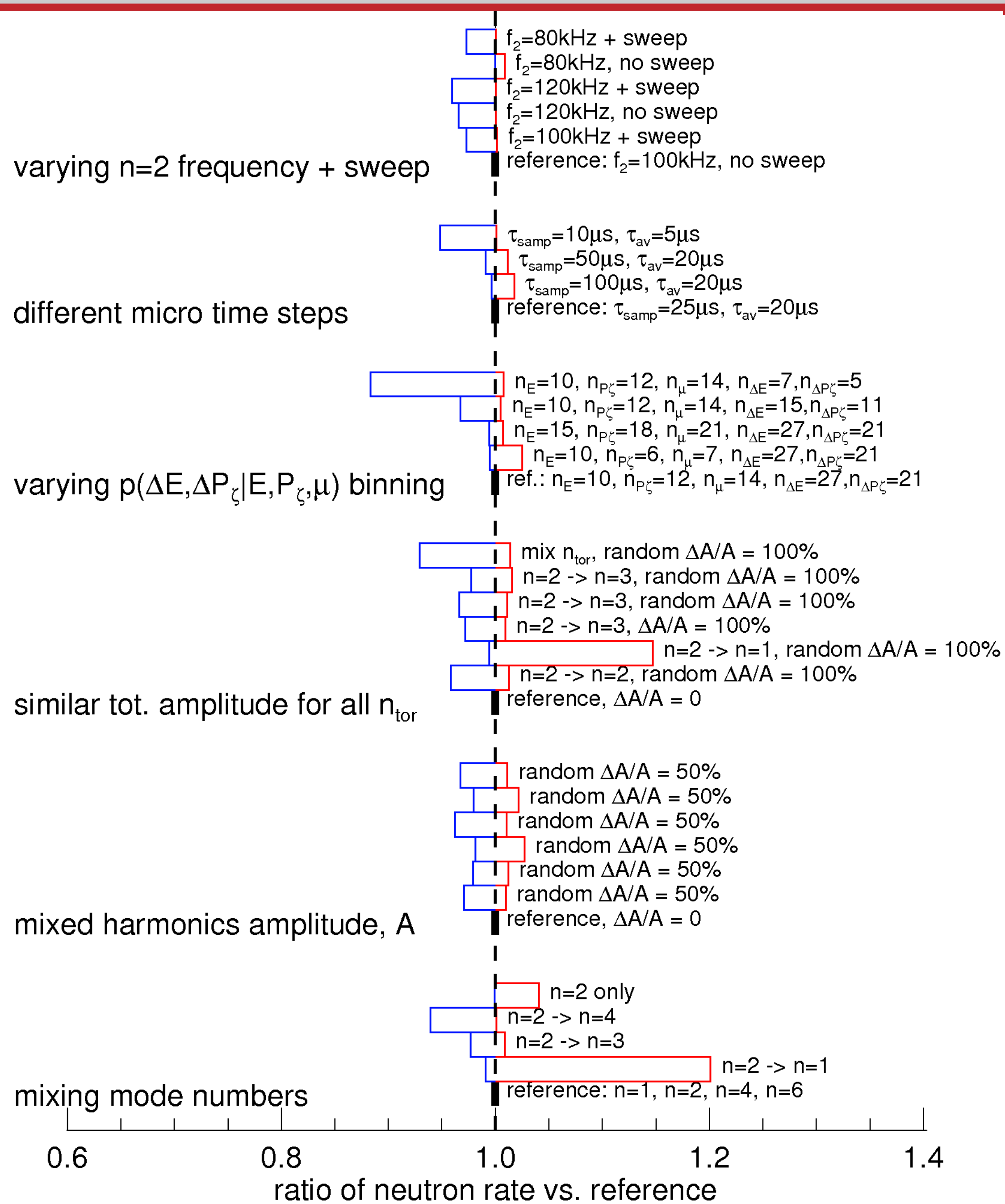
Neutrons:

- Mostly indicative of high-energy fast ions
- Limited/no information on other portions of phase space

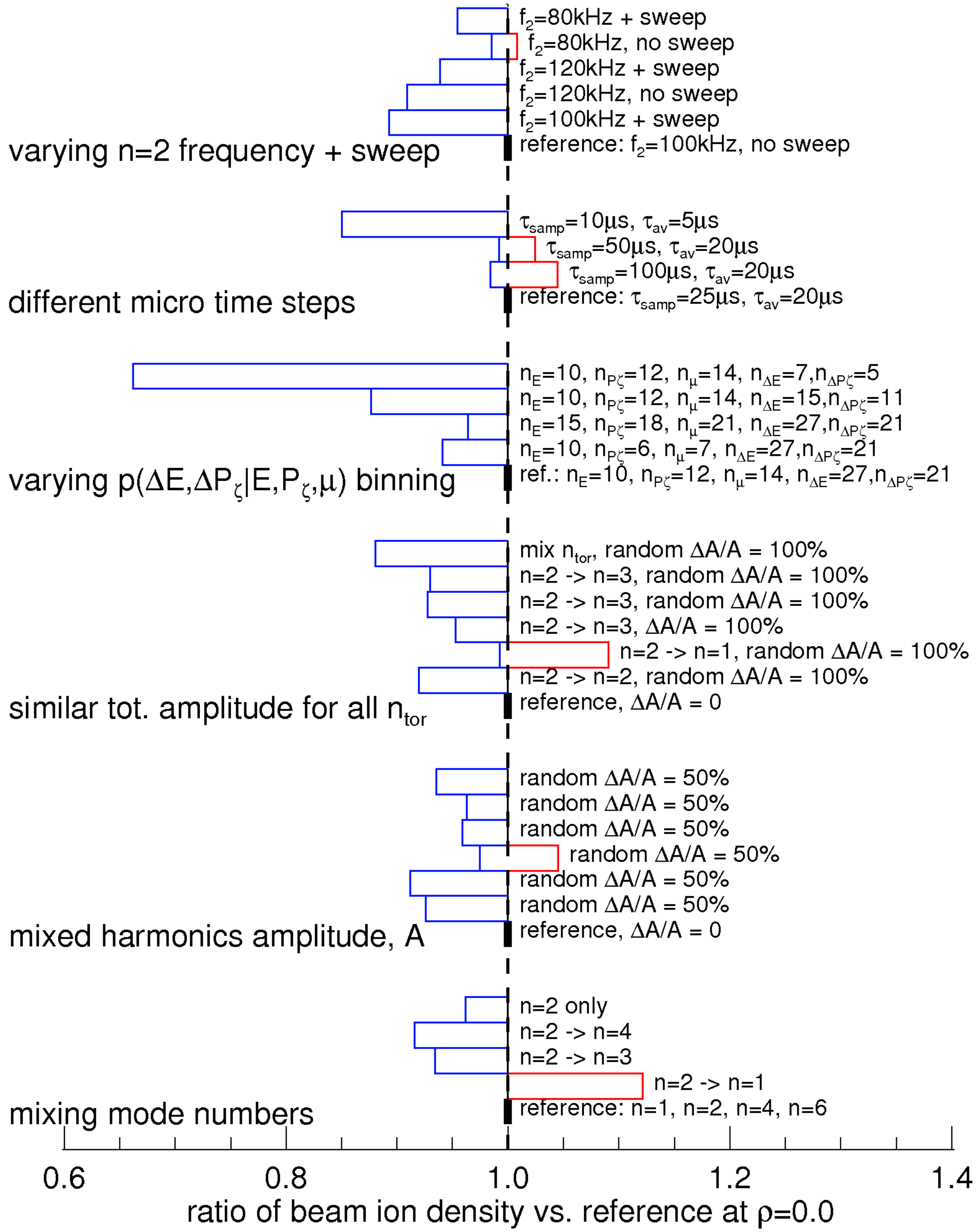
Density:

- All energies matter; more sensitive
- Accessible through FIDA, NPA diagnostics w/ spectral information

Summary of sensitivity study – neutron rate



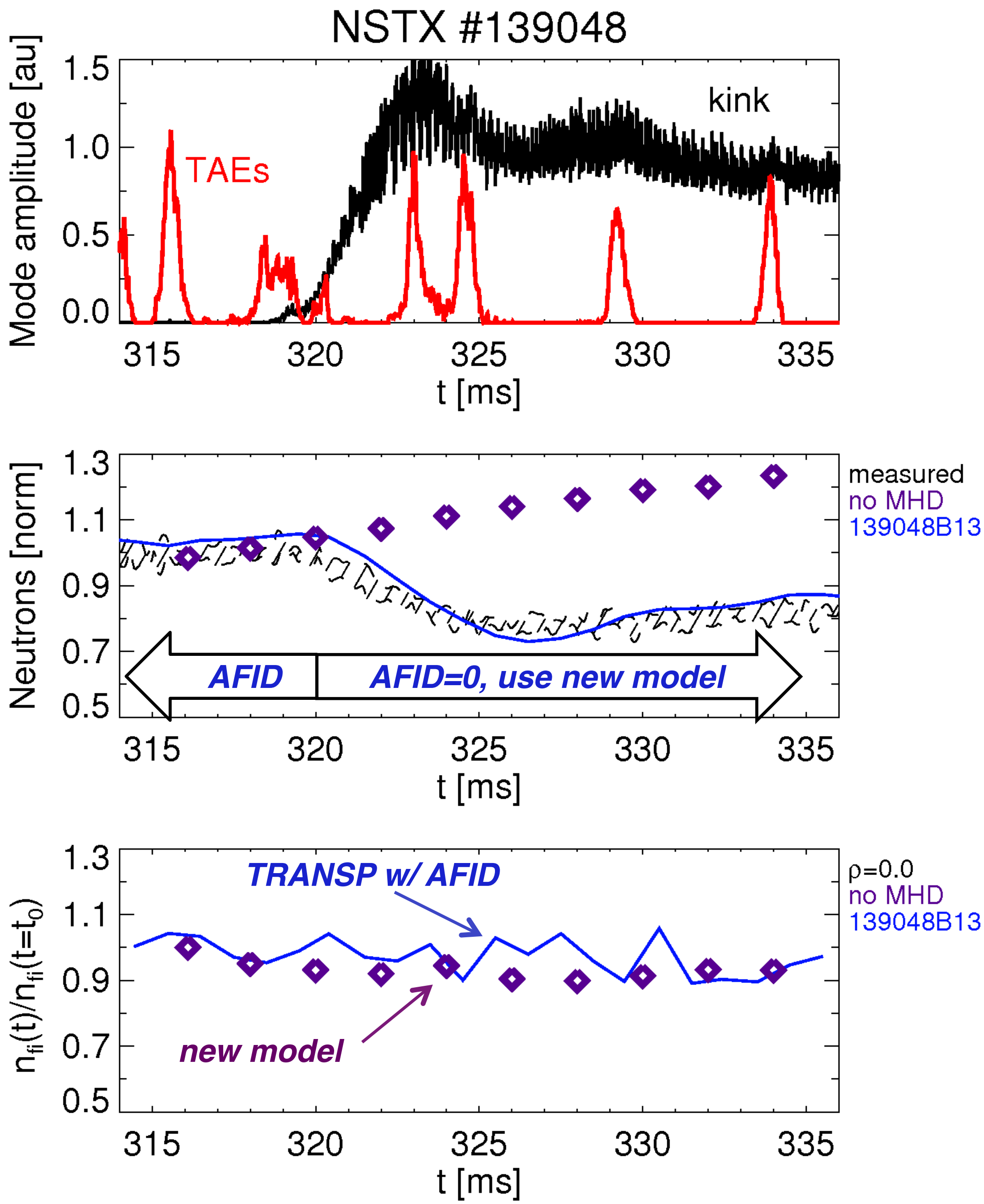
Summary of sensitivity study – beam ion density



Preliminary example: NSTX scenarios with simultaneous TAEs and kink-like modes

Baseline:

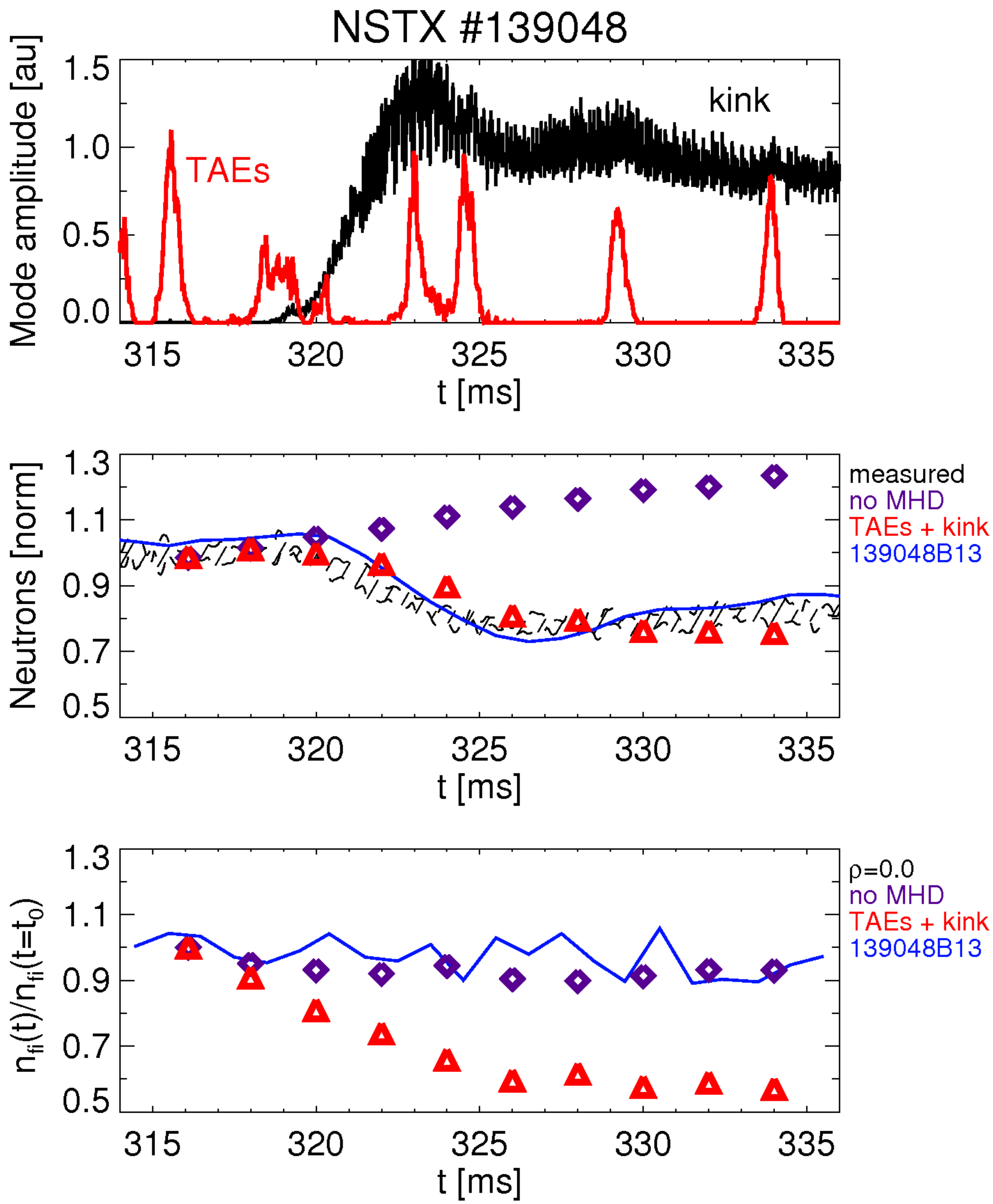
- Anomalous diffusivity (AFID) up to t=320ms to match neutrons
- Set AFID=0 after 320ms
- Keep profiles fixed



Preliminary example: NSTX scenarios with simultaneous TAEs and kink-like modes

TAEs + kink-like:

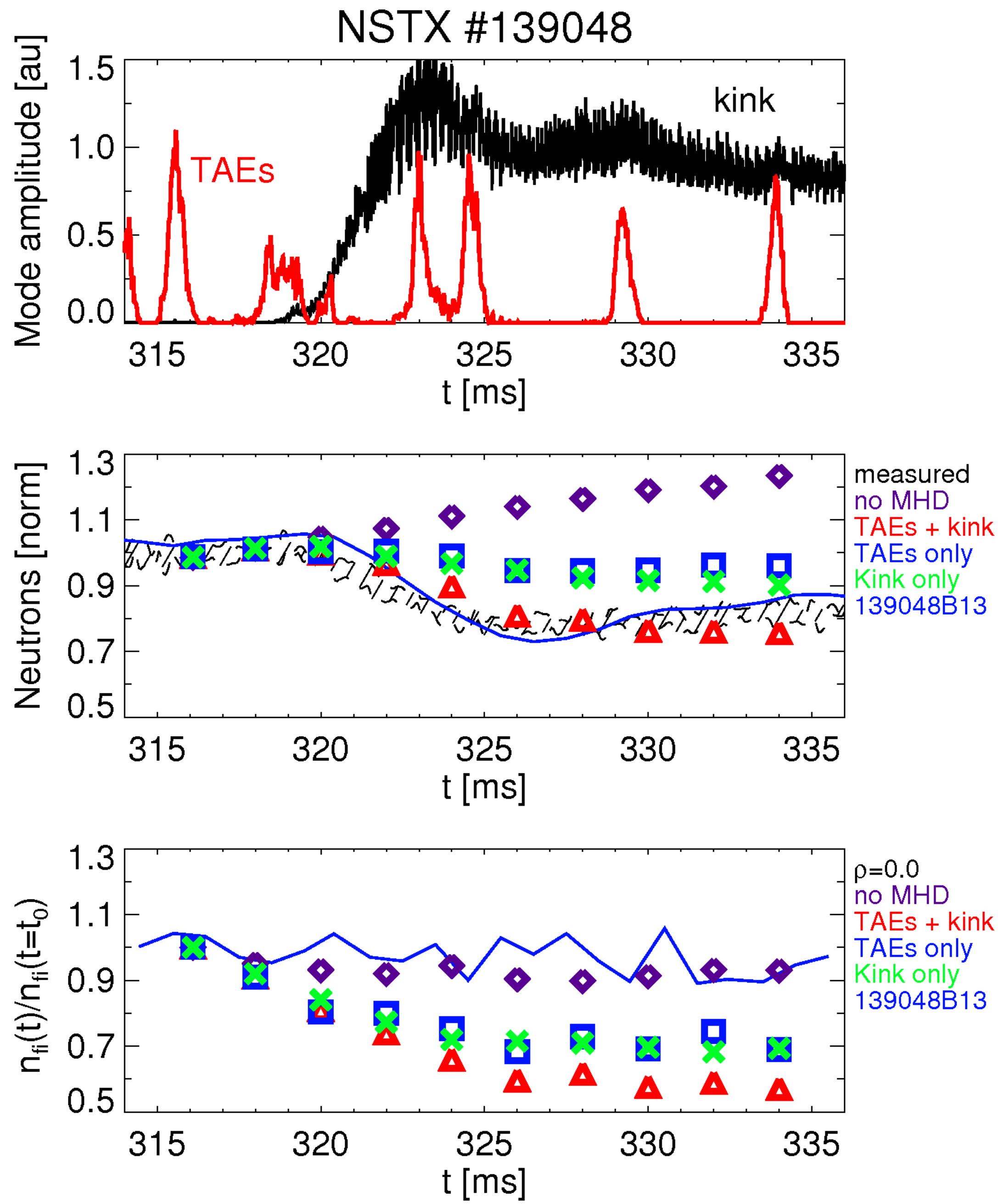
- Anomalous diffusivity (AFID) up to t=320ms to match neutrons
- Set AFID=0 after 320ms
- Keep profiles fixed
- Use previous $p_1(\Delta E, \Delta P_\zeta)$ for TAEs
- Use $p_2(\Delta E, \Delta P_\zeta)$ for kink-like modes
  - **Crude model w/ n=1,2,3**
  - *Just a test case!*
  - Analysis in progress to obtain "real" kink structure



Preliminary example: NSTX scenarios with simultaneous TAEs and kink-like modes

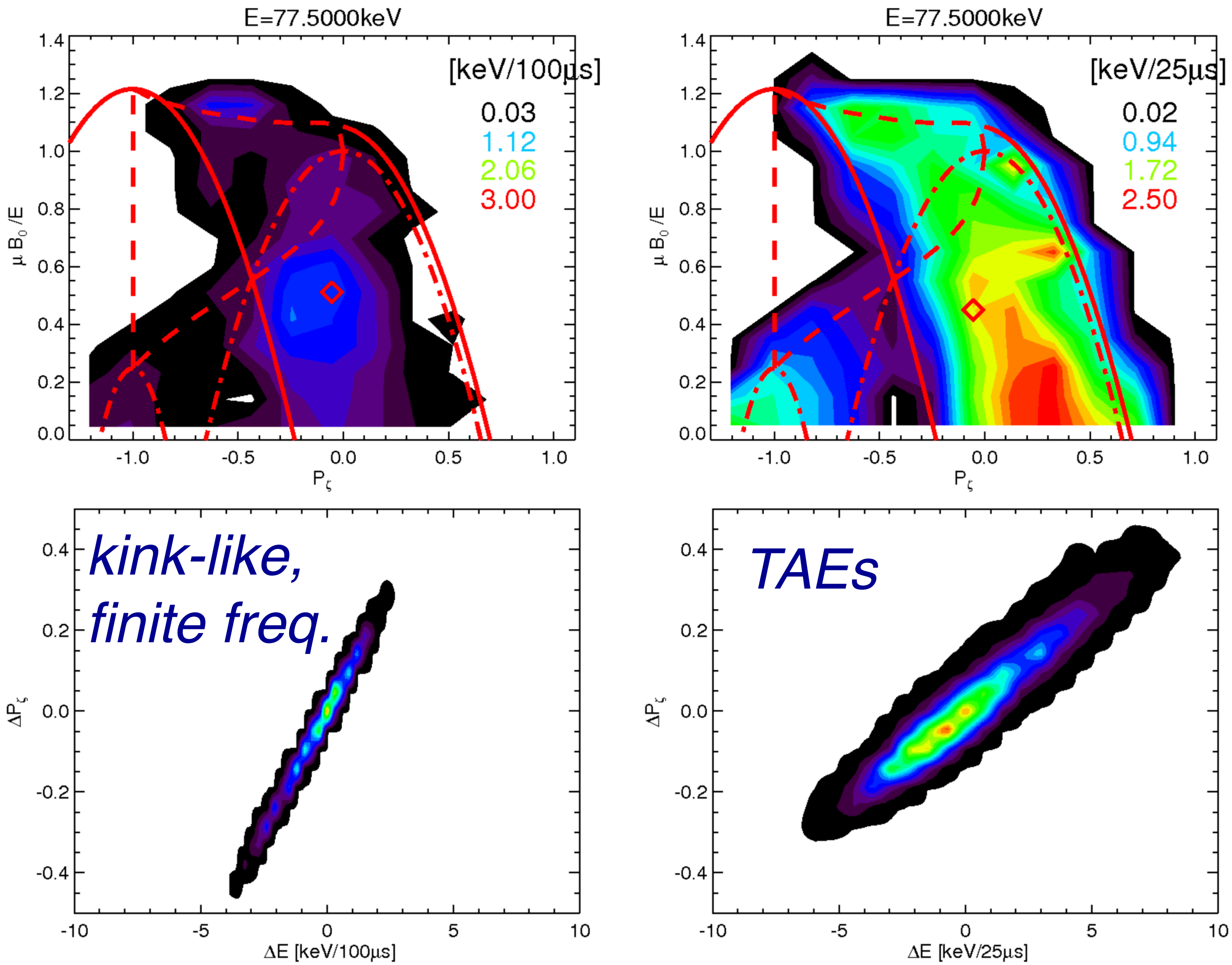
TAEs + kink-like:

- Anomalous diffusivity (AFID) up to t=320ms to match neutrons
- Set AFID=0 after 320ms
- Keep profiles fixed
- Use previous $p_1(\Delta E, \Delta P_\zeta)$ for TAEs
- Use $p_2(\Delta E, \Delta P_\zeta)$ for kink-like modes
  - **Crude model w/ n=1,2,3**
  - *<u>Just a test case!</u>*
  - Analysis in progress to obtain “real” kink structure



Preliminary example: NSTX scenarios with simultaneous TAEs and kink-like modes

– Multi-mode case can account for (some type of) synergy between different modes



Can the model be used in *predictive* mode?

- As it is, the model is OK to analyze ‘real’ discharges
  - Need mode structure to calculate $p(\Delta E, \Delta P_\zeta)$, e.g. from NOVA-K and reflectometers’ data + ORBIT
  - Need data (Mirnovs, neutrons, etc.) to infer $A_{modes}(t)$
- Two possibilities for ‘predictive’ runs:
  - Find reasonable guess for unstable modes (e.g. NOVA-K)
  - Explore different scenarios w/ scan of $A_{modes}(t)$: weak *AE activity, bursts/avalanches, etc.

  or:
  - Have an additional module to compute $A_{modes}(t)$ self-consistently?
  - Still requires mode structure, probably estimates for γ_{drive}, γ_{damp}
  - Let $A_{modes}(t)$ evolve according to F_{nb} evolution – but how?
  - *Couple to other reduced models, e.g. Critical Gradient Model?*